TOWARDS AN EXTENDED MACRO MODEL, THAT TAKES FINE-LAYERING INTO ACCOUNT

C.P.A. WAPENAAR, R.E. SLOT and F.J. HERRMANN

Centre for Technical Geoscience, Laboratory of Seismics and Acoustics, Delft University of Technology, P.O. Box 5046, 2600 GA Delft, The Netherlands.

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ABSTRACT

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Current macro models ignore the effect of fine-layering. The main effect of fine-layering on wave propagation is an angle-dependent dispersion of the transmitted wave field. In this paper it is shown that for a 1-D finely layered medium these effects can be mimicked by letting the wave field propagate through a *homogeneous* anisotropic medium with anelastic losses. The match between the true and the mimicked response is good up to a propagation angle of 45 degrees. For structurally complex media an *extended* macro model is proposed, in which frequency-dependent (complex) phase velocities c_V and c_H are assigned to each macro layer.

KEY WORDS: macro model, fine-layering, dispersion, anisotropy, anelastic losses, angle-dependency.

INTRODUCTION

The main process in seismic migration is the elimination of propagation effects from the seismic data. Usually these propagation effects are quantified by a *macro model*, which contains the main geological boundaries in the subsurface and the *average* velocities (and densities) between these boundaries. The effects of fine-layering on wave propagation are ignored altogether.

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In the seismic literature (O'Doherty and Anstey, 1971; Burridge and Chang, 1989; Folstad and Schoenberg, 1992) as well as in our consortium project DELPHI (Herrmann and Wapenaar, 1992), a lot of attention is paid to the theory of wave propagation through finely layered media. These studies show that internal multiple scattering in a finely layered medium effectively results in an *angle-dependent dispersion* of the transmitted wave field. In this paper we investigate whether it is possible to replace a finely layered medium by a homogeneous anisotropic medium with anelastic losses in such a way that the transmission response of this replacement medium is effectively the same as the transmission response of the original finely layered medium. The ultimate aim is to generalize the results of this paper for the development of 2-D or 3-D 'extended macro models'. By choosing for each macro layer a homogeneous medium with the appropriate anisotropic anelastic losses, a wave propagating through an extended macro model inherits the same angle-dependent dispersion effects as a wave propagating through the true 2-D or 3-D inhomogeneous medium with fine-layering. An extended macro model is essential for 2-D or 3-D *true amplitude* migration (Wapenaar and Herrmann, 1994).

The set-up of this paper is as follows: first, wave propagation through finely layered media is briefly reviewed. Independently from this, wave propagation through a homogeneous anisotropic medium with anelastic losses is considered. The link between the two theories is made in the subsequent section, where the expressions for the transmission responses of both theories are expanded into a series. By matching the coefficients in both series expansions the extended macro model parameters are found. The theory is illustrated with some simple examples.

THE TRANSMISSION RESPONSE OF A FINELY LAYERED MEDIUM

We describe the transmission response of a finely layered medium in terms of wave-field extrapolation operators in the rayparameter-frequency (p,ω) domain. To be more specific, for the response of a stack of layers between depth levels z_0 and z_m we write

$$\tilde{W}_g^+(z_m,z_0;p,\omega) = \tilde{W}_p^+(z_m,z_0;p,\omega)\tilde{C}(z_m,z_0;p,\omega) \quad , \tag{1}$$

where $\tilde{W}_p^+(z_m,z_0;p,\omega)$ is the extrapolation operator for the *primary* wave, $\tilde{C}(z_m,z_0;p,\omega)$ is a *correction* operator that accounts for the angle-dependent dispersion effects due to the fine-layering and $\tilde{W}_g^+(z_m,z_0;p,\omega)$ is the extrapolation operator for the *generalized primary* wave field in the finely layered medium. We first analyze these operators for vertical propagation $(p = 0)$ and later for oblique propagation $(p \neq 0)$.

