

WAVE-TYPE SEPARATION IN THE SPACE-FREQUENCY DOMAIN APPLIED TO VSP DATA

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ABSTRACT

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The particle velocity detectors distributed along the VSP borehole register compressional (P) and shear (S) waves without distinction; for various reasons, we may be interested in separating these two wave-types. After a brief review of already existing separation methods, we present how P and S waves can be separated by means of convolutional operations. This method can be considered as complementary to those already existing.

KEY WORDS: vector waves, wave-field decomposition, space-frequency domain, convolution, P- and S-waves.

INTRODUCTION

Wave-type separation applied to VSP data has already been successfully illustrated by two methods, the first one working in the wavenumber-frequency domain and the second one in the space-frequency domain. The first method assumes the restrictive condition of a homogeneous medium all along the VSP borehole, but the wave-field may be arbitrarily complex. The second method assumes the less restrictive condition of a locally homogeneous medium (along a few detector positions), but the wave-field must be sufficiently simple at each depth level to be well approximated by a small number of plane waves (typically

four). We propose an intermediate method that makes use of convolution operators in the space-frequency domain. We also assume a locally homogeneous medium but we can handle complex wave-fields.

REVIEW OF EXISTING METHODS

The wave-field decomposition per wave-type cannot be done uniquely when only based on the apparent velocity of the waves along the VSP well. We have to take into account the wave polarization. Hence, to do a proper wave-field decomposition, we need multi-component data and for a correct incorporation of the apparent velocities we have to consider a multi-channel approach. Multi-channel, multi-component wave-field decomposition of VSP data has already been investigated by some authors. Devaney and Oristaglio (1986) and Dankbaar (1987) propose to apply the decomposition in the wavenumber-frequency domain. With their method, the data expressed in the frequency domain, are decomposed into waves of distinct vertical wave-number k_z by Fourier transform. Assuming the medium is homogeneous all along the VSP well, this operation corresponds to a plane wave decomposition (for the 2-D situation). Then, to each plane-wave constituent a linear filter (based on the wave-type polarization vectors) is applied in order to separate the compressional and shear waves. The decomposed plane waves can then be transformed back to the space domain. The advantage of this method is that no restriction is made on the wave-field complexity; its main disadvantage lies in the assumption that the medium is vertically homogeneous all over the VSP well. Leaney (1990) proposes to apply the wave-type separation in the space-frequency domain, using a parametric inversion of the data.

With this approach, the wave-field is fitted at each depth level to a small number of plane waves (e.g., four: two downgoing P and S waves and two upgoing P and S waves). Under the assumption that the medium is locally homogeneous along a few detector positions, the data fitting is based on the relations that exist in a homogeneous medium between the particle velocities and the amplitude and the direction of propagation of the incident plane waves. The advantage of this method is that the medium is considered to be only locally homogeneous instead of all along the VSP well.

An important and interesting byproduct of this method is the P and S wave velocity model and the main directions of propagation of the incident waves. The disadvantage of this method is the need for an interpretational step and the computational cost of the parametric inversion. Its cost increases with the data complexity (more plane waves have to be considered to fit the data properly). In the following we propose to decompose the wave-field by means of convolutional operations.

WAVE-FIELD DECOMPOSITION IN THE SPACE-FREQUENCY DOMAIN

The method that is proposed here is based on previous work of the authors on wave-field decomposition of surface seismic data (Wapenaar et al., 1990; Herrmann, 1992). For simplicity, we present the theory for two-component data and 2-D wave propagation. The extension to three components and 3-D propagation is discussed in the first author's thesis.

