RECIPROCITY THEOREMS FOR SEISMOELECTRIC WAVES

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ABSTRACT

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We derive reciprocity theorems of the convolution type and of the correlation type for seismoelectric waves in dissipative inhomogeneous anisotropic fluid-saturated porous solids. They find their applications in forward and inverse seismoelectric wave problems, respectively. A special application of the reciprocity theorem of the convolution type leads to source-receiver reciprocity relations for a seismoelectric experiment. The reciprocity theorem of the correlation type yields as a special result the power balance for seismoelectric waves.

When the coupling tensor in the underlying equations vanishes, the reciprocity theorems decouple to theorems for decoupled elastodynamic and electromagnetic wave fields in porous solids.

KEY WORDS: reciprocity, seismoelectric, porosity, anisotropy, Biot theory.

INTRODUCTION

In this paper we derive reciprocity theorems for coupled elastodynamic and electromagnetic waves (also known as seismoelectric waves) in dissipative inhomogeneous anisotropic fluid-saturated porous solids. In general, a reciprocity theorem interrelates the quantities that characterize two admissible physical states that could occur in one and the same domain (de Hoop and Stam,

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1988). One can distinguish between convolution type and correlation type reciprocity theorems (Bojarski, 1983). Generally speaking, these two types of reciprocity theorems find their applications in forward and inverse problems, respectively. An extensive overview of reciprocity and its applications in seismic exploration is given by Fokkema and van den Berg (1993).

For the theory of seismoelectric wave propagation, we lean upon the work of Pride (1994). The main results are summarized in the next section. A reciprocity theorem of the convolution type for seismoelectric waves has previously been formulated by Pride and Haartsen (1996). Their reciprocity theorem interrelates two seismoelectric states in one and the same dissipative inhomogeneous anisotropic porous solid. In this paper we generalize this by considering different medium parameters in the two states [analogous to de Hoop (1987) for electromagnetic waves and de Hoop and Stam (1988) for elastodynamic waves in dissipative solids]. Hence, we obtain an extra integral containing the contrast parameters (i.e., the difference between the medium parameters in both states). This is relevant for example for the derivation of wave field representations containing these contrast parameters [like the Neumann series expansion, analogous as discussed by Fokkema and van den Berg (1993) for acoustic waves]. Another extension in this paper compared with the work of Pride and Haartsen (1996) is the derivation of a reciprocity theorem of the correlation type for seismoelectric waves, again for the situation of different medium parameters in both states. This is relevant for the derivation of inverse algorithms [analogous to de Hoop and Stam (1988) and Wapenaar and Haimé (1990) for elastodynamic waves].

BASIC EQUATIONS IN THE FREQUENCY DOMAIN

Reciprocity theorems can be derived in the time domain, the Laplace domain and the frequency domain. In this paper we only consider the frequency domain.

In this section we review the space-frequency domain equations for seismoelectric waves in a dissipative inhomogeneous anisotropic fluid-saturated porous solid. The Cartesian coordinate vector will be denoted as $\mathbf{x} = (x_1, x_2, x_3)$ and time as t. We use a subscript notation for vectorial and tensorial quantities and Einstein's summation convention applies to repeated subscripts.

We define the Fourier transform with respect to time of a real function as

$$\hat{u}(\omega) = \int_{-\infty}^{\infty} u(t)\exp(-j\omega t)dt \quad , \tag{1}$$

and its inverse as

$$u(t) = (1/\pi)\Re[\int_0^\infty \hat{u}(\omega)\exp(j\omega t)d\omega] \quad , \tag{2}$$

where j is the imaginary unit, ω the angular frequency and $\Re$ denotes that the real part is taken.

