APPLYING ONE-WAY RECIPROCITY THEOREMS IN TIME-LAPSE SEISMIC IMAGING

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ABSTRACT

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We formulate reciprocity theorems for time-lapse seismic methods, based on the full and the one-way wave equations. The latter form allows a straightforward physical interpretation of the various contributing terms. Unlike difference data taken at the acquisition surface, the boundary integral in the one-way reciprocity theorem represents the true time-lapse changes of the reflectivity of the top reflector of a reservoir below the boundary at which this integral is evaluated. Evaluation of the boundary integral yields therefore suited input for time-lapse AVO analysis.

KEY WORDS: reciprocity, time-lapse, one-way wave equation.

INTRODUCTION

A seismic difference section, obtained by subtracting the reference section from a monitor section in a time-lapse seismic experiment, should ideally reveal the changes that occurred in the subsurface in the elapsed time between the different measurements. However, even when the acquisition conditions of the seismic measurements were fully repeatable, the amplitudes in the difference section would be deteriorated as a result of traveltime shifts due to velocity changes in the reservoir (see Fig. 8). Hence, a seismic difference section is not a good measure for quantifying time-lapse changes in the subsurface.

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Fokkema et al. (1997) formulated a mathematical relation between the seismic reference and monitor sections in a time-lapse seismic experiment, based on acoustic reciprocity. Dillen (2000) elaborated on this and, in particular, he analysed the interaction integral between the wave fields of the seismic reference and monitor states at an arbitrary reference level (e.g., below a reservoir). He showed that the aforementioned amplitude deteriorations do not occur in this interaction integral.

Recently we derived reciprocity theorems for the one-way wave equations (Wapenaar and Grimbergen, 1996). These theorems formulate relations between down- and upgoing wave fields in two different acoustic states. When applied to the time-lapse seismic method, a similar interaction integral is obtained as the one analysed by Dillen (2000). This interaction integral now allows an interpretation in terms of downgoing and upgoing wave fields. It appears to formulate the difference between two virtual seismic experiments at the acquisition surface. In each of these virtual experiments the downgoing wave field propagates through the reference state whereas the upgoing wave field propagates through the monitor state. On the other hand, the reflection in one of the virtual experiments occurs in the reference state and in the other in the monitor state. Hence, the primary traveltimes in these virtual experiments are the same; the main difference between these experiments is the reflection amplitude. As a consequence, the interaction integral represents a new difference section, in which the amplitudes of some important reflectors are not deteriorated by time shifts. In the following we discuss the application of the one-way reciprocity theorem to time-lapse seismic imaging in more detail and point out some of its advantages and limitations.

RECIPROCITY THEOREM FOR THE FULL WAVE FIELD

In this section we review the scalar form of the acoustic reciprocity theorem. We closely follow de Hoop (1988) and Fokkema and van den Berg (1993). The former author derives reciprocity theorems in the time domain; the latter authors in the time domain, the Laplace domain and the frequency domain. Here we only consider the frequency domain.

Basic acoustic equations

In the space-frequency $(\mathbf{x},\omega)$ domain, the equations that govern linear acoustic wave motion read

$$\partial_k P + j\omega\rho V_k = F_k \ , \tag{1}$$

and

$$\partial_k V_k + j\omega\kappa P = Q \ , \tag{2}$$

where P is the acoustic pressure, V_k is the particle velocity, ρ is the volume density of mass, κ is the compressibility, F_k is the volume source density of volume force and Q is the volume source density of volume injection rate. The Latin subscripts take on the values 1 to 3 and the summation convention applies to repeated subscripts.

Reciprocity theorem for the full wave field

We introduce two acoustic states (i.e., wave fields, medium parameters and sources), that will be distinguished by the subscripts A and B. For these two states we consider the interaction quantity $\partial_k\{P_A V_{k,B} - V_{k,A} P_B\}$. Applying the product rule for differentiation, substituting equations (1) and (2) for states A and B, integrating the result over a volume V with boundary ∂V and outward pointing normal vector $\mathbf{n} = (n_1, n_2, n_3)$ (see Fig. 1) and applying the theorem of Gauss yields

$$\int_{\mathbf{x}\in\partial V} \{P_A V_{k,B} - V_{k,A} P_B\} n_k dA = -j\omega \int_{\mathbf{x}\in V} \{P_A \Delta\kappa P_B - V_{k,A} \Delta\rho V_{k,B}\} dV$$

$$+ \int_{\mathbf{x}\in V} \{P_A Q_B - V_{k,A} F_{k,B} + F_{k,A} V_{k,B} - Q_A P_B\} dV \quad , \tag{3}$$

where $\Delta\kappa = \kappa_B - \kappa_A$ and $\Delta\rho = \rho_B - \rho_A$. Equation (3) is Rayleigh's reciprocity theorem (Rayleigh, 1878).

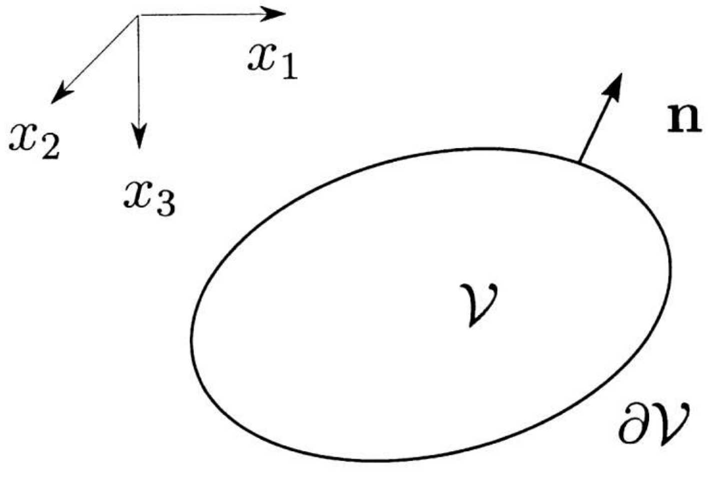

Fig. 1. Configuration for Rayleigh's reciprocity theorem.

We conclude this section by considering some special cases:

Unbounded media - Consider the situation in which the medium outside ∂V is homogeneous, unbounded and source-free in both states. Assume that the wave fields in both states are causally related to the sources in V. Then, if $\rho_A = \rho_B$ and $\kappa_A = \kappa_B$ outside ∂V, the boundary integral on the left-hand side of equation (3) vanishes (Bleistein, 1984; Fokkema and van den Berg, 1993).

