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REPLY TO COMMENT BY STEWART A. LEVIN AND STAN Y. C. LEE[1]

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We thank Dr Levin and Dr Lee for pointing out that the terminology "zero-offset single-fold redatumed sections" for our Fig. 17c is confusing. We fully agree and therefore this terminology is not used. In our paper we used the terminology "single-fold redatumed section", meaning: a section obtained by redatuming a single shot record. Levin and Lee are right in saying that stacking many of these sections does not necessarily imply constructive addition. We did similar kinematic exercises in the early stages of our project and we came to exactly the same conclusion.

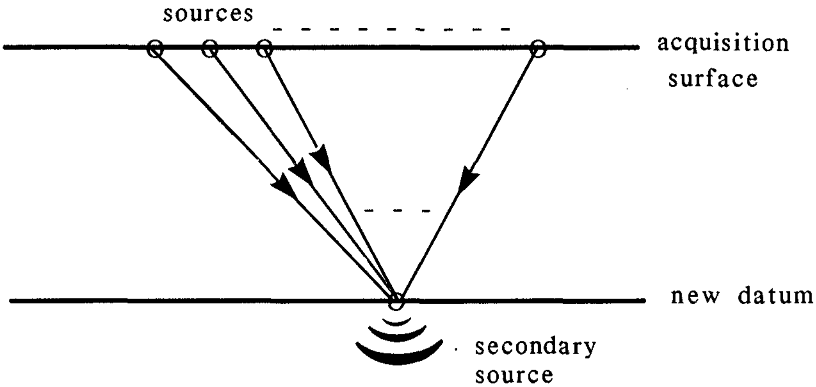

FIG. 1. By stacking single-fold redatumed sections, secondary sources are constructed at the new datum. Each secondary source has wide-angle directivity properties if it is illuminated under many angles by the true sources.

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However, independent of the interference phenomena during addition, stacking single-fold redatumed sections does give true zero-offset data. What happens during stacking is that at each point on the new datum, a secondary source is constructed with wide-angle directivity properties (Fig. 1). This means that at the new datum we not only recover true zero-offset data (Fig. 17d in the paper) but also complete shot records (true amplitude). For the mathematical proof we refer to the paper. In particular, note Fig. 15 which illustrates that our shot record oriented redatuming approach yields *identical* results to those of full prestack redatuming.

Finally, we would like to add that, in our acoustic microscopy project, a secondary point source is physically constructed by making the primary point sources part of one areal source which focusses its energy at the desired secondary source position.

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COMMENT ON "EFFICIENT 2D AND 3D SHOT RECORD REDATUMING" BY N. A. KINNEGING, V. BUDEJICKY, C. P. A. WAPENAAR AND A. J. BERKHOUT[1]

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We wish to point out a misleading impression that this recent paper makes concerning the meaning of the individual "zero-offset single-fold redatumed sections" as depicted in Fig. 17c. Both the terminology and the example make it appear that these are somewhat aperture-limited pieces of the genuine zero-offset response (Fig. 17d) and that they are summed in phase to construct the full redatumed zero-offset response underneath the survey line. Unfortunately, this is not the case. In general, these intermediate correlated time records are analogous to the uniform suite of dipping lines that superpose to form a point. Most of the addition is destructive rather than constructive.

To illustrate this, we use a kinematic approach. For a constant velocity earth, the forward extrapolated shot wavefield is an expanding spherical shell and its arrival at the datum forms a traveltime hyperbola. Correlated with a downward-continued record, the kinematic effect is to apply an offset-dependent time shift to the record. At the datum, this brings reflectors to time zero at the point that they intersect the datum. For times greater than zero, we now use ray tracing to deduce a reflector's image.

Referring to Fig. 1, let a reflector dipping at angle θ emerge at point $A = (a_x, 0)$. Select the origin $O = (0, 0)$ on the surface directly above the position where the reflector intersects the datum at depth z. Let the surface source position be at $S = (s_x, 0)$ and the emergent ray angle be denoted by γ. Let the point of reflection off the dipping reflector be $P = (p_x, p_z)$ and its arrival at the datum be $Q = (q_x, z)$. Finally, let the angle of incidence (and reflection) be α. The length we wish to compute is $\overline{SP} + \overline{PQ} - \overline{SQ}$.