The linearised equations of motion for elastodynamic waves coupled to electromagnetic waves in a dissipative anisotropic fluid-saturated porous solid read in the space-frequency domain

$$j\omega\hat{\rho}^b_{ij}\hat{v}^s_j + \quad j\omega\hat{\rho}^f_{ij}\hat{w}_j - \partial_j\hat{\tau}^b_{ij} = \hat{f}^b_i \quad , \tag{3}$$

$$j\omega\hat{\rho}^f_{jl}\hat{v}^s_l + \quad j\omega\hat{\rho}^E_{jl}(\hat{w}_l - \hat{L}_{lk}\hat{E}_k) + \partial_j\hat{p} = \hat{f}^f_j \quad , \tag{4}$$

with

$$\hat{w}_j = \phi(\hat{v}^f_j - \hat{v}^s_j) \quad . \tag{5}$$

Here $\hat{v}^s_j = \hat{v}^s_j(\mathbf{x},\omega)$ and $\hat{v}^f_j = \hat{v}^f_j(\mathbf{x},\omega)$ are the averaged solid and fluid particle velocities associated to the wave motion, $\hat{w}_j = \hat{w}_j(\mathbf{x},\omega)$ is the filtration velocity, $\phi = \phi(\mathbf{x})$ the porosity, $\hat{\tau}^b_{ij} = \hat{\tau}^b_{ij}(\mathbf{x},\omega)$ the averaged bulk stress, $\hat{p} = \hat{p}(\mathbf{x},\omega)$ the averaged fluid pressure and $\hat{E}_k = \hat{E}_k(\mathbf{x},\omega)$ the averaged electric field strength. The source functions $\hat{f}^b_i = \hat{f}^b_i(\mathbf{x},\omega)$ and $\hat{f}^f_j = \hat{f}^f_j(\mathbf{x},\omega)$ are the volume densities of external force on the bulk and on the fluid, respectively. The constitutive parameters $\hat{\rho}^b_{ij} = \hat{\rho}^b_{ij}(\mathbf{x},\omega)$ and $\hat{\rho}^f_{ij} = \hat{\rho}^f_{ij}(\mathbf{x},\omega)$ are the bulk and fluid mass densities, respectively. They are complex and frequency-dependent to account for anelastic losses (de Hoop, 1995). Moreover, they are defined as anisotropic tensors, which follows from effective medium theory (Schoenberg and Sen, 1983). In the following we assume that these tensors are symmetric, according to $\hat{\rho}^b_{ij} = \hat{\rho}^b_{ji}$ and $\hat{\rho}^f_{ij} = \hat{\rho}^f_{ji}$ (this is for example the case when the effective anisotropy is a result of parallel fine layering at a scale much smaller than the wavelength). For isotropic media these parameters reduce to $\hat{\rho}^b_{ij} = \hat{\rho}^b\delta_{ij}$ and $\hat{\rho}^f_{ij} = \hat{\rho}^f\delta_{ij}$. The tensor $\hat{\rho}^E_{jl} = \hat{\rho}^E_{jl}(\mathbf{x},\omega)$ is a complex frequency-dependent inertia tensor for the relative fluid motion, obeying the symmetry relation $\hat{\rho}^E_{jl} = \hat{\rho}^E_{lj}$. For an isotropic medium it reduces to $\hat{\rho}^E_{jl} = \hat{\rho}^E\delta_{jl}$, with $\hat{\rho}^E = \eta/j\omega\hat{k}$, where $\eta = \eta(\mathbf{x})$ is the fluid viscosity parameter and $\hat{k} = \hat{k}(\mathbf{x},\omega)$ the dynamic permeability. Finally, the complex frequency-dependent function $\hat{L}_{lk} = \hat{L}_{lk}(\mathbf{x},\omega)$ accounts for the coupling between the elastodynamic and electromagnetic waves. In the following we will assume that this tensor is symmetric, according to $\hat{L}_{lk} = \hat{L}_{kl}$ [Pride and Haartsen (1996) discuss the conditions for this symmetry]. It reduces to $\hat{L}_{lk} = \hat{L}\delta_{lk}$ in isotropic media.