Vertical propagation through a finely layered medium

The primary extrapolation operator for vertical propagation reads

$$\tilde{W}_p^+(z_m,z_0;p=0,\omega) = \exp\{-j\omega <1/c> \Delta z\} \quad , \tag{2}$$

where $\Delta z = z_m - z_0$. The correction operator reads

$$\tilde{C}(z_m,z_0;p=0,\omega) = \exp\{-A(\omega)\Delta z\} \quad , \tag{3}$$

where

$$A(\omega) = (1/2)S(\omega) + (1/2)j\mathcal{H}\{S(\omega)\} \quad , \tag{4}$$

$\mathcal{H}$ denoting the Hilbert transform. In equation (2), $<1/c>$ denotes the spatial average slowness over the interval (z_0,z_m). In equation (4), $S(\omega)$ represents the Fourier transform of the auto-covariance of the reflectivity as a function of travel time (O'Doherty and Anstey, 1971). It has been observed from well logs that in many situations the statistics of the fine-layering are described by fractal Brownian motion (Walden and Hosken, 1985). As a consequence, for $S(\omega)$ in equation (4) we may write

$$S(\omega) = \nu|\omega|^\alpha \quad , \tag{5}$$

with $0 < \alpha < 1$ and

$$\mathcal{H}\{S(\omega)\} = \nu \tan(\alpha\pi/2)\mathrm{sign}(\omega)|\omega|^\alpha \quad , \tag{6}$$

(Herrmann, 1991). Note that the dispersion effects are now fully parameterized by the parameters ν and α. In analogy with equation (5), we may write for $A(\omega)$

$$A(\omega) = \mu|\omega|^\alpha \quad , \tag{7}$$

where

$$\mu = (1/2)\nu\{1 + j\tan(\alpha\pi/2)\ \mathrm{sign}(\omega)\} \quad . \tag{8}$$

Note that $A(\omega)$ has the following scaling property:

$$A(\beta\omega) = |\beta|^\alpha A(\omega) \quad , \tag{9}$$

where β is an arbitrary scaling factor.

Oblique propagation through a finely layered medium

The primary extrapolation operator for oblique propagation through a finely layered medium reads

$$\tilde{W}^{+}_{p}(z_m,z_0;p,\omega) = \exp\{-j\omega \sum_{i=1}^{m} (c_i^{-2} - p^2)^{1/2} \, \Delta z_i\} \quad , \tag{10}$$

where p is the rayparameter and i is the layer index. Using a series expansion of the square-root around p = 0, we easily find

$$\tilde{W}^{+}_{p}(z_m,z_0;p,\omega) \approx \exp[-j\omega \sum_{i=1}^{m} \{c_i^{-1} - (1/2)c_i p^2 + \ldots\}\Delta z_i]$$

$$= \exp[-j\omega\{<1/c> - (1/2)<c>p^2 + \ldots\}\Delta z] \quad , \tag{11}$$

where

$$<1/c> = (1/\Delta z) \sum_{i=1}^{m} c_i^{-1} \Delta z_i \quad , \tag{12a}$$

$$<c> = (1/\Delta z) \sum_{i=1}^{m} c_i \Delta z_i \quad , \tag{12b}$$

and

$$\Delta z = \sum_{i=1}^{m} \Delta z_i = z_m - z_0 \quad . \tag{12c}$$

$\tilde{W}^{+}_{p}(z_m,z_0;p,\omega)$, as given by equation (11), may be seen as a series expansion around p = 0 of the following expression

$$\tilde{W}^{+}_{p}(z_m,z_0;p,\omega) = \exp\{-j\omega \cos\phi_{eff} <1/c> \Delta z\} \quad , \tag{13a}$$

where the angle ϕ_{eff} is related to the rayparameter p via the *effective velocity* c_{eff} of the layered medium, according to

$$\cos\phi_{eff} = (1 - c_{eff}^2 p^2)^{1/2} \quad , \tag{13b}$$

where

$$c_{eff}^2 = <c>/<1/c> \quad . \tag{13c}$$

Comparing equation (13a) with equation (2), we observe that ω has been replaced by $\omega \cos\phi_{eff}$. This is easily understood, since the apparent vertical wavelength λ_z of an oblique plane wave propagating under an angle ϕ with the vertical axis is given by $\lambda_z = \lambda/\cos\phi$, where λ is the wavelength in the propagation direction, see Fig. 1.

