Virtual reflector representation theorem (acoustic medium)

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Abstract: The virtual reflector method simulates new seismic signals by processing traces recorded by a plurality of sources and receivers. The approach is based on the crossconvolution of the recorded signals and makes it possible to obtain the Green's function of virtual reflected signals as if in the position of the receivers (or sources) there were a reflector, even if said reflector is not present. This letter presents the virtual reflector theory based on the Kirchhoff integral representation theorem for wave propagation in an acoustic medium with and without boundary and a generalization to variable reflection coefficients for scattered wavefields.

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1. Introduction

In recent years innovative techniques have been introduced in seismics and acoustics to simulate new signals by processing the measurements obtained by a plurality of sources and receivers. These methods are generally classified as "Green's function retrieval" or "seismic interferometry" (SI) and are essentially based on crosscorrelation of traces at different receivers and summation of the crosscorrelated signals over the space of the sources generating the wavefields. In this process, it is not necessary to know the sources and the medium. From a physical point of view, crosscorrelation (or convolution by time reversal) is equivalent to removing the common propagation effects.[1,2] A great advantage of this approach is that also signals from unknown random-phase sources are shaped into interpretable signals because of phase subtraction. In some applications, deconvolution is used instead of correlation to compensate also for source amplitude.[3] Under proper conditions, SI allows one to recover the signal ideally produced by a virtual source located at the position of a receiver.[4] This technique has important applications for ultrasonics,[5] underwater acoustics,[6] seismic exploration, and passive seismics. The method makes it possible to recover the local receiver-to-receiver Earth's response, i.e., the Green's function, and provides virtual sources where it is not possible or difficult to use a real seismic source, such as in borehole and sea-floor marine surveys. Interferometry has been the subject of several theoretical[1–3,5–7] and application studies, and nowadays it is used for seismic exploration purposes.[8,9] With an approach similar to interferometry, Poletto[10] proposed the virtual reflector (VR) method,[11] which also simulates new seismic signals by processing recorded traces from a plurality of sources and receivers, with the following differences with respect to interferometry. The method is based on crossconvolution (different from convolution with time reversal) of source (receiver) signals and subsequent summation in the domain of the receivers (sources). The VR method substantially performs the composition of the filtering effects in the wave propagation. To be effective, the method needs to deal with source signals with transient wavelets. The novel approach makes it possible to obtain virtual reflected signals as if at the

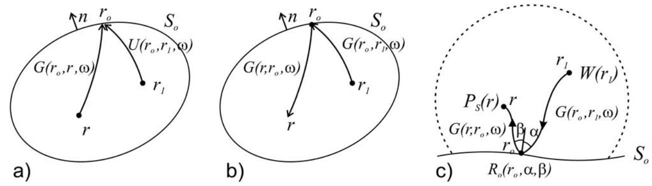

Fig. 1. (a) Acoustic model for VR representation. Two sources are located at r and r_1 encompassed by the surface S_o. (b) Composition of propagation effects and surface integration to obtain the VR signal. (c) The scattered field from reflector S_o is represented by a boundary integral over the reflector. The reflector coefficient $R_o(r_o,\alpha,\beta)$ depends on the reflector geometry and acquisition configuration.

position of receivers (or sources) there were a real reflector even if said reflector is not present there. In this letter we re-formulate the VR theory for an acoustic inhomogeneous medium by using the Kirchhoff–Helmholtz (KH) integral representation theorem.

2. Theory

The scalar fields in an inhomogeneous medium can be expressed by using the Green's integral theorem for scalar functions U and G. Let S be the total surface encompassing a volume of interest V, and n the outward normal direction to the surface. The Green's integral theorem states that

$$\int_V dV[G\nabla^2 U - U\nabla^2 G] = \int_S dS[G\,\nabla\,U - U\,\nabla\,G]\cdot n. \qquad (1)$$