From these definitions, we have

$$a_x = -z \cot \theta, \tag{1}$$

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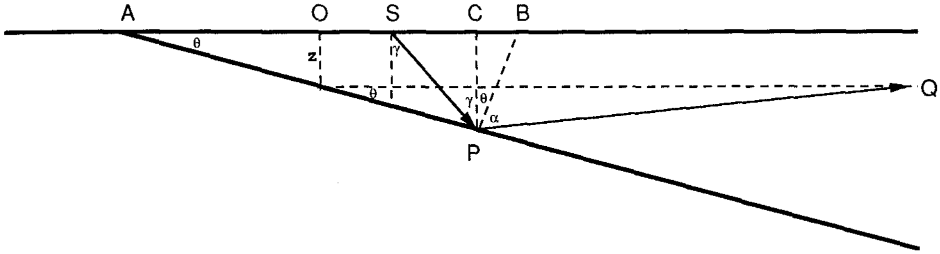

FIG. 1. Geometry for a survey over a dipping reflector with source S located on the surface and receiver Q on a datum plane at a fixed depth z. The origin (0, 0) of the coordinate system is at location O, directly above the intersection of the datum plane and the dipping reflector. Constant velocity is assumed in the wedge.

and, by similar triangles,

$$\frac{p_x - a_x}{-a_x} = \frac{p_z}{z}, \tag{2}$$

which simplifies to

$$p_z = z + p_x \tan\theta. \tag{3}$$

By definition,

$$\tan\gamma = \frac{p_x - s_x}{p_z}. \tag{4}$$

Also, by inspection

$$\alpha = \gamma + \theta, \tag{5}$$

whence

$$\frac{q_x - p_x}{p_x \tan\theta} = \tan(\gamma + 2\theta) = \frac{\tan\gamma + \tan 2\theta}{1 - \tan\gamma \tan 2\theta}. \tag{6}$$

If we substitute for $\tan\gamma$ and $\tan 2\theta$ in (6), we get

$$\begin{aligned}\frac{q_x - p_x}{p_x \tan\theta} &= \left(\frac{p_x - s_x}{z + p_x \tan\theta} + \frac{2\tan\theta}{1 - \tan^2\theta}\right) \Bigg/ \left(1 - \left(\frac{p_x - s_x}{z + p_x \tan\theta}\right)\left(\frac{2\tan\theta}{1 - \tan^2\theta}\right)\right),\\ &= \frac{(1 - \tan^2\theta)(p_x - s_x) + 2(z + p_x \tan\theta)\tan\theta}{(z + p_x \tan\theta)(1 - \tan^2\theta) - 2(p_x - s_x)\tan\theta},\\ &= \frac{2z\tan\theta - (1 - \tan^2\theta)s_x + p_x \sec^2\theta}{z(1 - \tan^2\theta) + 2s_x \tan\theta - p_x \sec^2\theta \tan\theta},\\ &= \frac{z\sin 2\theta - s_x \cos 2\theta + p_x}{z\cos 2\theta + s_x \sin 2\theta - p_x \tan\theta}.\end{aligned} \tag{7}$$

We can solve (7) for p_x in terms of q_x:

$$(q_x - p_x)(z \cos 2\theta + s_x \sin 2\theta - p_x \tan\theta) = p_x \tan\theta(z \sin 2\theta - s_x \cos 2\theta + p_x), \tag{8}$$

$$\begin{aligned} q_x(z \cos 2\theta + s_x \sin 2\theta) &= p_x z(\cos 2\theta + \tan\theta \sin 2\theta) \\ &\quad + p_x s_x(\sin 2\theta - \tan\theta \cos 2\theta) + p_x q_x \tan\theta, \\ &= p_x z + p_x s_x \tan\theta + p_x q_x \tan\theta. \end{aligned} \tag{9}$$

Hence

$$p_x = \frac{q_x(z \cos 2\theta + s_x \sin 2\theta)}{z + (q_x + s_x)\tan\theta}. \tag{10}$$

From this we also calculate

$$\begin{aligned} q_x - p_x &= q_x \frac{z + (q_x + s_x)\tan\theta - z\cos 2\theta - s_x \sin 2\theta}{z + (s_x + q_x)\tan\theta}, \\ &= q_x \frac{2z \sin^2\theta + q_x \tan\theta + s_x(\tan\theta - 2\sin\theta\cos\theta)}{z + (q_x + s_x)\tan\theta}, \\ &= q_x \tan\theta \, \frac{z \sin 2\theta + q_x - s_x \cos 2\theta}{z + (q_x + s_x)\tan\theta}. \end{aligned} \tag{11}$$

From Fig. 1 we have the relations

$$\overline{PQ} = \sqrt{(p_x \tan\theta)^2 + (q_x - p_x)^2}, \tag{12}$$

$$\overline{SP} = \sqrt{(z + p_x \tan\theta)^2 + (p_x - s_x)^2}, \tag{13}$$

and

$$\overline{SQ} = \sqrt{(q_x - s_x)^2 + z^2}. \tag{14}$$

Let us now examine how the traveltime

$$t(q_x) = (\overline{PQ} + \overline{SP} - \overline{SQ})/v \tag{15}$$

behaves in the vicinity of $q_x = 0$ where the reflector intersects the datum. In this case $p_x = 0$ and

$$\begin{aligned} t(0) &= \sqrt{z^2 + s_x^2} - \sqrt{z^2 + s_x^2}, \\ &= 0, \end{aligned} \tag{16}$$

affirming kinematically the imaging principle for shot profile migration.