In a homogeneous isotropic medium, 2-D incident monochromatic plane waves of angular frequency ω and vertical wavenumber k_z generate a particle velocity field $\tilde{\mathbf{v}}(k_z, \omega)$ along the VSP borehole, according to:

$$\tilde{\mathbf{v}}(k_z, \omega) = \tilde{\mathbf{L}}_1^+(k_z, \omega) \tilde{\mathbf{p}}^+(k_z, \omega) + \tilde{\mathbf{L}}_1^-(k_z, \omega) \tilde{\mathbf{p}}^-(k_z, \omega) . \quad (1)$$

The vector $\tilde{\mathbf{v}}$ contains the horizontal and vertical particle velocity components. The vectors $\tilde{\mathbf{p}}^\pm$ contain the complex amplitude of the P and S waves of vertical wavenumber k_z , propagating in the source-borehole direction (+) and in the borehole-source direction (-). $\tilde{\mathbf{L}}_1^\pm$ is a two-by-two matrix whose first column corresponds to the polarization vector of the incident P plane wave (+ or -) and whose second column corresponds to the polarization vector of the incident S plane wave (+ or -). The $\tilde{\mathbf{p}}^+$ and $\tilde{\mathbf{p}}^-$ vectors cannot both be recovered from the particle velocity field $\tilde{\mathbf{v}}$ alone, hence we assume $\tilde{\mathbf{p}}^- = \mathbf{0}$. This assumption is realistic for two main reasons:

- Except in the case of a complicated subsurface, all the waves emitted by the source reach the detectors as $\tilde{\mathbf{p}}^+$ waves (especially when the source to borehole offset is large).
- Since the diameter of the well is small compared to the wavelength of the incident body waves, its effect on any incident wave-field $\tilde{\mathbf{p}}^+$ can be neglected. Here we assume that the particle velocities associated to the tube waves have already been removed from the total particle velocity field.

Under this assumption, the wave-field decomposition operator per wave-type reads $[\tilde{\mathbf{L}}_1^+(k_z, \omega)]^{-1}$. We have:

$$\tilde{\mathbf{p}}^+(k_z, \omega) = [\tilde{\mathbf{L}}_1^+(k_z, \omega)]^{-1} \tilde{\mathbf{v}}(k_z, \omega) , \quad (2a)$$

with

$$[\tilde{\mathbf{L}}_1^+(k_z, \omega)]^{-1} = \omega / (k_z^2 + k_{r,p} k_{r,s}) \begin{pmatrix} k_{r,s} & k_z \\ -k_z & k_{r,p} \end{pmatrix} \quad (2b)$$

and

$$k_{r,p}^2 = (\omega^2/c_p^2) - k_z^2 , \quad k_{r,s}^2 = (\omega^2/c_s^2) - k_z^2 , \quad (2c)$$

c_p and c_s being the P and S wave velocities. In theory, the wave-field recorded at a given depth level z_m along the VSP well may be composed of a large number of plane waves, see Fig. 1. In the space domain (z), we would then have to design long decomposition convolutional operators that are able to properly decompose any incident plane wave. These operators would be given by the inverse Fourier transform (k_z to z) of the elements of matrix \tilde{L}_1^+ . To reduce the convolutional operators length (i.e., to make them as local as possible, such that the condition of the homogeneous medium is required only over the decomposition operator aperture), we reduce the vertical wavenumber bandwidth over which the operator must be accurate in accordance with the bandwidth over which incident P and S waves are expected.

Due to the discontinuity of the derivative of the operator $k_{r,p}$ at $k_z = \omega/c_p$ (vertical P-wave propagation), the convolutional decomposition operator will be long if this spectral position is included in the seismic bandwidth over which the operator has to be accurate. To avoid this discontinuity, instead of using $k_{r,p}$ in expression (2b) we use $\langle k_{r,p} \rangle$, which is a smooth bandpath version of $k_{r,p}$ which fits $k_{r,p}$ over the k_z bandwidth corresponding to the expected incident upgoing and downgoing P plane waves. The use of $\langle k_{r,p} \rangle$ instead of $k_{r,p}$ has the advantage of reducing the operator length in the space domain. Of course, the evanescent P wave-field will not be correctly treated ($k_z \geq \omega/c_p$).