The functions U and G are assumed to be the Fourier transforms of causal functions, and in the common use they represent the propagating wavefield and the medium response to an impulse, Green's function, respectively. Let S_o be the outer surface enclosing the volume of interest V. The VR signal is the result of the composition of scalar fields from different sources included in the volume V surrounded by receivers located at outer surface S_o (Ref. 11). Assume that two point sources at internal points r_1 and at r inside S_o [Fig. 1(a)] generate the scalar fields U and G, respectively. They can be expressed as[12–14]

$$U(r,r_1,\omega) - G(r_1,r,\omega) = \frac{1}{4\pi}\int_{S_o} dS_o\left[G(r_o,r,\omega)\frac{\partial U(r_o,r_1,\omega)}{\partial n} - U(r_o,r_1,\omega)\frac{\partial G(r_o,r,\omega)}{\partial n}\right], \qquad (2)$$

where $U(r_o,r_1,\omega)$ is the scalar field generated at r_1 and recorded along the surface S_o, r_o is location along the surface, $G(r_o,r,\omega)$ is the Green's function from r to r_o, $\partial/\partial n$ is the normal differentiation operator acting on r_o at surface S_o, and ω is the angular frequency. The advantage of the integral representation is that a suitable propagation Green's function G can be chosen to synthesize a particular U wavefield (and vice versa). Equation (2), with fields U and G satisfying different boundary conditions (BCs), was exploited in developing the reverse-time approach to reflection seismic imaging (e.g., Refs. 15 and 16). When U and G are subject to the same BCs on S_o, the integral vanishes. In this case from Eq. (2), we obtain $G=U$. Conversely, under proper BCs, different for U and G, the integral is in general different from zero. This result corresponds to the fact that, even when the wavefield represented by G from r to r_1 (from r_1 to r if we use reciprocity) is subtracted from U in Eq. (2), part of the synthesized wavefield still remains. If we assume that the medium outside S_o is homogeneous and there are no reflecting objects outside S_o (they may be on S_o), Eq. (2) can be used to represent a perfect VR at S_o. We define a new (VR) wavefield as $V=U-G$. This result comes from the assumption of reflecting BCs on the close surface S_o for G but not for U (or vice versa). In V we subtract direct arrivals and reflections. This reasoning is in agreement with the fact that the VR method provides *only*, however not necessarily *all*, reflected waves from S_o (Ref. 11). The scalar function V, when derived by Eq. (2), can be used to represent a perfect VR of unit (or constant) reflection coefficient. A generalization can be obtained by including a variable reflection coefficient function, according to a boundary integral representation for the response of a reflector.

Under the assumption that the scalar field $U(r_o,r_1,\omega)$ is produced from a unit source at point r_1 included in the space encompassed by S_o [Fig. 1(a)], said propagation function represents, by definition, the medium response. In this case we can express also the scalar quantity recorded on S_o as $U(r_o,r_1,\omega)=G(r_o,r_1,\omega)$, where $G(r_o,r_1,\omega)$ is the Green's function from r_1 to r_o, and substitute G for U in Eq. (2), by using V, which gives

$$V(r,r_1,\omega) = \frac{1}{4\pi}\int_{S_o} dS_o\left[G(r_o,r,\omega)\frac{\partial G(r_o,r_1,\omega)}{\partial n} - G(r_o,r_1,\omega)\frac{\partial G(r_o,r,\omega)}{\partial n}\right]. \tag{3}$$

In this representation it is intended that the reflecting BC on S_o is applied to only one of the two terms of the sum in the integrand at the right hand side.

3. Synthesis of virtual reflector

Equation (3) allows us to synthesize a VR wavefield with constant reflection coefficient under different BCs. We first analyze the case in which the model contains an equivalent medium with a reflecting interface at the boundary. We consider two cases in which $G(r_o,r,\omega)$ is chosen with a perfect reflecting BC on S_o. The first reflecting BC on S_o for $G(r_o,r,\omega)$ calculated for a given r is $\partial G(r_o,r,\omega)/\partial n=0$ (*Neumann* BC). In this case the contrast medium is perfectly rigid and reflecting (reflection coefficient $R=+1$). Equation (3) becomes

$$V_R(r,r_1,\omega) = \frac{-\imath\omega}{4\pi c}\int_{S_o} dS_o G(r_o,r,\omega)G(r_o,r_1,\omega)\cos(\alpha_o), \tag{4}$$

where we have used the approximation $\partial/\partial n \cong -\imath\omega\cos(\alpha_o)/c$ for the normal derivative, $\imath=\sqrt{-1}$, c is the medium velocity, and α_o is the angle between the ray of $G(r_o,r_1,\omega)$ and the normal at r_o on S_o. The second reflecting BC for $G(r_o,r,\omega)$ on S_o is $G(r_o,r,\omega)=0$ (*Dirichlet* BC). In this case, the contrast medium is vacuum, corresponding to a perfectly-reflecting free surface ($R=-1$). Equation (3) becomes (see Ref. 15)

$$V_F(r,r_1,\omega) = \frac{-1}{4\pi}\int_{S_o} dS_o G(r_o,r_1,\omega)\frac{\partial G(r_o,r,\omega)}{\partial n}. \tag{5}$$