Physical reciprocity - Assume that the above mentioned conditions are fulfilled and that $\rho_A = \rho_B$ and $\kappa_A = \kappa_B$ in V as well. Then the first volume integral on the right-hand side of equation (3) vanishes. Furthermore, consider point sources in states A and B at $\mathbf{x}_A \in V$ and $\mathbf{x}_B \in V$, respectively, according to $Q_A(\mathbf{x},\omega) = q_A(\omega)\delta(\mathbf{x}-\mathbf{x}_A)$, $Q_B(\mathbf{x},\omega) = q_B(\omega)\delta(\mathbf{x}-\mathbf{x}_B)$, with $q_A(\omega) = q_B(\omega)$ and $F_{k,A}(\mathbf{x},\omega) = F_{k,B}(\mathbf{x},\omega) = 0$. Equation (3) thus yields

$$P_A(\mathbf{x}_B|\mathbf{x}_A,\omega) = P_B(\mathbf{x}_A|\mathbf{x}_B,\omega) \quad . \tag{4}$$

This equation formulates the well-known fact that the acoustic pressure observed by a receiver at $\mathbf{x}_B$ due to a source at $\mathbf{x}_A$ is identical to the acoustic pressure observed by a receiver at $\mathbf{x}_A$ due to a source at $\mathbf{x}_B$. Note that this is a special case of the more general reciprocity theorem of equation (3). This general theorem was used by Fokkema et al. (1997) and Dillen (2000) to define a relation between the seismic reference and monitor sections in a time-lapse seismic experiment. This will be briefly reviewed in a later section.

RECIPROCITY THEOREM FOR ONE-WAY WAVE FIELDS

In this section we review the matrix-vector form of the acoustic reciprocity theorem for one-way wave fields (Wapenaar and Grimbergen, 1996).

One-way wave equation in matrix-vector form

We introduce a system of coupled equations for the one-way wave fields P^+ and P^-, propagating in the positive and negative depth direction, respectively, originating from sources S^+ and S^-:

$$\partial_3\mathbf{P} = \hat{\mathbf{B}}\mathbf{P} + \mathbf{S} \quad , \tag{5}$$

(the hat denotes a pseudo-differential operator), with

$$\mathbf{P} = \begin{pmatrix} P^+ \\ P^- \end{pmatrix} \quad \text{and} \quad \mathbf{S} = \begin{pmatrix} S^+ \\ S^- \end{pmatrix} \quad . \tag{6}$$

The one-way operator matrix $\hat{\mathbf{B}}$ is defined as

$$\hat{\mathbf{B}} = \begin{pmatrix} -\mathrm{j}\hat{\mathcal{H}}_1 & 0 \\ 0 & \mathrm{j}\hat{\mathcal{H}}_1 \end{pmatrix} + \begin{pmatrix} \hat{\mathrm{T}} & -\hat{\mathrm{R}} \\ -\hat{\mathrm{R}} & \hat{\mathrm{T}} \end{pmatrix} \quad , \tag{7}$$

where $\hat{\mathcal{H}}_1$ is the square-root of the Helmholtz operator and $\hat{\mathrm{R}}$ and $\hat{\mathrm{T}}$ are the reflection and transmission operators, respectively. See the Appendix for a further explanation.

Reciprocity theorem for one-way wave fields

We introduce two different states that will be distinguished by the subscripts A and B. For these two states we consider the interaction quantity $\partial_3\{\mathbf{P}_A^t\mathbf{N}\mathbf{P}_B\}$, with $\mathbf{N} = \left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)$ or, written alternatively, $\partial_3\{\mathrm{P}_A^+\mathrm{P}_B^- - \mathrm{P}_A^-\mathrm{P}_B^+\}$. The superscript ^{t} denotes transposition. Applying the product rule for differentiation, substituting the one-way wave equation (5) for states A and B, integrating the result over volume V with boundary $\partial V_0 \cup \partial V_1$ (where ∂V_0 represents the combination of two planar surfaces perpendicular to the x_3-axis and ∂V_1 a cylindrical surface with its axis parallel to the x_3-axis, see Fig. 2), applying the theorem of Gauss and using the symplectic relation $\hat{\mathbf{B}}^t\mathbf{N} = -\mathbf{N}\hat{\mathbf{B}}$, yields the following one-way reciprocity theorem

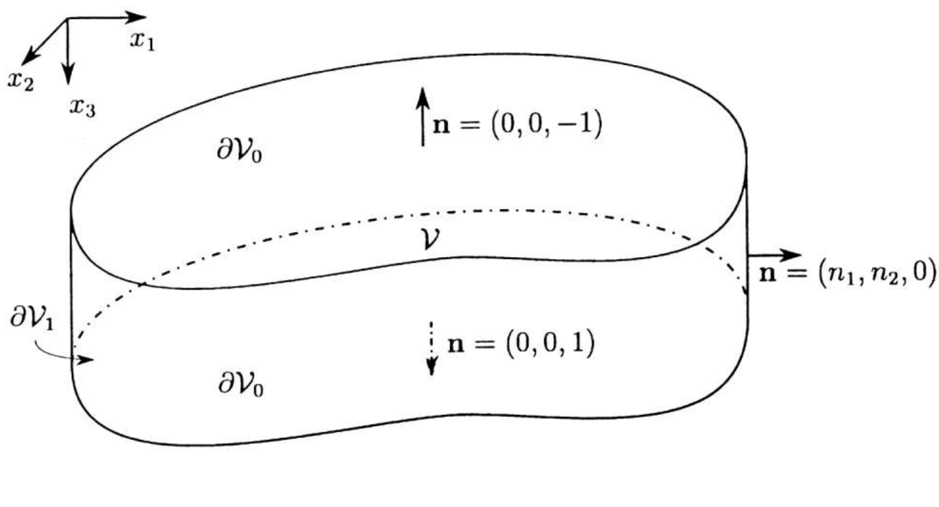

Fig. 2. Modified configuration for the one-way reciprocity theorem. The combination of the two planar surfaces is denoted by ∂V_0; the cylindrical surface is denoted by ∂V_1.