The linearised stress-strain relations read

$$-j\omega\hat{\tau}^{b}_{ij} + \hat{c}_{ijkl}\partial_l\hat{v}^{s}_{k} + \hat{C}_{ij}\partial_k\hat{w}_k = 0 \quad , \tag{6}$$

$$j\omega\hat{p} + \hat{C}_{kl}\partial_l\hat{v}^{s}_{k} + \hat{M}\partial_k\hat{w}_k = 0 \quad , \tag{7}$$

where $\hat{c}_{ijkl} = \hat{c}_{ijkl}(\mathbf{x},\omega)$, $\hat{C}_{ij} = \hat{C}_{ij}(\mathbf{x},\omega)$ and $\hat{M} = \hat{M}(\mathbf{x},\omega)$ are the complex frequency-dependent anisotropic stiffness parameters of the porous solid. Their symmetry properties are $\hat{c}_{ijkl} = \hat{c}_{jikl} = \hat{c}_{ijlk} = \hat{c}_{klij}$ and $\hat{C}_{ij} = \hat{C}_{ji}$. For isotropic porous solids these tensors read

$$\hat{c}_{ijkl} = [\hat{K}_G - (2/3)\hat{G}_{fr}]\delta_{ij}\delta_{kl} + \hat{G}_{fr}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \quad , \tag{8}$$

and $\hat{C}_{ij} = \hat{C}\delta_{ij}$. The isotropic parameters $\hat{K}_G$, $\hat{G}_{fr}$, $\hat{C}$ and $\hat{M}$ can be expressed in terms of Biot's parameters $\hat{N}$, $\hat{A}$, $\hat{Q}$ and $\hat{R}$ (Biot and Willis, 1957; Walton and Digby, 1987), according to

$$\hat{K}_G - (2/3)\hat{G}_{fr} = \hat{A} + 2\hat{Q} + \hat{R}, \quad \hat{G}_{fr} = \hat{N},$$

$$\hat{C} = (\hat{Q} + \hat{R})/\phi, \quad \hat{M} = \hat{R}/\phi^2. \tag{9}$$

Maxwell's electromagnetic field equations read

$$j\omega\hat{D}_i + \hat{J}_i - \epsilon_{ijk}\partial_j\hat{H}_k = -\hat{J}^{e}_{i} \quad , \tag{10}$$

$$j\omega\hat{B}_k + \epsilon_{kji}\partial_j\hat{E}_i = -\hat{J}^{m}_{k} \quad , \tag{11}$$

where $\hat{H}_k = \hat{H}_k(\mathbf{x},\omega)$ is the averaged magnetic field strength, $\hat{D}_i = \hat{D}_i(\mathbf{x},\omega)$ and $\hat{B}_k = \hat{B}_k(\mathbf{x},\omega)$ are the averaged electric and magnetic flux density, $\hat{J}_i = \hat{J}_i(\mathbf{x},\omega)$ is the averaged induced electric current density, $\hat{J}^{e}_{i} = \hat{J}^{e}_{i}(\mathbf{x},\omega)$ and $\hat{J}^{m}_{k} = \hat{J}^{m}_{k}(\mathbf{x},\omega)$ are source functions in terms of external electric and magnetic current densities and, finally, ϵ_{ijk} is the alternating tensor ($\epsilon_{123} = \epsilon_{312} = \epsilon_{231} = -\epsilon_{213} = -\epsilon_{321} = -\epsilon_{132} = 1$; $\epsilon_{ijk} = 0$ when not all subscripts i,j,k are different). Note that $\epsilon_{ijk}\partial_j\hat{H}_k$ is the subscript representation of $\nabla \times \hat{\mathbf{H}}$, etc.

According to Pride (1994), the constitutive relations are given by

$$\hat{D}_i = \hat{\epsilon}_{ik}\hat{E}_k \quad , \tag{12}$$

$$\hat{B}_k = \hat{\mu}_{kj}\hat{H}_j \quad , \tag{13}$$

$$\hat{J}_i = \hat{\sigma}_{ik}\hat{E}_k - \hat{L}_{ij}(\partial_j\hat{p} + j\omega\hat{\rho}^{f}_{jl}\hat{v}^{s}_{l} - \hat{f}^{f}_{j}) \quad , \tag{14}$$

where $\hat{\epsilon}_{ik} = \hat{\epsilon}_{ik}(\mathbf{x},\omega)$, $\hat{\mu}_{kj} = \hat{\mu}_{kj}(\mathbf{x},\omega)$ and $\hat{\sigma}_{ik} = \hat{\sigma}_{ik}(\mathbf{x},\omega)$ are the permittivity, permeability and conductivity, respectively. In general these are complex