Deriving an equation similar to Eq. (4) from Eq. (5) is strictly not valid for the free surface condition because $G(r_o,r,\omega)$ vanishes on S_o. Conversely, if we substitute the wavefield of the corresponding model without boundary at S_o for the wavefield of the model with boundary, to avoid vanishing of G, and use the same normal-derivative approximation, we obtain as an approximation a virtual relation similar to Eq. (4) with opposite sign.

From Eq. (4) we obtain the VR signal for the acoustic medium *with* reflecting boundary. According to the concept of the VR method,[10] the integral in Eq. (4) is the integral on S_o of the crossconvolution[1] of the non-zero scalar quantities (a factor of 4 is here neglected) measured by receivers located on S_o and produced by unit sources located at r and r_1, respectively [Fig. 1(a)]. Because we consider measured wavefields in an unknown medium configuration, we ignore the factor $\cos(\alpha_o)$. Moreover, to interpret Eq. (4) in terms of propagating wavefields, we use reciprocity and exchange source (ideal image of a reflected source) and receiver at r and r_o (or equivalently at r_1 and r_o). Using the reciprocity condition $G(r,r_o,\omega)=G(r_o,r,\omega)$, the VR representation equation for the equivalent medium with reflecting boundary becomes

$$V_{RF}(r,r_1,\omega) = \frac{\mp \imath\omega}{4\pi c}\int_{S_o} dS_o G(r,r_o,\omega)G(r_o,r_1,\omega), \tag{6}$$

which means that the VR signal is obtained by composing the propagated signals from a unit source at point r_1 to receiver in r_o and from r_o to r (or vice versa) and by integrating over the receiver surface the convolution results [Fig. 1(b)]. Consider now the case in which a reflector is *not* present at S_o boundary (transparent surface). The function $\hat{G}$ *without* reflecting boundary at S_o is related to the function G of the model with boundary by $G(r_o,r_j,\omega)=\hat{G}(r_o,r_j,\omega)+\Delta G(r_o,r_j,\omega)$, where ΔG represents the additional effects due to the presence of the reflecting boundary. In other words, $\hat{G}$ is an approximation of the Green's function G with reflecting boundary, in the sense that $\hat{G}$ does not contain reflections from S_o. The virtual reflections $\hat{V}(r,r_1,\omega)$ from S_o without reflecting boundary are synthesized by substituting the signal $\hat{G}$ for G in Eq. (6). Result (4) [or (6)] and its unbounded approximations are equivalent to the summation of crossconvolutions of the measured signals over the receiver (or source) space as proposed by Ref. 10. For selected events it holds $\hat{V}(r,r_1,\omega) \propto V(r,r_1,\omega)$ since the VR method synthesizes only but not all the reflections from S_o (Ref. 11), without the effects of the additional term ΔG.

The derivation of a generalization of Eq. (6) for the scattered wavefield is based on work of Refs. 17, 12, and 18. For the configuration in Fig. 1(c), the scattered wavefield at an observation point r above the reflector is given by

$$P_S(r,r_1,\omega) = \int_{S_o} dS_o G(r,r_o,\omega)R_o(r_o,\alpha,\beta)G(r_o,r_1,\omega)W(r_1,\omega), \tag{7}$$