$$\int_{\mathbf{x}\in\partial V_0} \mathbf{P}_A^t \mathbf{N} \mathbf{P}_B n_3 dA = \int_{\mathbf{x}\in V} \mathbf{P}_A^t \mathbf{N} \hat{\boldsymbol{\Delta}} \mathbf{P}_B dV + \int_{\mathbf{x}\in V} \{\mathbf{P}_A^t \mathbf{N} \mathbf{S}_B + \mathbf{S}_A^t \mathbf{N} \mathbf{P}_B\} dV \quad , \tag{8}$$

where the contrast operator $\hat{\boldsymbol{\Delta}}$ is given by

$$\hat{\boldsymbol{\Delta}} = \hat{\mathbf{B}}_B - \hat{\mathbf{B}}_A \quad . \tag{9}$$

Note that the boundary integral over ∂V_1 vanished. For bounded ∂V_1 this occurs when $\mathbf{P}_A$ and $\mathbf{P}_B$ satisfy homogeneous Dirichlet or Neumann boundary conditions on ∂V_1. On the other hand, when ∂V_1 is unbounded this boundary contribution also vanishes under the condition that $\mathbf{P}_A$ and $\mathbf{P}_B$ have sufficient decay at infinity.

We conclude this subsection by analyzing reciprocity theorem (8) for some special cases:

Unbounded media - Consider the situation in which the medium at and outside ∂V_0 is homogeneous, unbounded and source-free in both states. Assume that the wave fields in both states are causally related to the sources in V. Then in both states the wave fields are outgoing at ∂V_0 (i.e., $P_A^+ = P_B^+ = 0$ at the upper surface and $P_A^- = P_B^- = 0$ at the lower surface) and it is easily seen that $\mathbf{P}_A^t \mathbf{N} \mathbf{P}_B = P_A^+ P_B^- - P_A^- P_B^+ = 0$ at ∂V_0, so the boundary integral on the left-hand side of equation (8) vanishes. Apparently it is not required that the medium parameters at and outside ∂V_0 are identical in both states, unlike the conditions for the vanishing of the boundary integral in reciprocity theorem (3).

Physical reciprocity - Assume that the above mentioned conditions are fulfilled and that $\rho_A = \rho_B$ and $\kappa_A = \kappa_B$ inside as well as outside V. Then the first volume integral on the right-hand side of equation (8) vanishes. Furthermore, consider point sources in states A and B at $\mathbf{x}_A \in V$ and $\mathbf{x}_B \in V$, respectively, according to $\mathbf{S}_A(\mathbf{x},\omega) = \mathbf{s}_A(\omega)\delta(\mathbf{x}-\mathbf{x}_A)$ and $\mathbf{S}_B(\mathbf{x},\omega) = \mathbf{s}_B(\omega)\delta(\mathbf{x}-\mathbf{x}_B)$. Equation (8) thus yields

$$\mathbf{P}_A^t(\mathbf{x}_B|\mathbf{x}_A,\omega)\mathbf{N}\mathbf{s}_B(\omega) = -\mathbf{s}_A^t(\omega)\mathbf{N}\mathbf{P}_B(\mathbf{x}_A|\mathbf{x}_B,\omega) \quad . \tag{10}$$

For the special case that $\mathbf{s}_A = \binom{s_A^+}{0}$ and $\mathbf{s}_B = \binom{s_B^+}{0}$, with $s_A^+ = s_B^+$, this reduces to

$$P_A^-(\mathbf{x}_B|\mathbf{x}_A,\omega) = P_B^-(\mathbf{x}_A|\mathbf{x}_B,\omega) \quad , \tag{11}$$

see Fig. 3. This equation formulates the fact that the acoustic upgoing wave field observed by a receiver at $\mathbf{x}_B$ due to a source for a downgoing wave field at $\mathbf{x}_A$ is identical to the acoustic upgoing wave field observed by a receiver at $\mathbf{x}_A$ due to a source for a downgoing wave field at $\mathbf{x}_B$. Note that this is a special case of the more general one-way reciprocity theorem of equation (8). This general theorem will be used in this paper to define a relation between the seismic reference and monitor sections in a time-lapse seismic experiment.

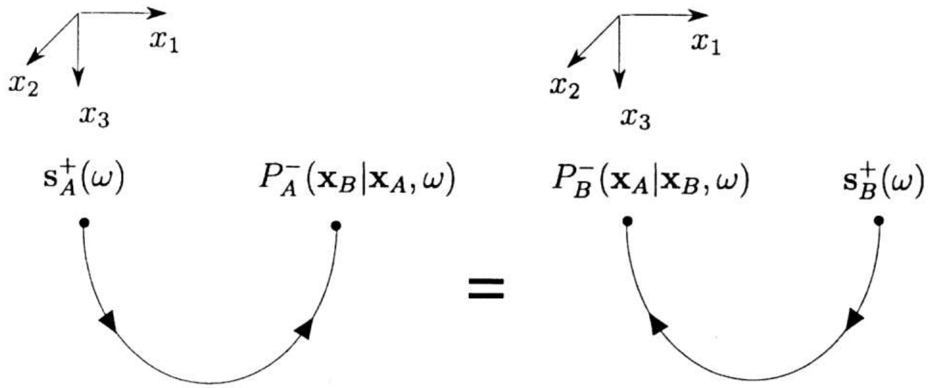

Fig. 3. Physical reciprocity for one-way sources and receivers.

RECIPROCITY THEOREMS FOR THE TIME-LAPSE SEISMIC METHOD

Since in a reciprocity theorem two states interact, it is optimally fitted to formulate the relation between two measurements in a time-lapse seismic experiment. State A is associated with the reference wave field at, say, $t = t_1$, while state B is associated with the monitoring wave field at, say, $t = t_2 > t_1$. It is noted that $t_2 - t_1$ is much longer than the seismic experiment time. In our analysis, IR^3 is divided in three domains (Fig. 4): V_0 is the domain where there are no differences between the material parameters in the two states, mostly associated with the domain above the reservoir; the domain V_c, for example associated with the reservoir, where there is a difference between the material parameters in the two states mostly due to the reservoir production history; and V' denotes the complement of $V_0 \cup V_c$; the material parameters in this domain may or may not be different; a possible difference in this domain is taken into account in a subsequent step. The domains are specified as follows

$$
\begin{aligned}
V_0 &= \{\mathbf{x} \in \mathrm{IR}^3,\ x_3 \le x_3^1\} \ , \\
V_c &= \{\mathbf{x} \in \mathrm{IR}^3,\ x_3^1 < x_3 < x_3^2\} \ , \qquad (12) \\
V' &= \{\mathbf{x} \in \mathrm{IR}^3,\ x_3 \ge x_3^2\} \ .
\end{aligned}
$$

In the next subsections we will discuss the matter for the two reciprocity theorems discussed above.