frequency-dependent anisotropic tensors, with symmetry relations $\hat{\epsilon}_{ik} = \hat{\epsilon}_{ki}$, $\hat{\mu}_{kj} = \hat{\mu}_{jk}$ and $\hat{\sigma}_{ik} = \hat{\sigma}_{ki}$. For isotropic media they reduce to $\hat{\epsilon}_{ik} = \epsilon_0\hat{\epsilon}_r\delta_{ik}$, $\hat{\mu}_{kj} = \mu_0\hat{\mu}_r\delta_{kj}$ and $\hat{\sigma}_{ik} = \hat{\sigma}\delta_{ik}$. The subscripts 0 refer to the parameters in vacuum and the subscripts r denote relative parameters. Pride (1994) gives for $\hat{\epsilon}_r$ and $\hat{\mu}_r$

$$\hat{\epsilon}_r = (\phi/\alpha_\infty)(\hat{\kappa}^f - \hat{\kappa}^s) + \hat{\kappa}^s \ , \tag{15}$$

$$\hat{\mu}_r \approx 1 \ , \tag{16}$$

where $\hat{\kappa}^f$ and $\hat{\kappa}^s$ are the dielectric parameters of the fluid and solid, respectively, and α_∞ is the tortuosity at infinite frequency. Substituting the constitutive relations (12), (13) and (14) into the Maxwell equations (10) and (11), and adding $\hat{L}_{i\,j}$ times equation (4) to equation (10) in order to compensate for the term $-\hat{L}_{i\,j}(\partial_j\hat{p} + j\omega\hat{\rho}^f_{jl}\hat{v}^s_l - \hat{f}^f_j)$, yields

$$j\omega\hat{E}_{ik}\hat{E}_k + j\omega\hat{L}_{i\,j}\hat{\rho}^E_{j\,l}\hat{w}_l - \epsilon_{ijk}\partial_j\hat{H}_k = -\hat{J}^e_i \ , \tag{17}$$

$$j\omega\hat{\mu}_{kj}\hat{H}_j + \epsilon_{kji}\partial_j\hat{E}_i = -\hat{J}^m_k \ , \tag{18}$$

with

$$\hat{E}_{ik} = \hat{\epsilon}_{ik} + (\hat{\sigma}_{ik}/j\omega) - \hat{L}_{i\,j}\hat{\rho}^E_{j\,l}\hat{L}_{l\,k} \ . \tag{19}$$

The system of equations (3), (4), (6), (7), (17) and (18) governs the coupled propagation of elastodynamic and electromagnetic waves in dissipative anisotropic fluid-saturated porous solids.

At any position in space where the medium parameters are discontinuous, the wave quantities should obey boundary conditions. For interfaces between porous media one can distinguish between open, partially open and closed pores (Gurevich and Schoenberg, 1999). Here we consider the situation with open pores (Deresiewicz and Skalak, 1963). At an interface with normal vector $\mathbf{n} = (n_1,n_2,n_3)$ the open pore boundary conditions require continuity of $\hat{\tau}^b_{ij}n_j$, $\hat{p}$, $\hat{v}^s_i$, $\hat{w}_jn_j$, $\epsilon_{kji}\hat{E}_in_j$ and $\epsilon_{ijk}\hat{H}_kn_j$.

RECIPROCITY THEOREM OF THE CONVOLUTION TYPE

In this section we derive a reciprocity theorem that interrelates two seismoelectric states in one and the same domain D with boundary ∂D and outward pointing normal vector $\mathbf{n}$, see Fig. 1. The two seismoelectric states (i.e., wave fields, medium parameters and source functions) will be distinguished by the subscripts A and B. We assume that the medium parameters in states A and B are piecewise continuous in D and that the open pore boundary conditions hold at interfaces in D.

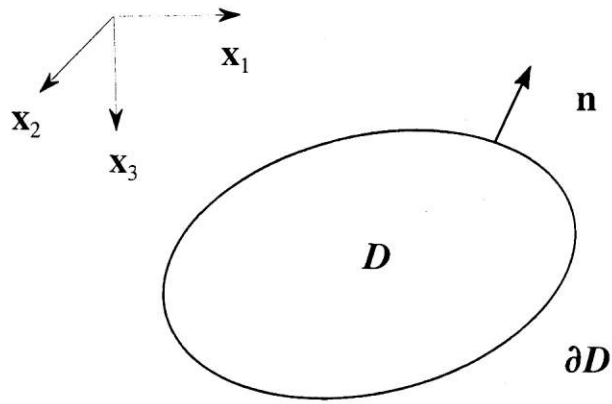

Fig. 1. Configuration for the reciprocity theorems.