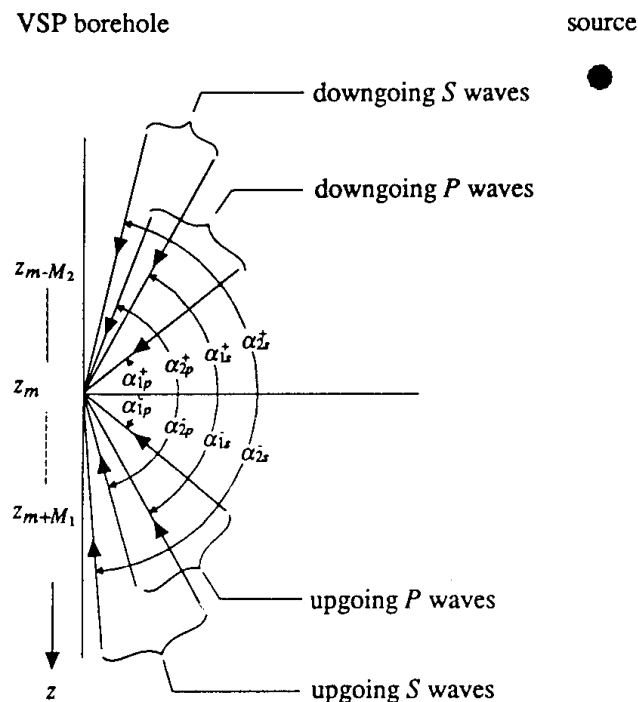


Fig. 1. Theoretically a lot of waves may reach depth level z_m along the VSP well. In order to reduce the length of the decomposition convolutional operator in the space domain, we reduce the domain over which the operator has to be accurate to the angle range in which the incident P and S plane waves are expected.

COMPUTATION OF THE CONVOLUTIONAL DECOMPOSITION OPERATOR

For one frequency component, we describe how the four convolutional decomposition operators may be computed. In the following $F(k_z, \omega)$ stands for one of the four elements of the decomposition operator. As in the space domain, the data are discretized; we consider a discretized convolutional operator of $M_2 + M_1 + 1$ complex points, contained in the vector $f(\omega)$:

$$f(\omega) = [f_{-M_2}(\omega), \dots, f_0(\omega), \dots, f_{M_1}(\omega)]^T, \quad (3a)$$

To determine the vector coefficients, we proceed as follows: a unit plane wave of vertical wavenumber k_z recorded at the $M_2 + M_1 + 1$ vertical borehole positions $z_{m-M_2}, \dots, z_m, \dots, z_{m+M_1}$, (Fig. 1), leads to $M_2 + M_1 + 1$ complex values $d(k_z, z_i, \omega)$, that can be written in a vector $d(k_z, \omega)$ according to:

$$d(k_z, \omega) = [d(k_z, z_{m-M_2}, \omega), \dots, d(k_z, z_m, \omega), \dots, d(k_z, z_{m+M_1}, \omega)]^T, \quad (3b)$$

with,

$$d(k_z, z_i, \omega) = e^{-jk_z(z_i - z_m)}$$

Now the operator vector $f(\omega)$ must be designed such that the vector product of the vector $f(\omega)$ with the data vector $d(k_z, \omega)$ leads to the filtered version of $d(k_z, z_m, \omega)$, so that:

$$d^T(k_z, \omega) f(\omega) = F(k_z, \omega), \quad (3c)$$

for any k_z in the specified band. In agreement with (3c) and with the requirements previously developed, the coefficients $f_i(\omega)$ are computed to minimize, in a least-squares sense, the function $S(\omega)$, defined as

$$S(\omega) = \sum_{k_z = -k_z \text{ Nyquist}}^{+k_z \text{ Nyquist}} W(k_z, \omega) e^*(k_z, \omega) e(k_z, \omega), \quad (3d)$$