To find the slope of the traveltime curve at $q_x = 0$, we take partial derivatives with respect to q_x and evaluate them at $q_x = 0$. Starting with $\overline{PQ}$, we substitute (10) and (11) into (12) to get

$$\overline{PQ} = \frac{q_x \tan\theta}{z + (s_x + q_x)\tan\theta}\,[z^2\cos^2 2\theta + 2zs_x \sin 2\theta\cos 2\theta + s_x^2\sin^2 2\theta$$
$$+ z^2\sin^2 2\theta + 2zq_x\sin 2\theta + q_x^2 - 2zs_x\cos 2\theta\sin 2\theta$$
$$-2s_x q_x\cos 2\theta + s_x^2\cos^2 2\theta]^{1/2},$$
$$= \frac{q_x\tan\theta}{z + (s_x + q_x)\tan\theta}\,[z^2 + 2q_x(z\sin 2\theta - s_x\cos 2\theta) + q_x^2 + s_x^2]^{1/2}, \tag{17}$$

whence

$$\left.\frac{\partial\overline{PQ}}{\partial q_x}\right|_{q_x=0} = \frac{\sqrt{z^2 + s_x^2}\tan\theta}{z + s_x\tan\theta}. \tag{18}$$

For $\overline{SP}$, we have

$$\frac{\partial\overline{SP}}{\partial q_x} = \frac{(z + p_x\tan\theta)\tan\theta + (p_x - s_x)}{\sqrt{(z + p_x\tan\theta)^2 + (p_x - s_x)^2}}\,\frac{\partial p_x}{\partial q_x}. \tag{19}$$

From (10) we find

$$\left.\frac{\partial p_x}{\partial q_x}\right|_{q_x=0} = \frac{z\cos 2\theta + s_x\sin 2\theta}{z + s_x\tan\theta}, \tag{20}$$

whence

$$\left.\frac{\partial\overline{SP}}{\partial q_x}\right|_{q_x=0} = \frac{(z\tan\theta - s_x)(z\cos 2\theta + s_x\sin 2\theta)}{(z + s_x\tan\theta)\sqrt{z^2 + s_x^2}}. \tag{21}$$

Finally,

$$\left.\frac{\partial\overline{SQ}}{\partial q_x}\right|_{q_x=0} = -\frac{s_x}{\sqrt{z^2 + s_x^2}}. \tag{22}$$

Combining, we have

$$\left.\frac{\partial t(q_x)}{\partial q_x}\right|_{q_x=0} = \frac{1}{v}\left(\frac{\sqrt{z^2 + s_x^2}\tan\theta}{z + s_x\tan\theta} + \frac{(z\tan\theta - s_x)(z\cos 2\theta + s_x\sin 2\theta)}{(z + s_x\tan\theta)\sqrt{z^2 + s_x^2}}\right.$$
$$\left. + \frac{s_x}{\sqrt{z^2 + s_x^2}}\right),$$
$$= \frac{1}{v}\left(\frac{2z^2\tan\theta\cos^2\theta + 4s_x z\sin^2\theta + 2s_x^2\tan\theta\sin^2\theta}{(z + s_x\tan\theta)\sqrt{z^2 + s_x^2}}\right),$$
$$= \frac{1}{v}\left(\frac{\sin 2\theta(z + s_x\tan\theta)^2}{(z + s_x\tan\theta)\sqrt{z^2 + s_x^2}}\right),$$
$$= \frac{(z + s_x\tan\theta)\sin 2\theta}{v\sqrt{z^2 + s_x^2}}. \tag{23}$$

In the 'zero-offset' case $s_x = 0$, (23) becomes

$$\left.\frac{\partial t(q_x)}{\partial q_x}\right|_{q_x=0,\, s_x=0} = \frac{\sin 2\theta}{v}.$$

On the other hand, a zero-offset survey over the reflector at the datum would have time slope $2 \sin \theta/v$. From this we conclude that our assumption, that the redatumed image is a zero-offset time section, is at best approximate.

Looking now at the 'far-offset' limit $s_x \to \infty$, (23) becomes

$$\left.\frac{\partial t(q_x)}{\partial q_x}\right|_{q_x=0,\, s_x\to\infty} = \frac{2 \sin^2 \theta}{v},$$

at which point the slope of the reflector's image is nearly flat for small θ. A similar, though much easier, calculation applies to flat reflectors directly below the datum. This leads us to conclude that the superposition of the various images of the reflector must be generally destructive in order to obtain the true zero-offset result predicted by theory. In our experience this causes significant problems when applying the theory to field data which are necessarily finite aperture recordings. Without sufficiently dense coverage of both positive, negative and near-zero offsets, the image formed by this process becomes significantly skewed away from true zero-offset recording towards images that are too flat.