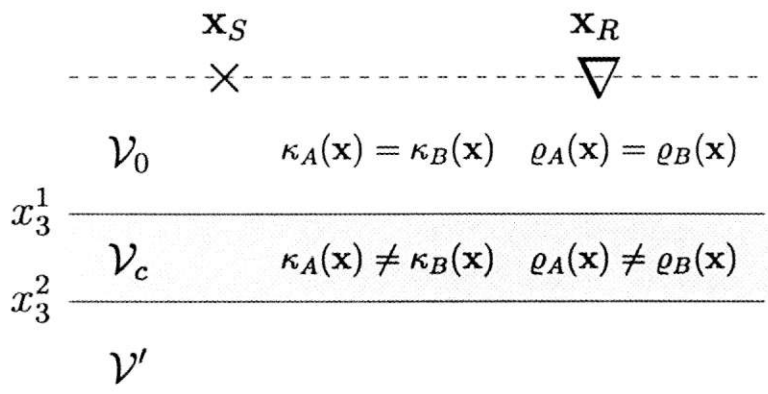

Fig. 4. Configuration for the time-lapse seismic method.

Full wave equation

In order to simplify the analysis we only consider point sources of the volume injection type. The source of state A is taken at $\mathbf{x} = \mathbf{x}_S$, while the source of state B is taken at $\mathbf{x} = \mathbf{x}_R$, according to

$$Q_A(\mathbf{x},\omega) = q_A(\omega)\delta(\mathbf{x} - \mathbf{x}_S) \quad , \tag{13}$$

$$Q_B(\mathbf{x},\omega) = q_B(\omega)\delta(\mathbf{x} - \mathbf{x}_R) \quad . \tag{14}$$

Application of reciprocity theorem (3) to domain $V = V_0 \cup V_c$ yields

$$\begin{aligned}
&\int_{x_3=x_3^2} \{P_A(\mathbf{x}|\mathbf{x}_S)V_{3,B}(\mathbf{x}|\mathbf{x}_R) - V_{3,A}(\mathbf{x}|\mathbf{x}_S)P_B(\mathbf{x}|\mathbf{x}_R)\}dA \\
&= -j\omega \int_{\mathbf{x}\in V_c} \{P_A(\mathbf{x}|\mathbf{x}_S)\Delta\kappa(\mathbf{x})P_B(\mathbf{x}|\mathbf{x}_R) - V_{k,A}(\mathbf{x}|\mathbf{x}_S)\Delta\rho(\mathbf{x})V_{k,B}(\mathbf{x}|\mathbf{x}_R)\}dV \\
&+ q_B(\omega)P_A(\mathbf{x}_R|\mathbf{x}_S) - q_A(\omega)P_B(\mathbf{x}_S|\mathbf{x}_R) \quad .
\end{aligned} \tag{15}$$

Using physical reciprocity (equation 4) we arrive at

$$\begin{aligned}
&q_B(\omega)P_A(\mathbf{x}_R|\mathbf{x}_S) - q_A(\omega)P_B(\mathbf{x}_R|\mathbf{x}_S) \\
&= j\omega \int_{\mathbf{x}\in V_c} \{P_A(\mathbf{x}|\mathbf{x}_S)\Delta\kappa(\mathbf{x})P_B(\mathbf{x}|\mathbf{x}_R) - V_{k,A}(\mathbf{x}|\mathbf{x}_S)\Delta\rho(\mathbf{x})V_{k,B}(\mathbf{x}|\mathbf{x}_R)\}dV \\
&+ \int_{x_3=x_3^2} \{P_A(\mathbf{x}|\mathbf{x}_S)V_{3,B}(\mathbf{x}|\mathbf{x}_R) - V_{3,A}(\mathbf{x}|\mathbf{x}_S)P_B(\mathbf{x}|\mathbf{x}_R)\}dA \quad .
\end{aligned} \tag{16}$$

The boundary integral on the right-hand side of equation (16) takes into account a possible difference of the material parameters in V', below the reservoir; it vanishes when there is no difference between the two states in V'.

One-way wave equation

In the one-way analysis we consider point-sources for downgoing waves in both states, according to

$$\mathbf{S}_A(\mathbf{x},\omega) = \begin{pmatrix} s_A^+(\omega) \\ 0 \end{pmatrix} \delta(\mathbf{x} - \mathbf{x}_S) \ , \tag{17}$$

$$\mathbf{S}_B(\mathbf{x},\omega) = \begin{pmatrix} s_B^+(\omega) \\ 0 \end{pmatrix} \delta(\mathbf{x} - \mathbf{x}_R) \ . \tag{18}$$

Application of reciprocity theorem (8) to domain $V = V_0 \cup V_c$ yields

$$\int_{x_3 = x_3^2} \{P_A^+(\mathbf{x}|\mathbf{x}_S)P_B^-(\mathbf{x}|\mathbf{x}_R) - P_A^-(\mathbf{x}|\mathbf{x}_S)P_B^+(\mathbf{x}|\mathbf{x}_R)\}dA$$
$$= \int_{x \in V_c} \mathbf{P}_A^t(\mathbf{x}|\mathbf{x}_S)\mathbf{N}\hat{\mathbf{\Delta}}(\mathbf{x})\mathbf{P}_B(\mathbf{x}|\mathbf{x}_R)dV - s_B^+(\omega)P_A^-(\mathbf{x}_R|\mathbf{x}_S) +$$
$$s_A^+(\omega)P_B^-(\mathbf{x}_S|\mathbf{x}_R) \ . \tag{19}$$

Using physical reciprocity (equation 11) we arrive at

$$s_B^+(\omega)P_A^-(\mathbf{x}_R|\mathbf{x}_S) - s_A^+(\omega)P_B^-(\mathbf{x}_R|\mathbf{x}_S) = \int_{x \in V_c} \mathbf{P}_A^t(\mathbf{x}|\mathbf{x}_S)\mathbf{N}\hat{\mathbf{\Delta}}(\mathbf{x})\mathbf{P}_B(\mathbf{x}|\mathbf{x}_R)dV$$
$$+ \int_{x_3 = x_3^2} \{P_A^-(\mathbf{x}|\mathbf{x}_S)P_B^+(\mathbf{x}|\mathbf{x}_R) - P_A^+(\mathbf{x}|\mathbf{x}_S)P_B^-(\mathbf{x}|\mathbf{x}_R)\}dA \ . \tag{20}$$

As in the previous case, the boundary integral on the right-hand side of equation (20) vanishes when there is no difference between the two states in V' (i.e., below $x_3 = x_3^2$).

