Deblending by direct inversion

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ABSTRACT

Deblending of simultaneous-source data is usually considered to be an underdetermined inverse problem, which can be solved by an iterative procedure, assuming additional constraints like sparsity and coherency. By exploiting the fact that seismic data are spatially band-limited, deblending of densely sampled sources can be carried out as a direct inversion process without imposing these constraints. We applied the method with numerically modeled data and it suppressed the crosstalk well, when the blended data consisted of responses to adjacent, densely sampled sources.

INTRODUCTION

The simultaneous-source method involves the recording and processing of responses to sources that are ignited with relatively short time intervals (i.e., shorter than the time it takes to record the reflections from the deepest reflectors of interest). After the pioneering work by Garotta (1983), Womack et al. (1990) and Beasley et al. (1998), research of this method has gained real momentum in the past five years. Several approaches have been developed to deal with the crosstalk that occurs when processing simultaneous-source data. These methods involve the use of phase-encoded sources (Bagaini, 2006; Ikelle, 2007; Neelamani et al., 2010), randomization of the time interval between the sources (Akerberg et al., 2008; Hampson et al., 2008), prediction and subtraction of the crosstalk (Spitz et al., 2008; Mahdad et al., 2011), or unraveling the simultaneous-source data using sparseness constraints (Berkhout, 2008; Abma et al., 2010, Mansour et al., 2011).

Simultaneous-source acquisition is also known as blended acquisition; hence, the process of unraveling the data is also called deblending (Berkhout, 2008). In the situation of deterministic transient sources, deblending of simultaneous-source data into single-source responses seems to be an underdetermined problem.

Therefore the methods mentioned above use additional constraints, like sparseness and coherency, which implies that those methods are iterative. Our aim is to show that sparsity or coherency constraints are not necessary if one utilizes the fact that seismic data are spatially band-limited, and if the sources are densely sampled. With intuitive arguments and a numerical example, we show that by taking the spatial band-limitation into account, deblending of densely sampled sources can be implemented as a direct (i.e., noniterative) inversion of the blending operator.

THE BLENDING MATRIX

We define the unblended data as $P(\mathbf{x}_R^{(k)}, \mathbf{x}_S^{(i)}, t)$, where $\mathbf{x}_S^{(i)}$ denotes the ith source position, $\mathbf{x}_R^{(k)}$ the kth receiver position, and t denotes time. For simultaneous-source acquisition we define source groups $\boldsymbol{\sigma}^{(m)}$, each containing a subset of the source positions $\mathbf{x}_S^{(i)}$. The sources within each group are ignited with relatively short delay times t_i. For source-group $\boldsymbol{\sigma}^{(m)}$, the simultaneous-source response is thus given by

$$P_{\text{sim}}(\mathbf{x}_R^{(k)}, \boldsymbol{\sigma}^{(m)}, t) = \sum_{\mathbf{x}_S^{(i)} \in \boldsymbol{\sigma}^{(m)}} P(\mathbf{x}_R^{(k)}, \mathbf{x}_S^{(i)}, t - t_i), \quad (1)$$

where $\mathbf{x}_S^{(i)} \in \boldsymbol{\sigma}^{(m)}$ denotes that the summation takes place over all source positions $\mathbf{x}_S^{(i)}$ in group $\boldsymbol{\sigma}^{(m)}$. After a temporal Fourier transformation, equation 1 becomes

$$P_{\text{sim}}(\mathbf{x}_R^{(k)}, \boldsymbol{\sigma}^{(m)}, \omega) = \sum_{\mathbf{x}_S^{(i)} \in \boldsymbol{\sigma}^{(m)}} P(\mathbf{x}_R^{(k)}, \mathbf{x}_S^{(i)}, \omega) \exp(-j\omega t_i), \quad (2)$$

where ω denotes the angular frequency, and $j = \sqrt{-1}$. Using Berkhout's matrix notation, we can write for each frequency component,

$$\mathsf{P}_{\text{sim}} = \mathsf{PB}. \quad (3)$$

The element at the kth row and ith column of matrix P contains the unblended response $P(\mathbf{x}_R^{(k)}, \mathbf{x}_S^{(i)}, \omega)$. When there are K receivers and