We consider the interaction quantity $\partial_j[\epsilon_{ijk}\hat{E}_{i,A}\hat{H}_{k,B} - \epsilon_{ijk}\hat{H}_{k,A}\hat{E}_{i,B} - \hat{v}^s_{i,A}\hat{\tau}^b_{ij,B} + \hat{\tau}^b_{ij,A}\hat{v}^s_{i,B} + \hat{w}_{j,A}\hat{p}_B - \hat{p}_A\hat{w}_{j,B}]$. Applying the product rule for differentiation, substituting equations (3), (4), (6), (7), (17) and (18) for states A and B, using the symmetry relations for the medium parameters and the alternating tensor ϵ_{ijk}, integrating the result over D and applying the theorem of Gauss yields

$$\begin{aligned}
&\oint_{\partial D} [\epsilon_{ijk}\hat{E}_{i,A}\hat{H}_{k,B} - \epsilon_{ijk}\hat{H}_{k,A}\hat{E}_{i,B} - \hat{v}^s_{i,A}\hat{\tau}^b_{ij,B} + \hat{\tau}^b_{ij,A}\hat{v}^s_{i,B} + \hat{w}_{j,A}\hat{p}_B - \hat{p}_A\hat{w}_{j,B}]n_j d^2\mathbf{x} \\
&\quad = j\omega \int_D [(\hat{\mu}_{jk,A} - \hat{\mu}_{jk,B})\hat{H}_{j,A}\hat{H}_{k,B} - (\hat{E}_{ki,A} - \hat{E}_{ki,B})\hat{E}_{k,A}\hat{E}_{i,B} \\
&\quad - (\hat{L}_{ij,A}\hat{\rho}^E_{jl,A} - \hat{L}_{ij,B}\hat{\rho}^E_{jl,B})(\hat{w}_{l,A}\hat{E}_{i,B} + \hat{E}_{i,A}\hat{w}_{l,B}) + (\hat{\rho}^b_{ij,A} - \hat{\rho}^b_{ij,B})\hat{v}^s_{i,A}\hat{v}^s_{j,B} \\
&\quad + (\hat{\rho}^f_{ij,A} - \hat{\rho}^f_{ij,B})(\hat{w}_{i,A}\hat{v}^s_{j,B} + \hat{v}^s_{i,A}\hat{w}_{j,B}) + (\hat{\rho}^E_{jl,A} - \hat{\rho}^E_{jl,B})\hat{w}_{j,A}\hat{w}_{l,B}]d^3\mathbf{x} \\
&\quad + (1/j\omega) \int_D [(\hat{c}_{ijkl,A} - \hat{c}_{ijkl,B})\partial_j\hat{v}^s_{i,A}\partial_l\hat{v}^s_{k,B} + (\hat{M}_A - \hat{M}_B)\partial_j\hat{w}_{j,A}\partial_k\hat{w}_{k,B} \\
&\quad + (\hat{C}_{ij,A} - \hat{C}_{ij,B})(\partial_k\hat{w}_{k,A}\partial_i\hat{v}^s_{j,B} + \partial_j\hat{v}^s_{i,A}\partial_k\hat{w}_{k,B})]d^3\mathbf{x} \\
&\quad + \int_D [\hat{J}^m_{k,A}\hat{H}_{k,B} - \hat{H}_{k,A}\hat{J}^m_{k,B} - \hat{J}^e_{i,A}\hat{E}_{i,B} + \hat{E}_{i,A}\hat{J}^e_{i,B} \\
&\quad - \hat{f}^b_{i,A}\hat{v}^s_{i,B} + \hat{v}^s_{i,A}\hat{f}^b_{i,B} - \hat{f}^f_{j,A}\hat{w}_{j,B} + \hat{w}_{j,A}\hat{f}^f_{j,B}]d^3\mathbf{x} \quad . \qquad (20)
\end{aligned}$$

This is the reciprocity theorem of the convolution type for seismoelectric waves (we speak of convolution type since the products in the frequency domain ($\hat{E}_{i,A}\hat{H}_{k,B}$ etc.) correspond to convolutions in the time domain). It has the usual form of a boundary integral containing convolutions of wave fields, domain integrals containing the contrast functions of the medium parameters and a domain integral containing the source functions. If one of the states is a Green's state (i.e., the wave field of a point source in a background medium) and the other state is the actual wave field, then equation (20) yields a representation for the actual wave field in terms of a boundary and a domain integral, similar as for uncoupled elastodynamic or electromagnetic wave fields.