Let us analyse this boundary integral, however, for the situation in which there are changes below the reference level $x_3 = x_3^2$. Fig. 5 shows a configuration with two regions in which changes occur (the grey areas). Fig. 5a shows some primary wave-paths in the integral $\int_{x_3=x_3^2} P_A^-(\mathbf{x}|\mathbf{x}_S)P_B^+(\mathbf{x}|\mathbf{x}_R)dA$. If P_B^+ is interpreted as a Green's function for state B [multiplied by the source function $s_B^+(\omega)$], then it is understood that this integral performs an upward extrapolation of P_A^- in state A from the reference level x_3^2 to $\mathbf{x}_R$ at the acquisition surface. This results in a virtual experiment (see Fig. 6a), in which the downgoing waves propagate from $\mathbf{x}_S$ through the medium in state A (before the changes took place), reflection at the second reservoir occurs in state A, and the upgoing waves propagate through the medium in state B (after the changes took place) to $\mathbf{x}_R$. A similar interpretation is shown in Figs. 5b and 6b for the integral $\int_{x_3=x_3^2} P_A^+(\mathbf{x}|\mathbf{x}_S)P_B^-(\mathbf{x}|\mathbf{x}_R)dA$. It represents a virtual experiment with the same propagation paths, except with reflection taking place at the second reservoir in state B. Hence, since the traveltimes in these virtual experiments are the same, the difference of these terms [as expressed by the boundary integral in equation (20)] is proportional to the time-lapse changes of the reflectivity of the top reflector of the second reservoir. From this difference reflectivity, the time-lapse changes in the second reservoir can be estimated.

Of course reflections from any boundary below the second reservoir, including its bottom reflector, will cause traveltime differences in the boundary integral [albeit smaller than in a difference section at the surface, as expressed by the left-hand side of equation (20)]. In a subsequent step, the reference level x_3^2 could be lowered to a level below the second reservoir. The two terms of the boundary integral will cancel when no changes occur below the new reference level x_3^2 and when $P_A^\pm$ and $P_B^\pm$ at x_3^2 are obtained correctly. This yields a verification criterion for the estimated changes above x_3^2 (Dillen, 2000; Dillen, et al., 2000; Scherpenhuijsen, 2000).

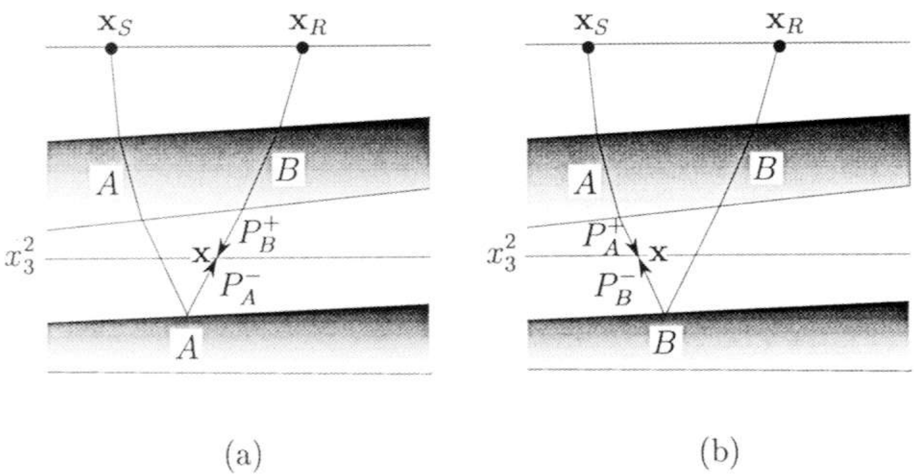

Fig. 5. Analysis of the two terms in the boundary integral in equation (20). Both terms accomplish a forward extrapolation of upgoing waves from x_3^2 to the surface. The results are shown in Fig. 6.

Note that in the discussion above, we concentrated on some primary wave paths. In theory, however, the boundary integral on the right-hand side of equation (20) vanishes for the primaries as well as the multiples in $P_A^{\pm}$ and $P_B^{\pm}$ at $x_3 = x_3^2$ when no time-lapse changes occur below this level. Of course this is no longer true when in practice $P_A^{\pm}$ and $P_B^{\pm}$ at $x_3 = x_3^2$ are obtained by one-way wave field extrapolation. In this case it should be assumed that surface related multiples have been eliminated prior to applying equation (20) and that internal multiples are negligible.

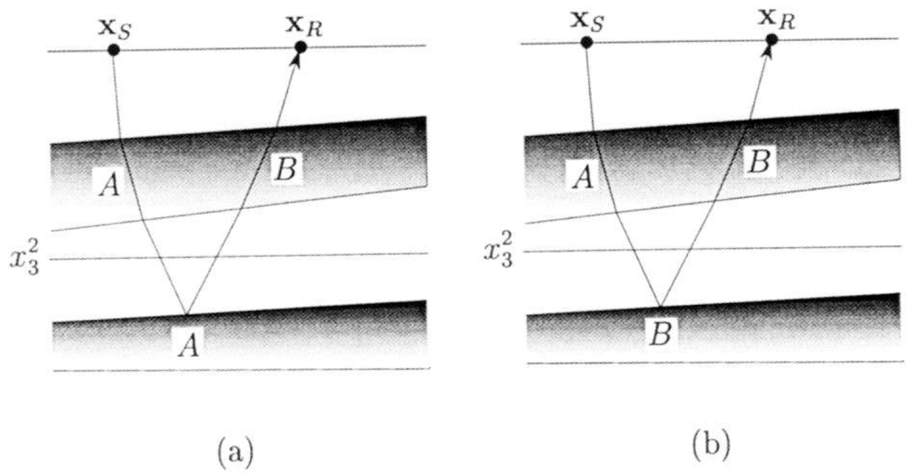

Fig. 6. Virtual experiments, corresponding to Figs. 5a and b. A and B: situations before and after the changes took place.