(* stands for complex conjugate), with the error function e defined as:

$$e(k_z, \omega) = d^T(k_z, \omega) f(\omega) - F(k_z, \omega), \quad (3e)$$

and the weighting function $W(k_z, \omega)$ being a real positive function defined according to:

$$W(k_z, \omega) = 1 \text{ when } k_z \in B(\omega) ,$$

$$W(k_z, \omega) = \epsilon \text{ (} 0 < \epsilon < < 1 \text{) when } k_z \notin B(\omega) ,$$

with $B(\omega)$ defining the band where P and S plane waves are expected; hence

$$B(\omega) = [(\omega/c_p)\sin(\alpha_{1p}^+), (\omega/c_p)\sin(\alpha_{2p}^+)] \cup$$

$$[-(\omega/c_p)\sin(\alpha_{2p}^-), -(\omega/c_p)\sin(\alpha_{1p}^-)] \cup$$

$$[(\omega/c_s)\sin(\alpha_{1s}^+), (\omega/c_s)\sin(\alpha_{2s}^+)] \cup$$

$$[-(\omega/c_s)\sin(\alpha_{2s}^-), -(\omega/c_s)\sin(\alpha_{1s}^-)] , \quad (3f)$$

see Fig. 1 for the meaning of the angles α .

FIELD DATA EXAMPLE

We present an example of wave-field decomposition of real VSP data using the decomposition convolutional operators. The data, provided by the Compagnie Générale de Géophysique (CGG), have been shot in the Paris basin. The data were recorded with the following parameters:

detector spacing	: 10 m
first detector	: 220 m
borehole offset	: 650 m
sample interval	: 2 ms
frequency content	: 10-80 Hz

Figs. 2a,b represent the horizontal and vertical particle velocity components recorded along the well. We used 5 points convolutional operators for the wavefield decomposition. As can be seen on the original sections, P and S wave energy is recorded both on the horizontal and vertical particle velocity components. At small depths (220 - 600 m), the data are mainly composed of refracted horizontally propagating P and S waves. We can also notice the presence of upgoing reflected waves (with high angles of incidence) coming from the deeper layers.