We conclude this section by considering some special situations. When the medium parameters in both states are identical in D, then the domain integrals containing the contrast parameters vanish. This leaves

$$\oint_{\partial D} [\epsilon_{ijk}\hat{E}_{i,A}\hat{H}_{k,B} - \epsilon_{ijk}\hat{H}_{k,A}\hat{E}_{i,B} - \hat{v}^s_{i,A}\hat{\tau}^b_{ij,B} + \hat{\tau}^b_{ij,A}\hat{v}^s_{i,B} + \hat{w}_{j,A}\hat{p}_B - \hat{p}_A\hat{w}_{j,B}]n_j d^2\mathbf{x}$$
$$= \int_D [\hat{J}^m_{k,A}\hat{H}_{k,B} - \hat{H}_{k,A}\hat{J}^m_{k,B} - \hat{J}^e_{i,A}\hat{E}_{i,B} + \hat{E}_{i,A}\hat{J}^e_{i,B}$$
$$- \hat{f}^b_{i,A}\hat{v}^s_{i,B} + \hat{v}^s_{i,A}\hat{f}^b_{i,B} - \hat{f}^f_{j,A}\hat{w}_{j,B} + \hat{w}_{j,A}\hat{f}^f_{j,B}]d^3\mathbf{x} \quad . \tag{21}$$

This is the reciprocity theorem that was previously derived by Pride and Haartsen (1996).

In addition to the condition of identical medium parameters in both states, consider the situation in which the medium at and outside ∂D is an unbounded homogeneous isotropic lossless solid and assume that the wave fields in both states are causally related to the sources in D. In this case the elastodynamic and electromagnetic components of the wave field outside ∂D have the same asymptotic behaviour as in the equivalent uncoupled situation. Hence, for this situation the boundary integral on the left-hand side of equation (21) vanishes (Pao and Varatharajulu, 1976; de Hoop, 1995).

From the remaining integral it is straightforward to derive source-receiver reciprocity properties. For example, let the source in state A be an electric current oriented in the x_m-direction at $\mathbf{x}_A \in D$, according to $\hat{J}^e_{i,A}(\mathbf{x},\omega) = \hat{s}_A(\omega)\delta(\mathbf{x}-\mathbf{x}_A)\delta_{im}$, and let the source in state B be a force oriented in the x_n-direction at $\mathbf{x}_B \in D$, according to $\hat{f}^b_{i,B}(\mathbf{x},\omega) = \hat{f}^f_{i,B}(\mathbf{x},\omega) = \hat{s}_B(\omega)\delta(\mathbf{x}-\mathbf{x}_B)\delta_{in}$. Assuming all other sources are zero, we thus obtain

$$\hat{E}_{m,B}(\mathbf{x}_A,\omega)/\hat{s}_B(\omega) = [\hat{v}^s_{n,A}(\mathbf{x}_B,\omega) + \hat{w}_{n,A}(\mathbf{x}_B,\omega)]/\hat{s}_A(\omega) \quad . \tag{22}$$

The left-hand side of this equation is the electric field strength in the x_m-direction at $\mathbf{x}_A$, due to a force oriented in the x_n-direction at $\mathbf{x}_B$, divided by the source spectrum of this force; the right-hand side is the velocity in the x_n-direction at $\mathbf{x}_B$, due to an electric current oriented in the x_m-direction at $\mathbf{x}_A$, divided by the source spectrum of this current. The reciprocity properties for other source-receiver combinations can be derived in a similar way.