EXAMPLE 1

Fig. 7 shows a subsurface model, including a reservoir layer in which changes take place. The propagation velocity in the reservoir in state A (before the changes took place) is given by $c_A = 2500$ m/s; in state B (after the changes took place) it is given by $c_B = 2580$ m/s. Fig. 8 shows a shot record in state A (the 'reference' shot gather) and in state B (the 'monitor' shot gather) as well as the difference of these two shot gathers. Note that the first event in the difference shot gather is representative for the time-lapse changes of the reflectivity of the top of the reservoir. The other events in this difference shot gather, however, are spurious events: they are caused by the traveltime differences of the waves propagating through the reservoir and bear no relation with the time-lapse differences with the reflectors below the reservoir (as a matter of fact there are no time-lapse differences below the reservoir, so we would like these difference amplitudes to be zero).

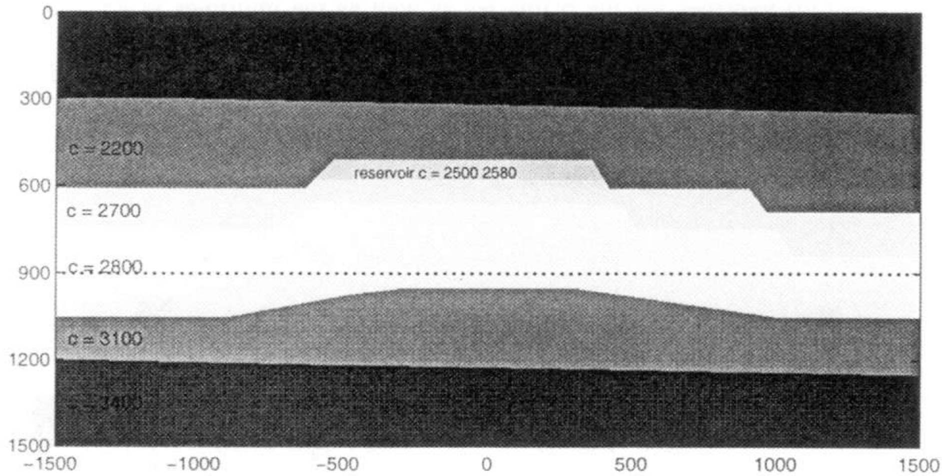

Fig. 7. Subsurface model, including a reservoir in which changes take place: $c_A = 2500$ m/s, $c_B = 2580$ m/s. The boundary integral (20) is evaluated at $x_3^2 = 900$ m.

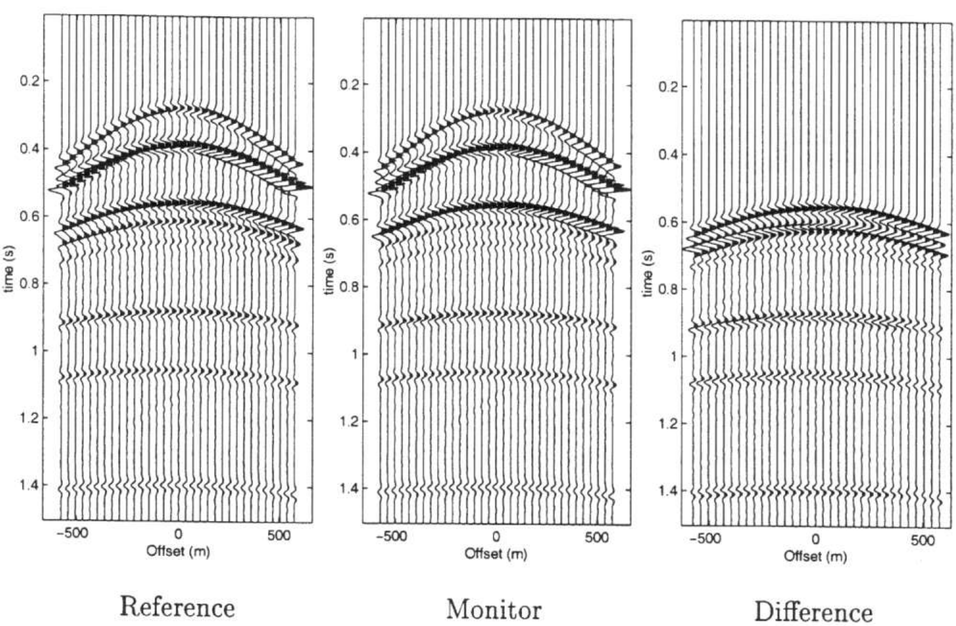

Fig. 8. Shot gather for the model of Fig. 7 in state A (the 'reference' shot gather) and in state B (the 'monitor' shot gather) and the difference of these two shot gathers.

Next we consider the boundary integral in equation (20). Let $x_3 = x_3^2 = 900$ m denote the dotted line in Fig. 7 below the reservoir. Following the explanation in the previous section, the two terms in the integral in equation (20) should cancel, because they can be seen as virtual experiments with the same propagation paths and with the same reflectors below the dotted line in Fig. 7. Fig. 9 shows the two contributions of this integral (anti-causal events have been muted) as well as their difference (Scherpenhuijsen, 2000). Unlike the difference section in Fig. 8, the difference section in Fig. 9 is indeed zero.