N sources, P is a $K \times N$ matrix; B is the blending matrix. As a point of clarification, we note that Berkhout (2008) uses the symbol Γ to denote the blending matrix; however, to avoid confusion with the point-spread matrix Γ introduced in equation 9 (which is also used in our interferometry papers) we denote the blending matrix by B. When each source group contains n sources, B is a $N \times (N/n)$ matrix. For example, for source groups of two adjacent sources (i.e., $n = 2$), matrix B is defined as:

$$\mathsf{B} = \begin{pmatrix} b_1 & 0 & \cdots & 0 \\ b_2 & 0 & \cdots & 0 \\ 0 & b_3 & \cdots & 0 \\ 0 & b_4 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & b_{N-1} \\ 0 & 0 & \cdots & b_N \end{pmatrix}, \qquad (4)$$

where $b_i = \exp(-j\omega t_i)$. The matrix product PB in equation 3 yields the $K \times (N/n)$ matrix $\mathsf{P}_{\mathrm{sim}}$, its elements containing the blended response $P_{\mathrm{sim}}(\mathbf{x}_R^{(k)}, \boldsymbol{\sigma}^{(m)}, \omega)$.

DEBLENDING BY BAND-LIMITED LEAST-SQUARES INVERSION

Deblending involves solving equation 3 for P. Formally, deblending is formulated as

$$\hat{\mathsf{P}} = \mathsf{P}_{\mathrm{sim}} \hat{\mathsf{B}}_{\mathrm{inv}}, \qquad (5)$$

where $\hat{\mathsf{B}}_{\mathrm{inv}}$ is in some sense the inverse of the blending matrix B. The number of elements in P is n times as large as in $\mathsf{P}_{\mathrm{sim}}$. If we were to follow the least-squares approach for underdetermined systems (Menke, 1989), $\hat{\mathsf{B}}_{\mathrm{inv}}$ would be defined as

$$\hat{\mathsf{B}}_{\mathrm{inv}} = (\mathsf{B}^\dagger \mathsf{B})^{-1} \mathsf{B}^\dagger, \qquad (6)$$

where the dagger (†) denotes transposition and complex conjugation. Mahdad et al. (2011) observe that, if the blending matrix B only contains phase terms, the least-squares inverse corresponds to the transpose complex conjugate $\mathsf{B}^\dagger$. They call this the pseudo-deblending operator. We may conclude that straightforward least-squares inversion of blending equation 3 does not give a satisfactory solution. It aims to find an inverse for the blending matrix B; hence, it aims to unravel blended point sources into independent point sources. The question arises whether we really need to retrieve point sources: The answer is no. Because the response to a point source is, in the far-field, spatially band-limited, it suffices to retrieve spatially band-limited point sources.

In the wavenumber-frequency domain, the spatial bandwidth of the far-field response is limited by plus and minus $|\omega|/c_a$, with $c_a = c/\sin \alpha_{\max}$, where c is the propagation velocity and $\alpha_{\max}$ is the maximum propagation angle in the upper layer. The corresponding filter in the space-frequency domain is given by the following sinc function:

$$\gamma(x_1, \omega) = \sin(|\omega| x_1/c_a)/(\pi x_1). \qquad (7)$$

Here we consider the 2D situation; in 3D we would have a Bessel function divided by its argument. Analogous to the terminology in seismic interferometry, we call $\gamma(x_1, \omega)$ the basic point-spread function. The response to a point source $\delta(x_1 - x_{1,S})$, spatially convolved with $\gamma(x_1, \omega)$, is in the far-field indistinguishable from the point-source response. Hence, instead of aiming to unravel the blended point sources into independent point sources, it suffices to unravel the blended point-source responses into independent responses to smeared point sources, characterized by the point-spread function $\gamma(x_1 - x_{1,S}^{(i)}, \omega)$. Because seismic data are always spatially band-limited, exploiting the band-limitation is essentially different from imposing additional coherency or sparsity constraints.

The band-limitation helps us to solve the seemingly underdetermined problem of equation 3, i.e., to resolve the $K \times N$ matrix P from the $K \times (N/n)$ matrix $\mathsf{P}_{\mathrm{sim}}$. This can be intuitively understood as follows: Assuming the blended source groups are formed of n adjacent sources, then the source-group interval equals $n\Delta_s$, where Δ_s is the unblended source interval. The source groups are sampled unaliased when

$$\frac{|\omega|}{c_a} < \frac{\pi}{n\Delta_s}. \qquad (8)$$

Unaliased sampling allows interpolation. Hence, when equation 8 is fulfilled, interpolation between the source groups is possible, meaning that the $K \times (N/n)$ matrix $\mathsf{P}_{\mathrm{sim}}$ could be interpolated to form a $K \times N$ matrix, from which the $K \times N$ matrix P could subsequently be resolved. This explains why, for spatially band-limited data, equation 3 is not an underdetermined problem. In our implementation, however, we do not first interpolate $\mathsf{P}_{\mathrm{sim}}$ and then solve for P, but we construct a band-limited version of the deblending matrix $\hat{\mathsf{B}}_{\mathrm{inv}}$ and apply this directly to the blended data matrix $\mathsf{P}_{\mathrm{sim}}$, according to equation 5.