RECIPROCITY THEOREM OF THE CORRELATION TYPE

In this section we derive a second reciprocity theorem for seismoelectric waves. This time we consider the interaction quantity $\partial_j[-\epsilon_{ijk}\hat{E}^*_{i,A}\hat{H}_{k,B} - \epsilon_{ijk}\hat{H}^*_{k,A}\hat{E}_{i,B} - \hat{v}^{s*}_{i,A}\hat{\tau}^b_{ij,B} - \hat{\tau}^{b*}_{ij,A}\hat{v}^s_{i,B} + \hat{w}^*_{j,A}\hat{p}_B + \hat{p}^*_A\hat{w}_{j,B}]$, where * denotes complex conjugation. Following the same procedure as in the previous section we obtain

$$
\begin{aligned}
\oint_{\partial D} & [-\epsilon_{ijk}\hat{E}^*_{i,A}\hat{H}_{k,B} - \epsilon_{ijk}\hat{H}^*_{k,A}\hat{E}_{i,B} - \hat{v}^{s*}_{i,A}\hat{\tau}^b_{ij,B} - \hat{\tau}^{b*}_{ij,A}\hat{v}^s_{i,B} + \hat{w}^*_{j,A}\hat{p}_B + \hat{p}^*_A\hat{w}_{j,B}]n_j d^2\mathbf{x} \\
&= j\omega \int_D [(\hat{\mu}^*_{jk,A} - \hat{\mu}_{jk,B})\hat{H}^*_{j,A}\hat{H}_{k,B} + (\hat{E}^*_{ki,A} - \hat{E}_{ki,B})\hat{E}^*_{k,A}\hat{E}_{i,B} \\
&\quad + (\hat{L}^*_{ij,A}\hat{\rho}^{E*}_{jl,A} + \hat{L}_{ijl,B}\hat{\rho}^E_{jl,B})(\hat{w}^*_{l,A}\hat{E}_{i,B} - \hat{E}^*_{i,A}\hat{w}_{l,B}) + (\hat{\rho}^{b*}_{ij,A} - \hat{\rho}^b_{ij,B})\hat{v}^{s*}_{i,A}\hat{v}^s_{j,B} \\
&\quad + (\hat{\rho}^{f*}_{ij,A} - \hat{\rho}^f_{ij,B})(\hat{w}^*_{i,A}\hat{v}^s_{j,B} + \hat{v}^{s*}_{i,A}\hat{w}_{j,B}) + (\hat{\rho}^{E*}_{jl,A} - \hat{\rho}^E_{jl,B})\hat{w}^*_{j,A}\hat{w}_{l,B}]d^3\mathbf{x} \\
&\quad + (1/j\omega) \int_D [(\hat{c}^*_{ijkl,A} - \hat{c}_{ijkl,B})\partial_j\hat{v}^{s*}_{i,A}\partial_l\hat{v}^s_{k,B} + (\hat{M}^*_A - \hat{M}_B)\partial_j\hat{w}^*_{j,A}\partial_k\hat{w}_{k,B} \\
&\quad + (\hat{C}^*_{ij,A} - \hat{C}_{ij,B})(\partial_k\hat{w}^*_{k,A}\partial_i\hat{v}^s_{j,B} + \partial_j\hat{v}^{s*}_{i,A}\partial_k\hat{w}_{k,B})]d^3\mathbf{x} \\
&\quad + \int_D [-\hat{J}^{m*}_{k,A}\hat{H}_{k,B} - \hat{H}^*_{k,A}\hat{J}^m_{k,B} - \hat{J}^{e*}_{i,A}\hat{E}_{i,B} - \hat{E}^*_{i,A}\hat{J}^e_{i,B} \\
&\quad\quad + \hat{f}^{b*}_{i,A}\hat{v}^s_{i,B} + \hat{v}^{s*}_{i,A}\hat{f}^b_{i,B} + \hat{f}^{f*}_{j,A}\hat{w}_{j,B} + \hat{w}^*_{j,A}\hat{f}^f_{j,B}]d^3\mathbf{x} \quad . \qquad (23)
\end{aligned}
$$

This is the reciprocity theorem of the correlation type for seismoelectric waves (we speak of correlation type since the products in the frequency domain ($\hat{E}^*_{i,A}\hat{H}_{k,B}$ etc.) correspond to correlations in the time domain). Its applications are found in inverse problems, similar as for the correlation type reciprocity theorems for uncoupled elastodynamic or electromagnetic wave fields (de Hoop and Stam, 1988; Wapenaar and Haimé, 1990).