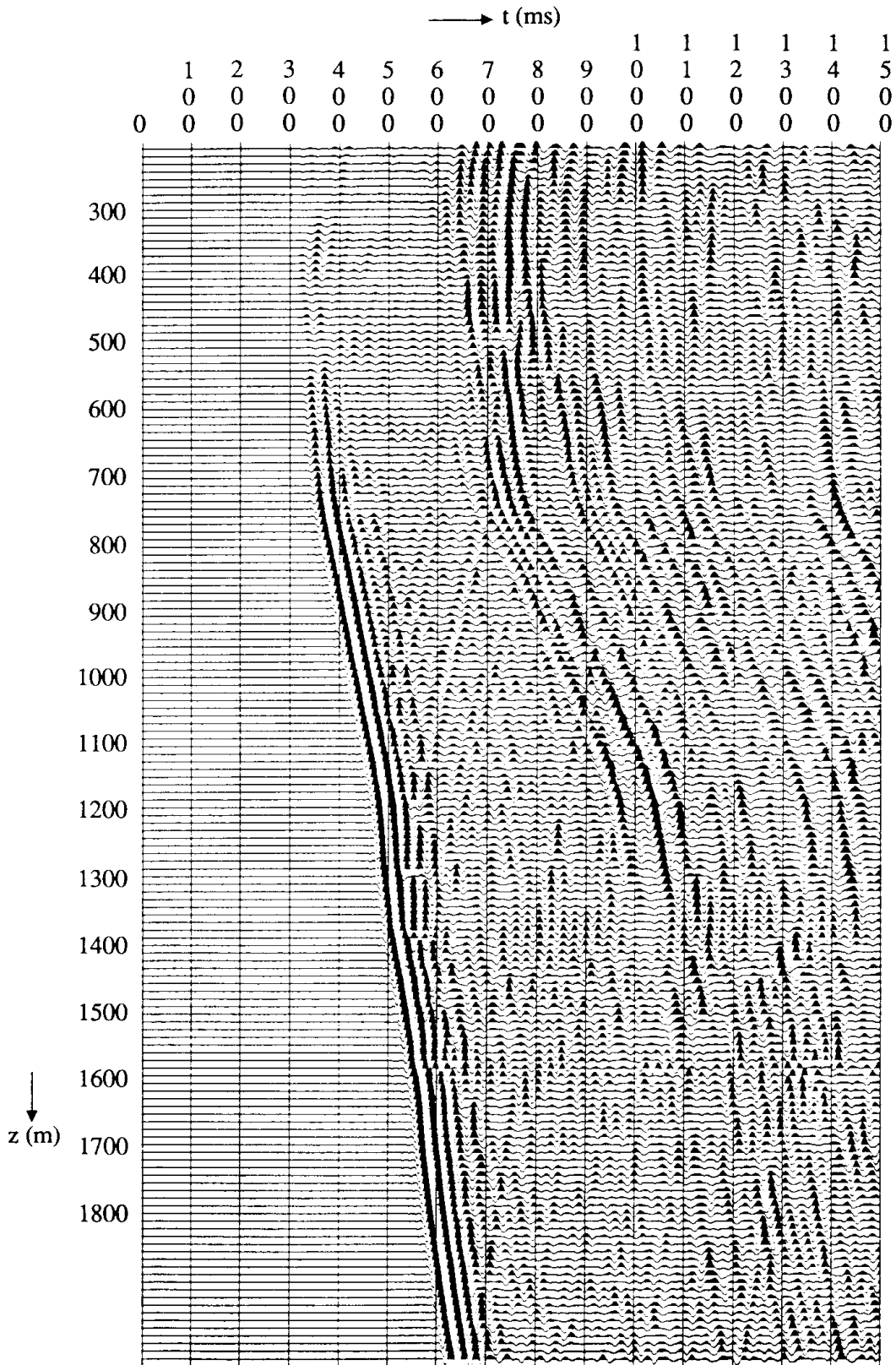


Fig. 2a. Vertical particle velocity component.