We define the band-limited deblending matrix $\hat{\mathsf{B}}_{\mathrm{inv}}$ by inserting the point-spread function $\gamma(x_1, \omega)$ in the definition of the deblending matrix (equation 6), as follows:

$$\hat{\mathsf{B}}_{\mathrm{inv}} = (\mathsf{B}^\dagger \Gamma \mathsf{B})^{-1} \mathsf{B}^\dagger \Gamma, \qquad (9)$$

where Γ contains the discretized version of the point-spread function, according to

$$\Gamma = \begin{pmatrix} \gamma_0 & \gamma_1 & \gamma_2 & \cdots & & \\ \gamma_{-1} & \gamma_0 & \gamma_1 & \gamma_2 & \cdots & \\ \gamma_{-2} & \gamma_{-1} & \gamma_0 & \gamma_1 & \gamma_2 & \cdots \\ & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix}, \qquad (10)$$

with $\gamma_p = \gamma(p\Delta_s, \omega)$. The specific form of the band-limited deblending operator in equation 9 is motivated by the analogy between seismic interferometry and the simultaneous-source method (Wapenaar et al., 2012). We call $N \times N$ matrix Γ the basic point-spread matrix. We introduce a $(N/n) \times (N/n)$ point-spread matrix Γ_{sim} for simultaneous-source acquisition as

$$\Gamma_{\mathrm{sim}} = \mathsf{B}^\dagger \Gamma \mathsf{B}. \qquad (11)$$

With this definition we rewrite equation 9 as

$$\hat{\mathsf{B}}_{\mathrm{inv}} = \Gamma_{\mathrm{sim}}^{-1} \mathsf{B}^\dagger \Gamma. \qquad (12)$$

As an example, we analyze matrix Γ_{sim} where the blending matrix B creates identical source groups of two adjacent sources with a

constant time interval. Hence, B is again defined by equation 4, but with $b_1 = b_3 = \cdots = b_{N-1} = \exp(-j\omega t_1)$ and $b_2 = b_4 = \cdots = b_N = \exp(-j\omega t_2)$. Upon substitution of equations 4 and 10 into equation 11, we obtain

$$\Gamma_{\text{sim}} = 2 \begin{pmatrix} \gamma_0 & \gamma_2 & \gamma_4 & \cdots & \\ \gamma_{-2} & \gamma_0 & \gamma_2 & \gamma_4 & \cdots \\ \gamma_{-4} & \gamma_{-2} & \gamma_0 & \gamma_2 & \gamma_4 & \cdots \\ & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix}$$

$$+ \exp(-j\omega\Delta t) \begin{pmatrix} \gamma_1 & \gamma_3 & \gamma_5 & \cdots & \\ \gamma_{-1} & \gamma_1 & \gamma_3 & \gamma_5 & \cdots \\ \gamma_{-3} & \gamma_{-1} & \gamma_1 & \gamma_3 & \gamma_5 & \cdots \\ & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix}$$

$$+ \exp(+j\omega\Delta t) \begin{pmatrix} \gamma_{-1} & \gamma_1 & \gamma_3 & \cdots & \\ \gamma_{-3} & \gamma_{-1} & \gamma_1 & \gamma_3 & \cdots \\ \gamma_{-5} & \gamma_{-3} & \gamma_{-1} & \gamma_1 & \gamma_3 & \cdots \\ & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix}, \quad (13)$$

with $\Delta t = t_2 - t_1$. The first term on the right-hand side is a re-sampled version of the basic point-spread matrix Γ of equation 10 (sampling interval $2\Delta_s$). The second and third terms are also re-sampled point-spread matrices, but shifted over a distance $\pm\Delta_s$. Moreover, the factors $\exp(\mp j\omega\Delta t)$ account for a temporal shift $\pm\Delta t$ of the basic point-spread function in the time domain. The second and third terms in equation 13 account for the crosstalk of the simultaneous-source method. For this analysis of Γ_{sim}, we considered source groups of two adjacent sources. For source groups of n adjacent sources we would obtain an expression similar to equation 13, but with $2n - 1$ shifted, and resampled basic point-spread matrices on the right-hand side, each sampled with an interval $n\Delta_s$. Hence, as long as equation 8 is fulfilled, Γ_{sim} is sampled unaliased.