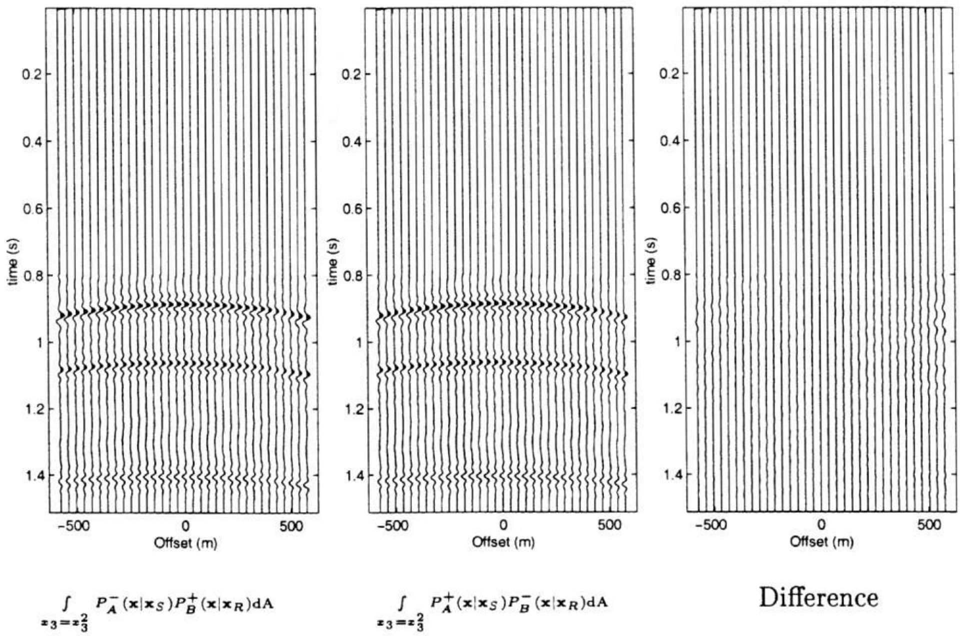

Fig. 9. Evaluation of the two terms of the integral in equation (20) (anti-causal events have been muted) and their difference.

EXAMPLE 2

Fig. 10 shows a subsurface model, including two reservoirs in which changes take place. In reservoir 1 the velocity changes from $c_A = 2500$ m/s to $c_B = 2580$ m/s; in reservoir 2 from $c_A = 3100$ m/s to $c_B = 3000$ m/s. The same analysis is done as in the previous example. Fig. 11a shows the straightforward difference of two shot gathers. This result is contaminated by traveltime differences. Fig. 11b is the result of evaluating the boundary integral (20) at the reference level $x_3^2 = 900$ m (after muting non-causal events). The latter result contains, around $t = 0.9$ s, the true time-lapse changes of the reflectivity of the top reflector of the second reservoir. Note the significant difference with the same event in Fig. 11a. The difference reflection around t = 0.9 s in Fig. 11b is suited for AVO analysis to estimate the time-lapse changes in the second reservoir. The reflections around t = 1.1 s and t = 1.4s in Fig. 11b are contaminated by traveltime differences (but less than in Fig. 11a). In a next step the boundary integral could be evaluated at a new reference level below the second reservoir.

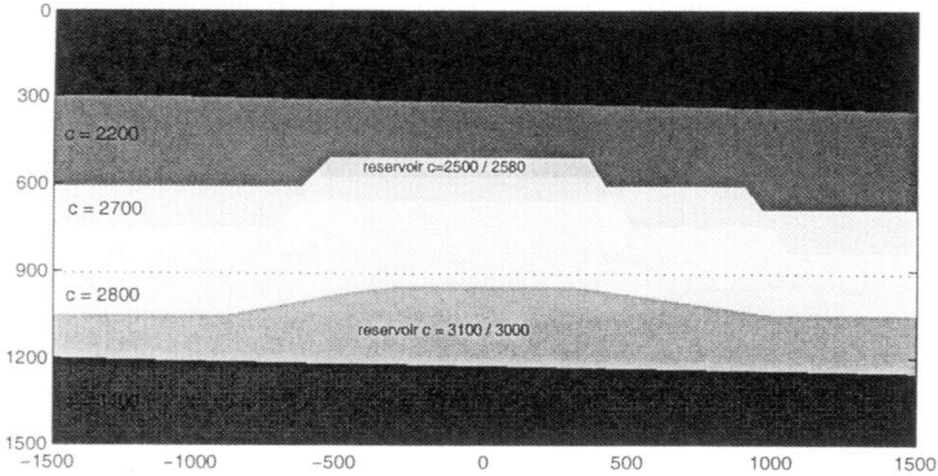

Fig. 10. Subsurface model, including two reservoirs in which changes take place. Reservoir 1: $c_A = 2500$ m/s, $c_B = 2580$ m/s. Reservoir 2: $c_A = 3100$ m/s, $c_B = 3000$ m/s. The boundary integral (20) is evaluated at $x_3^2 = 900$ m.

CONCLUSIONS

We have formulated reciprocity theorems for time-lapse seismic methods, based on the full and the one-way wave equations. The latter form allows a straightforward physical interpretation of the various contributing terms. Its implementation requires wave field decomposition and one-way wave field extrapolation of down- and upgoing waves. We have illustrated the evaluation of the boundary integral in the one-way reciprocity theorem. Unlike difference data taken at the acquisition surface, the boundary integral represents the true time-lapse changes of the reflectivity of the first 'time-lapse interface' below the boundary at which the integral is evaluated ($x_3 = x_3^2$). The advantage is two-fold:

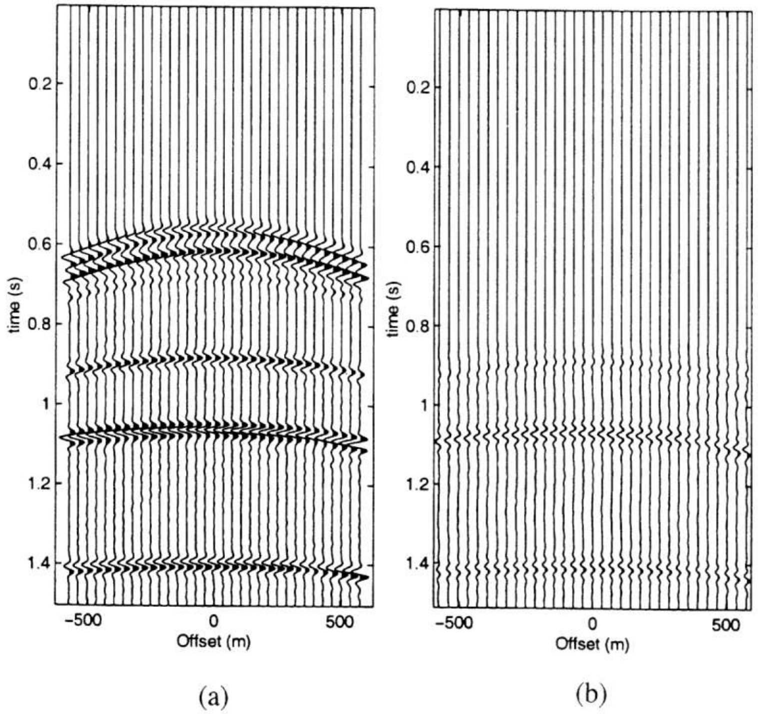

Fig. 11. Results for the stacked reservoir model of Fig. 10. (a) Difference of two shot gathers, contaminated by traveltime differences. (b) Evaluation of the boundary integral (20). This result contains the true time-lapse changes of the reflectivity of the second reservoir.