We conclude this section by analysing equation (23) for the situation in which both states (i.e., wave fields, medium parameters and source functions) are identical. Omitting the subscripts A and B, we thus obtain

$$\Re \int_D [-\hat{J}_k^{m*}\hat{H}_k - \hat{J}_i^{e*}\hat{E}_i + \hat{f}_i^{b*}\hat{v}_i^s + \hat{f}_j^{f*}\hat{w}_j]d^3\mathbf{x}$$

$$= \Re \oint_{\partial D} [\epsilon_{jik}\hat{E}_i^*\hat{H}_k - \hat{\tau}_{ij}^{b*}\hat{v}_i^s + \hat{p}^*\hat{w}_j]n_j d^2\mathbf{x}$$

$$+ \omega \int_D [-\Im(\hat{\mu}_{jk})\hat{H}_j^*\hat{H}_k - \Im(\hat{E}_{ki})\hat{E}_k^*\hat{E}_i + 2\Re(\hat{L}_{ij}\hat{\rho}_{jl}^E)\Im(\hat{w}_l^*\hat{E}_i) - \Im(\hat{\rho}_{jl}^E)\hat{w}_j^*\hat{w}_l$$

$$- \Im(\hat{\rho}_{ij}^b)\hat{v}_i^{s*}\hat{v}_j^s - 2\Im(\hat{\rho}_{ij}^f)\Re(\hat{w}_i^*\hat{v}_j^s)]d^3\mathbf{x}$$

$$+ (1/\omega) \int_D [\Im(\hat{c}_{ijkl})\partial_j\hat{v}_i^{s*}\partial_l\hat{v}_k^s + \Im(\hat{M})\partial_j\hat{w}_j^*\partial_k\hat{w}_k$$

$$+ 2\Im(\hat{C}_{ij})\Re(\partial_k\hat{w}_k^*\partial_i\hat{v}_j^s)]d^3\mathbf{x} \quad , \tag{24}$$

where $\Im$ denotes the imaginary part. The domain integral on the left-hand side represents the power, generated by the sources in D. The boundary integral on the right-hand side represents the power-flux propagating outward through ∂D and the domain integrals on the right-hand side represent the dissipated power in D. Hence, equation (24) is the power balance for seismoelectric waves in its global form for the domain D [Pride and Haartsen (1996) derived the power balance in its local form]. For this reason, equation (23) is also referred to as the power reciprocity theorem for seismoelectric waves.

CONCLUSIONS

We have derived reciprocity theorems of the convolution type and of the correlation type for coupled elastodynamic and electromagnetic waves (also known as seismoelectric waves) in dissipative inhomogeneous anisotropic fluid-saturated porous solids. In both theorems, which are formulated in the space-frequency domain, we assumed that the wave fields, medium parameters and source functions may be different in both states. Hence, both theorems are expressed in terms of a boundary integral containing products of wave fields, domain integrals containing the contrast functions of the medium parameters and a domain integral containing the source functions. When coupling tensor $\hat{L}_{lk}$ is taken equal to zero, the underlying system of equations decouples to the Biot equations for elastodynamic waves in fluid-saturated porous solids [equations (3) through (7) with $\hat{L}_{lk} = 0$] and Maxwell's equations for electromagnetic waves [equations (17) through (19) with $\hat{L}_{lk} = 0$]. As a consequence, the reciprocity theorems (20) and (23) decouple to reciprocity theorems for these decoupled wave fields. Moreover, when the tensors $\hat{\rho}_{ij}^f$, $\hat{\rho}_{jl}^E$, $\hat{C}_{ij}$ and $\hat{M}$ are taken equal to zero, the Biot equations reduce to the equation of motion [equation (3) with $\hat{\rho}_{ij}^f = 0$] and the stress-strain relation [equation (6) with $\hat{C}_{ij} = 0$] for elastodynamic waves in non-porous solids and equations (20) and (23) reduce analogously to reciprocity theorems for elastodynamic waves in non-porous solids.

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