Note that the insertion of Γ between $B^\dagger$ and B in equation 11 has entirely changed the character of this matrix product. Observe that $B^\dagger B$ in equation 6 is nothing but n-times an identity matrix, which plays no role in the deblending process. On the contrary, $\Gamma_{\text{sim}} = B^\dagger\Gamma B$ contains shifted versions of the basic point-spread function, which account for the crosstalk of the simultaneous-source method. Hence, the inversion of Γ_{sim} in equation 12 suppresses the effects of crosstalk.

NUMERICAL EXAMPLE

We illustrate the method with a numerical example of irregularly blended data. Figure 1a shows a subsurface configuration with a laterally and vertically varying propagation velocity. The lowest velocity (which occurs in the upper layer) is $c = 2000$ m/s. We model the responses to 384 sources at the surface, with a source spacing Δ_s of 5 m. The source function is a Ricker wavelet with a central frequency of 23 Hz. The reflection responses are registered at the surface by 128 receivers with a receiver spacing of 15 m. We blend the data by forming 48 source groups of eight adjacent sources (hence, $n = 8$). The ignition times within each source group are chosen randomly from a uniform distribution between 0 and 2 s. Figure 1b shows the blended response to a source group in the middle of the acquisition surface.

With the chosen parameters, the Nyquist wavenumber related to the blended source-group sampling is $\pi/n\Delta_s \approx 0.08$ m^{-1}. The maximum propagation angle in the upper layer is 78°; hence, $c_a = 2045$ m/s. At the central frequency f_c, we have $\omega_c/c_a = 2\pi f_c/c_a \approx 0.07$ m^{-1}, which is smaller than the Nyquist wavenumber. Hence, for the central frequency equation 8 is fulfilled. At the maximum frequency (60 Hz), we have $\omega_{\text{max}}/c_a \approx 0.18$ m^{-1}; hence, equation 8 is not fulfilled. We will see that, despite the violation of the source-group sampling criterion for the higher frequencies, the deblending algorithm performs remarkably well.

Now we construct the spatially band-limited point-spread function needed to deblend the data of Figure 1b. Figure 2a shows the passband in the wavenumber-frequency domain, and Figure 2b shows the corresponding filter $\gamma(x_1, \omega)$ (i.e., the basic point-spread function) in the space-frequency domain (equation 7). For convenience we took $c_a = c = 2000$ m/s; hence, the filter passes all propagating waves and suppresses the evanescent waves. This filter is stored in the basic point-spread matrix Γ according to equation 10 (one matrix per frequency component). Next, using equation 11, the point-spread matrix Γ_{sim} for simultaneous-source acquisition is constructed. This matrix is shown in Figure 2c for the central frequency
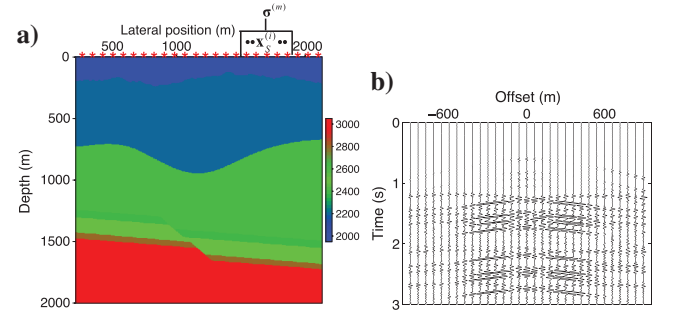