1. Interfaces between $x_3 = x_3^2$ and the first time-lapse interface below $x_3 = x_3^2$ should vanish in the result of the boundary integral evaluation; this yields a verification criterion for the estimated velocity changes above $x_3 = x_3^2$.

2. The first time-lapse interface below $x_3 = x_3^2$ (for instance the top of a reservoir) is recovered with its true time-lapse AVO reflectivity and is thus proper input for time-lapse AVO analysis.

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REFERENCES

Anno, P., Cohen, J.K. and Bleistein, N., 1992. Waves and rays in acoustic variable density media. Expanded Abstr., 62nd Ann. Internat. SEG Mtg., New Orleans: 1332-1336.

Bleistein, N., 1984. Mathematical Methods for Wave Phenomena. Academic Press, Inc., Orlando.

de Hoop, A.T., 1988. Time-domain reciprocity theorems for acoustic wave fields in fluids with relaxation. J. Acoust. Soc. Am., 84: 1877-1882.

de Hoop, M.V., 1992. Directional decomposition of transient acoustic wave fields. Ph.D. thesis, Delft University of Technology.

Dillen, M.W.P., Fokkema, J.T. and Wapenaar, C.P.A., 2000. Convolution type interaction of time-lapse acoustic wave fields. Expanded Abstr., 70th Ann. Internat. SEG Mtg., Calgary, Alberta.

Dillen, M.W.P., 2000. Time-lapse seismic monitoring of subsurface stress dynamics. Ph.D. thesis, Delft University of Technology.

Fokkema, J.T. and van den Berg, P.M., 1993. Seismic Applications of Acoustic Reciprocity. Elsevier Science Publishers, Amsterdam.

Fokkema, J.T., Dillen, M.W.P. and Wapenaar, C.P.A., 1997. A proposal for 4D seismic imaging. Extended Abstr., 59th EAGE Conf., Geneva: B041.

Messiah, A., 1962. Quantum Mechanics, Vol. II. North-Holland Publishing Company, Amsterdam.

Rayleigh, J.W.S., 1878. The Theory of Sound, Vol. II (Reprint 1945). Dover Publications, Inc.

Scherpenhuijsen, P.C., 2000. Application of the one-way reciprocity theorem on 4D seismic data. M.Sc. thesis, Delft University of Technology.

Wapenaar, C.P.A. and Berkhout, A.J., 1989. Elastic Wave Field Extrapolation. Elsevier Science Publishers, Amsterdam.

Wapenaar, C.P.A. and Grimbergen, J.L.T., 1996. Reciprocity theorems for one-way wave fields. Geophys. J. Int., 127: 169-177.

APPENDIX

OPERATORS IN THE ONE-WAY WAVE EQUATION

The one-way wave equation (equations 5 to 7) can be written as a coupled system of scalar equations for the down- and upgoing one-way wave fields P^+ and P^-, according to

$$\partial_3 P^+ = -j\hat{\mathcal{H}}_1 P^+ + \hat{T}P^+ - \hat{R}P^- + S^+ \;, \tag{A-1}$$

$$\partial_3 P^- = j\hat{\mathcal{H}}_1 P^- + \hat{T}P^- - \hat{R}P^+ + S^- \;. \tag{A-2}$$

The operator $\hat{\mathcal{H}}_1$ is the square-root of the Helmholtz operator, according to

$$\hat{\mathcal{H}}_1 = [(\omega/c)^2 + \partial_\alpha \partial_\alpha]^{1/2} \;. \tag{A-3}$$

(Greek subscripts take on the values 1 and 2 and the summation convention applies to repeated subscripts). For media with constant density (and variable compressibility), c is the propagation velocity, defined as $c = (\kappa\rho)^{-1/2}$. For variable density media, c obeys the Klein-Gordon dispersion relation known from relativistic quantum mechanics (Messiah, 1962; Wapenaar and Berkhout, 1989; de Hoop, 1992; Anno, 1992), according to

$$(\omega/c)^2 = \omega^2\kappa\rho - 3(\partial_\alpha\rho)(\partial_\alpha\rho)/4\rho^2 + \partial_\alpha\partial_\alpha\rho/2\rho \quad . \tag{A-4}$$

The reflection and transmission operators $\hat{R}$ and $\hat{T}$ in equations (A-1) and (A-2) are given by

$$\hat{R} = 1/2(\hat{\mathcal{H}}_1^{1/2}\rho^{-1/2}\partial_3\rho^{1/2}\hat{\mathcal{H}}_1^{-1/2} - \hat{\mathcal{H}}_1^{-1/2}\rho^{1/2}\partial_3\rho^{-1/2}\hat{\mathcal{H}}_1^{1/2}) \quad , \tag{A-5}$$

$$\hat{T} = -1/2(\hat{\mathcal{H}}_1^{1/2}\rho^{-1/2}\partial_3\rho^{1/2}\hat{\mathcal{H}}_1^{-1/2} + \hat{\mathcal{H}}_1^{-1/2}\rho^{1/2}\partial_3\rho^{-1/2}\hat{\mathcal{H}}_1^{1/2}) \quad , \tag{A-6}$$

which follows from the explicit expression for the one-way operator matrix $\hat{\mathbf{B}}$, given in, e.g., Wapenaar and Grimbergen (1996). Note that for media in which the medium parameters do not vary in the depth (x_3) direction, the reflection and transmission operators vanish and hence equations (A-1) and (A-2) decouple.