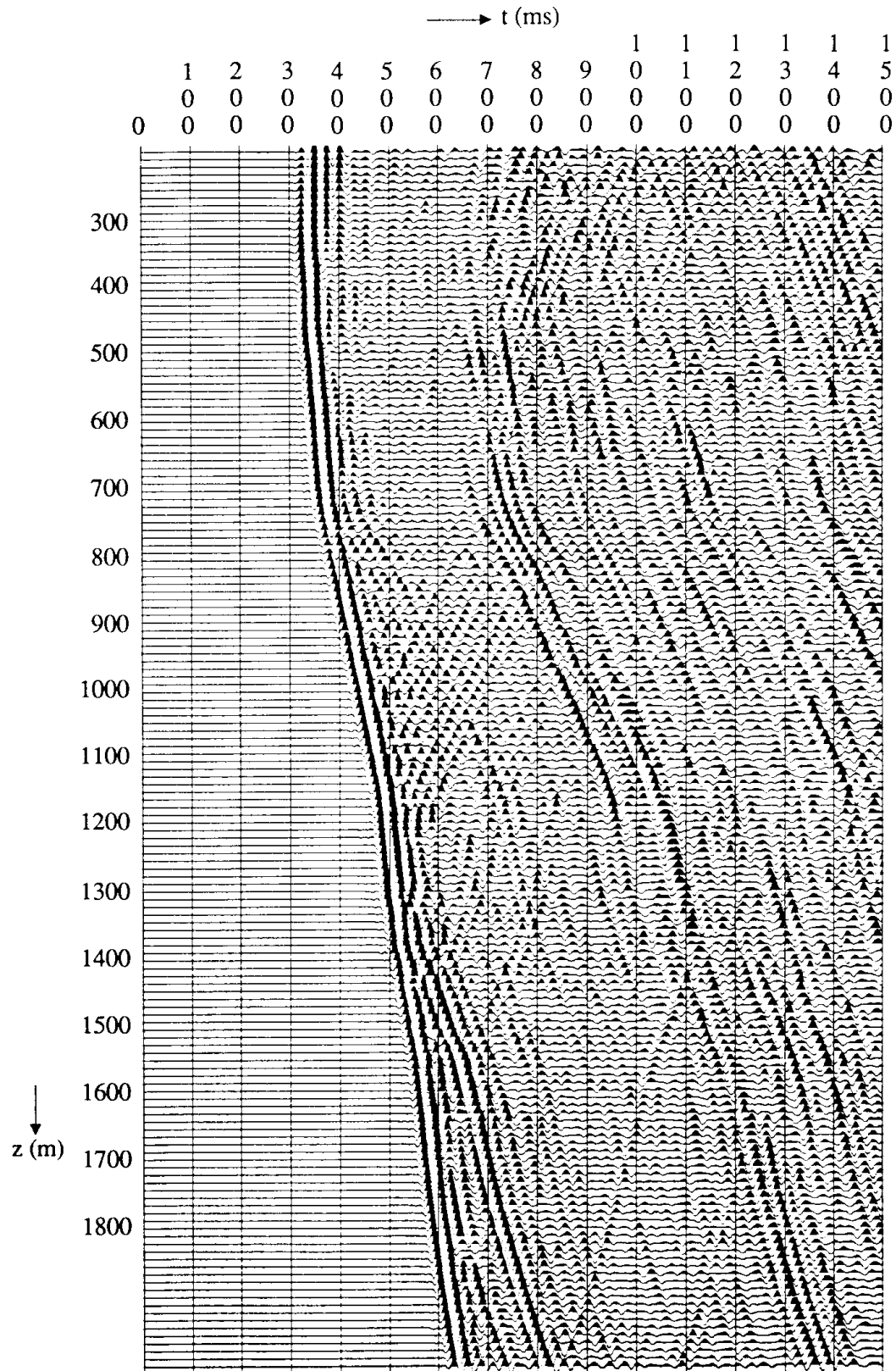


Fig. 2b. Horizontal particle velocity component.

This part of the data will enable us to test if our decomposition operators can correctly treat P and S waves with the same apparent vertical velocity. In this case, the separation is based only on the difference of polarization of the two wave-types. This part of the data is also interesting to check if we can treat a wide range of angles of incidence. From 1000 m depth a lot of wave reflections and conversions occur, because here the medium contains strong vertical velocity variations. It will enable us to test if the local decomposition operators can handle a complex medium. One can see from the horizontal and vertical particle velocity sections the effect of the sensitivity of the particle velocity detectors to the angle of incidence of the waves. It is interesting to check if these effects are removed on the decomposed sections. The result of the decomposition (Figs. 2c,d) is considered to be impressive. The following comments can be made: the strong downgoing direct P wave has been completely removed in the decomposed S section. The effect of the detector directivity has been removed and the continuity of the P and S waves is much better on the decomposed section than on the particle velocity sections. We can also note that all the dips have been correctly treated, and that the spatial resolution of the decomposed S section is high, even in the highly inhomogeneous part of the medium around 1000 m depth.

DISCUSSION

Including the VSP wave-type separation method presented in this study, we have three methods at our disposal. To conclude, we discuss their optimum domain of application.

The optimum domain of application of the method proposed by Devaney and Oristaglio (1986) and Dankbaar (1987) is the moderate and large offset VSP with the restriction of small vertical velocity variations along the VSP well.

The optimum domain of application of the method proposed by Leaney (1990) is the near and moderate offset VSP data. Looking at such data, we see at each depth level the wave-field can be well approximated with just four plane waves, two downgoing and two upgoing P and S waves, thus reducing the costs of parametric inversion. Another reason is that for such data with high angles of incidence, the wave-type separation is very sensitive to the choice of the P- and S-wave velocity model. It is then preferable to use this algorithm that does not need the velocity model as input but provides it as output.

The optimum domain of application of our method is the moderate and far offset VSP data. Looking at such data, we see that the wave-field is more complex meaning that it cannot always be properly modeled locally with a few plane waves. We have to consider a larger range of plane waves. Another reason is that such data have smaller angles of incidence than near-offset data;