Figure 1. (a) Inhomogeneous subsurface model. (b) Simultaneous-source response to a group of eight adjacent sources in the middle of the acquisition surface (only every fourth trace is shown).
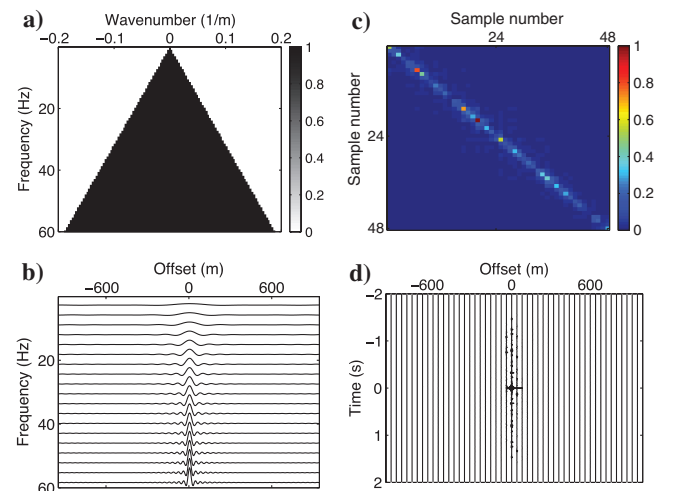

Figure 2. Construction of the point-spread function. (a) Pass band of the basic point-spread function in the wavenumber-frequency domain. (b) Basic point-spread function $\gamma(x_1, \omega)$ in the space-frequency domain. (c) Point-spread matrix Γ_{sim} at 23 Hz. (d) Same point-spread function, in space-time domain.
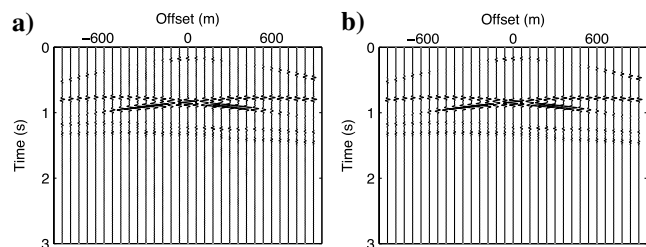
Figure 3. (a) Deblended data for the central source position at the acquisition surface in Figure 1a (only every fourth trace is shown). (b) For comparison, directly modeled response of the central source.

of 23 Hz. Figure 2d shows another cross section of the point-spread function. It is obtained by taking the central column of Γ_{sim} for all frequency components and applying an inverse Fourier transform from the frequency domain to the time domain. The event around $t = 0$ and zero offset is the basic point-spread function $\gamma(x_1, t)$ (i.e., the inverse Fourier transform of equation 7), sampled with $n\Delta_s = 40$ m. The dispersed events between -2 and $+2$ s account for the crosstalk and are essential in the deblending process. Note that these dispersed events would be absent if the filter in Figure 2a would be an all-pass filter.

Next we add a small frequency-independent stabilization parameter to the diagonal of the point-spread matrix Γ_{sim}. This parameter is 10^{-4} times the maximum of this matrix over all frequencies. The stabilized matrix is inverted and the deblending matrix $\hat{B}_{\text{inv}}$ is formed for each frequency component, using equation 12. This matrix (for each frequency component) is applied to the blended data matrix P_{sim}, giving, according to equation 5, the deblended data $\hat{P}$. Taking the central column of $\hat{P}$ for all frequency components and applying an inverse temporal Fourier transform gives the deblended data for the central source position in the space-time domain (see Figure 3a). This result accurately resembles the directly modeled response of the central source, shown in Figure 3b. Some noise remains, but this is negligible compared with the blending noise in Figure 1b.

CONCLUSIONS

Solving the deblending problem by standard least-squares inversion is equivalent to pseudodeblending: it unfolds the blended data and moves events to their correct position in space and time, but it does not remove the blending noise (i.e., the crosstalk). We have inserted a filter in the least-squares inversion algorithm, which honors the spatial band-limitation of the seismic response. This filter transforms the pseudodeblending operator into a true deblending operator, in the sense that it also suppresses the crosstalk. Unlike iterative deblending methods discussed in the literature, our method is implemented as a direct matrix inversion and does not make assumptions about coherency or sparsity of the blending noise. Instead, our method puts restrictions on the source sampling. With a numerical example, we have shown that the method suppresses the crosstalk well when the blended data consist of responses to adjacent densely sampled sources. Because the method requires densely sampled sources, it can be applied in situations where the simultaneous-source method is used to improve quality (rather than to reduce acquisition time) by inserting sources between the regular source positions. The method breaks down when the point-spread matrix looses its band structure, which is, for instance, the case when the sources in each group are randomly distributed along the acquisition surface. The blending conditions and the regularization of the inversion of the point-spread matrix need further investigation. For practical situations, the method will probably benefit from imposing additional constraints to the matrix inversion.

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