where $R_o(r_o,\alpha,\beta)$ is the angle-dependent reflection operator and $W(r_1,\omega)$ is the source signal injected at r_1. In the most general case the reflection operator is a pseudodifferential operator (Ref. 17), but for the VR method we approximate it by a high-frequency angle-dependent reflection coefficient (Refs. 12 and 18). Here $P_S(r,r_1,\omega)$ is the scattered wavefield of the true medium, including the reflector, whereas $G(r,r_o,\omega)$ and $G(r_o,r_1,\omega)$ are Green's functions in one-and-the-same reference medium, which is identical to the true medium above the reflector but continues without a jump in the medium parameters below the reflector. The step to the VR principle is again easily made: assume that in the true medium, S_o is not a reflector but a surface with receivers (or sources), whereas r and r_1 both denote sources (or receivers), then the Green's functions with interchanged coordinates in one of them are obtained by measurements as proposed in the previous sections. Substituting these measurements in Eq. (7), inserting a user defined reflection coefficient, and evaluating the integral gives the VR response P_S. In its simplest form $R_o(r_o,\alpha,\beta)$ can be taken equal to $2i\omega R(r_o)/[c(r_o)\rho(r_o)]$, where ρ is the density of the medium. In particular, by using $R=\mp 1$ with normal incidence approximation and unit source signal $W(r_1,\omega)=1$, from Eq. (7) we obtain again Eq. (6), apart from a scaling factor.

All the reasoning made for the VR representation by sources surrounded by receivers holds, using reciprocity, also for the representation with receivers surrounded by sources.

4. Examples, summary, and conclusions

Figure 2 shows the model used for a two dimensional acoustic simulation by a finite-differences code, where a background homogeneous medium with compressional velocity 2 km/s includes a diffraction body at point D. Synthetic signals are calculated with and without the presence of a reflecting boundary at the circumference (surface S_o) of a circle of radius of 1.8 km. The outer hard-contrast medium velocity is 20 km/s. The signals of two sources, S_1 and S_2, are recorded by 360 receivers located on S_o. Figure 2(c) compares the simulated signals from source S_1 to a control receiver at the position of the source S_2 in the model (1) without and (2) with circular

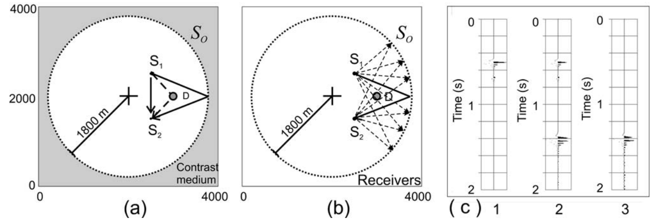

Fig. 2. Synthetic model: (a) With contrast medium at circular boundary S_o. The arrows indicate the direct boundary reflection and diffraction (dashed line) arrivals from the source S_1 to a control receiver at S_2. (b) Representation of the VR arrival (solid line) from the source S_1 to the source S_2. The VR signal is reflected by a VR at S_o and is obtained by processing the signals (dashed arrows) recorded by the receivers on the circle without the contrast medium. (c) Synthetic traces. (1) Control signal without reflector. (2) Control signal with reflector. (3) VR signal from S_1 to S_2.

reflecting boundary and (3) the VR signal calculated by performing the crossconvolution of the signals from the two sources and by integrating the crossconvolutions over the receiver space S_o in the model without reflection boundary.

To summarize, we calculated the KH integral representation of the virtual signal produced by a reflecting surface surrounding two points where sources are located. We showed that the reflection representation is equivalent to composing the wavefields of the two sources recorded on the surface. The analysis demonstrated that halfside of the KH integral represents the crossconvolution term of the VR signal. We generalized the representation for the scattered wavefield from a VR with variable reflection coefficients. Note that the VR method is complementary to SI. The main feature of SI is that the response of a virtual source can be generated without knowing the medium; all that is required is a receiver illuminated from many directions, at the location where one wants to create the virtual source. The VR method enables the generation of the reflection response of a VR without knowing the medium, as long as there are receivers (or sources) distributed along the surface at which one wants to create the VR.

The VR signal synthesis with an ideal reflector has several applications with exploration data.[11] Reference 19 shows examples of identification of and removal of surface reflection events in synthetic marine data and performs the analysis of virtual signals with real borehole data. Other potential applications are signal phase analysis and recovery of the source delay. In these applications the VR signal is used in combination with the interferometry one, provided that the VR and the SI results be obtained by using data with similar source-receiver geometry. For these methods a sufficient spatial sampling of receivers (sources) is required. From a practical point of view, this condition can be reasonably achieved in exploration seismics.

References and links

[1]The product $G(r_o,r,\omega)G(r_o,r_1,\omega)$ is the Fourier transform of a temporal crossconvolution. In this paper we loosely refer to such products as (cross-)convolutions.

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