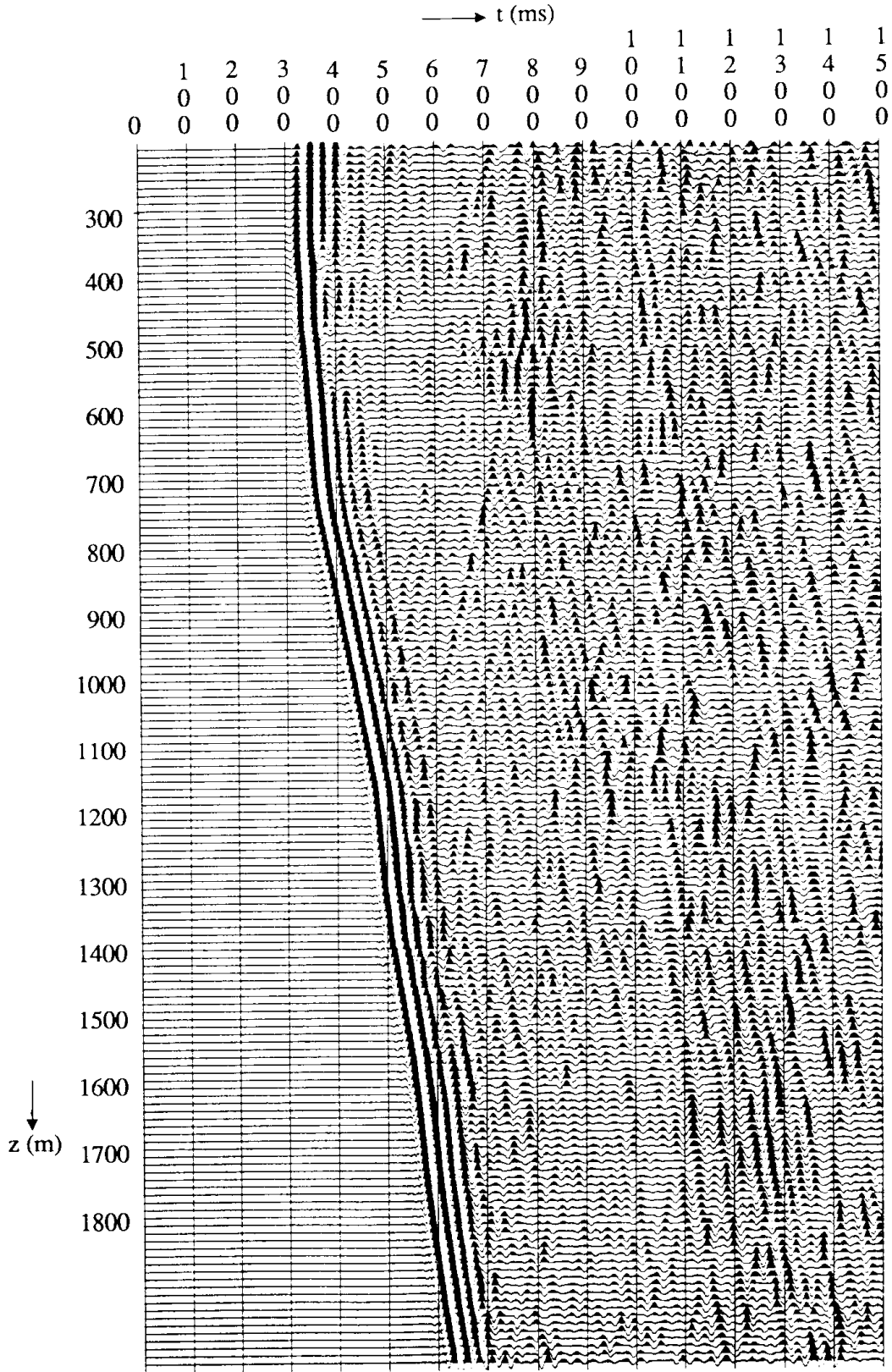


Fig. 2c. Decomposed P wave-field.

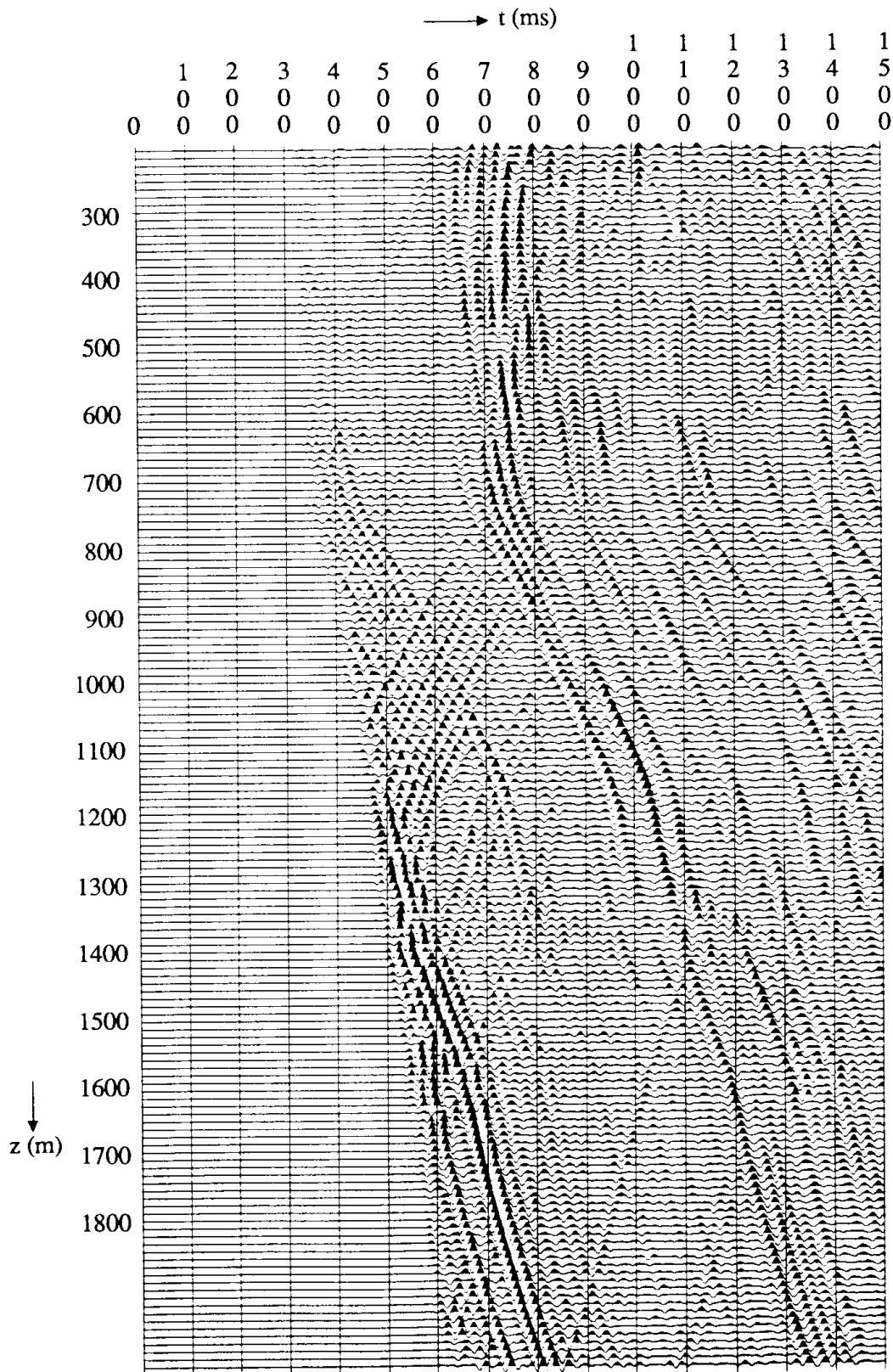


Fig. 2d. Decomposed S wave-field.

the sensitivity of the wave-type separation decreases for decreasing angles of incidence, hence the input P- and S-velocity model does not need to be as accurate as for near-offset data.

CONCLUSION

We have presented a VSP wave-field decomposition procedure that operates in the space-frequency domain and that is particularly suited for moderate to large offset VSP applications. The real data example indicates that the operators effectively separate P- and S-waves in a true amplitude sense, also under complex field conditions (many overlapping events, similar apparent velocities, fast-medium velocity variations).

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