Instead of giving a strict derivation of the correction operator $\tilde{C}(z_m,z_0;p,\omega)$, we directly introduce the apparent frequency $\omega \cos\phi_{eff}$ in the expression for $\tilde{C}(z_m,z_0;p=0,\omega)$. We consider two special cases:

- *Density contrasts only*

In this case, the reflectivity of each interface is *in*dependent of the propagation angle ϕ_{eff}: $r(\phi_{eff}) = r(0)$. As a consequence, in the 'apparent frequency domain' the Fourier transform of the auto-covariance of the reflectivity is also independent of the propagation angle. Hence, for the correction operator $\tilde{C}(z_m,z_0;p,\omega)$ we only need to replace ω in the right-hand side of equation (3) by $\omega\cos\phi_{eff}$, yielding

$$\tilde{C}(z_m,z_0;p,\omega) = \exp\{-A(\omega\cos\phi_{eff})\Delta z\} \quad . \tag{14}$$

- *Velocity contrasts only*

In this case, the reflectivity of each interface depends on the propagation angle ϕ_{eff}, according to $r(\phi_{eff}) \approx r(0)/\cos^2\phi_{eff}$ (Aki and Richards, 1980). As a consequence, in the 'apparent frequency domain' the Fourier transform of the auto-covariance of the reflectivity should be scaled by $(1/\cos^2\phi_{eff})^2$. Hence, for the correction operator we now obtain

$$\tilde{C}(z_m,z_0;p,\omega) = \exp\{-[A(\omega\cos\phi_{eff})\Delta z \,/\, (\cos^4\phi_{eff})]\} \quad . \tag{15}$$

Equations (14) and (15) may be seen as special cases of the more general expressions derived by Burridge and Chang (1989).

For the series expansion in a later section, it is essential that the ϕ_{eff}-dependency is removed from $A(\omega\cos\phi_{eff})$. This is easily accomplished by using the scaling property (9), which applies to fractal Brownian motion. This yields

$$\tilde{C}(z_m,z_0;p,\omega) = \exp\{-A(\omega)(\cos\phi_{eff})^{\alpha-n}\Delta z\} \quad , \tag{16}$$

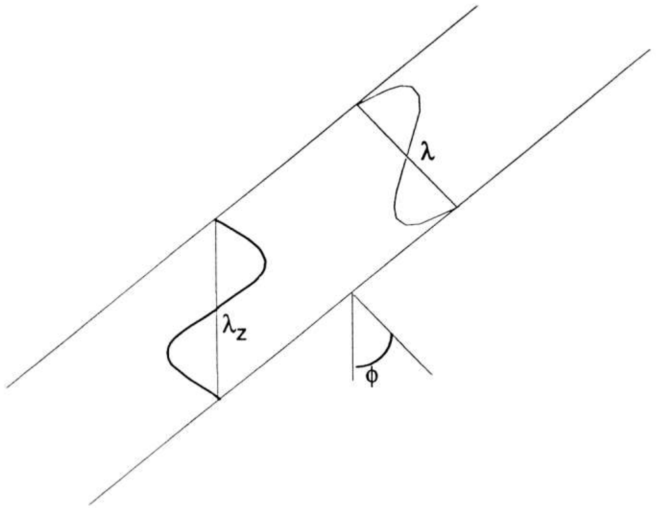

Fig. 1. The apparent vertical wavelength of an oblique plane wave is given by $\lambda_z = \lambda/\cos\phi$. Accordingly, the apparent frequency is given by $\omega\cos\phi$.

where

$$n = \begin{cases} 0 & \text{density contrasts only} \\ 4 & \text{velocity contrasts only} \quad . \end{cases} \tag{17}$$

A discussion for the more general situation of velocity as well as density contrasts is straightforward, but is beyond the scope of this paper.

THE TRANSMISSION RESPONSE OF A HOMOGENEOUS ANISOTROPIC MEDIUM WITH LOSSES

The basic equations for acoustic wave propagation in an anisotropic medium read

$$\nabla P = -j\omega \mathbf{M}\mathbf{V} \quad , \tag{18}$$

where $\mathbf{M}$ is a 3×3 mass density tensor, and

$$\nabla \cdot \mathbf{V} = -j\omega\varkappa P \quad , \tag{19}$$

where $\varkappa$ is the compressibility. When the medium is lossy, $\mathbf{M}$ and $\varkappa$ are complex and frequency-dependent, with the constraint that their inverse Fourier transforms are causal. Eliminating the particle velocity vector $\mathbf{V}$ from equations (18) and (19) yields the anisotropic wave equation for the acoustic pressure P, according to

$$\nabla \cdot (\mathbf{M}^{-1}\nabla P) + \varkappa\omega^2 P = 0 \quad . \tag{20}$$

In the following we assume that the medium is transversely isotropic (remember that we want to mimic the response of a horizontally layered medium). This means that for $\mathbf{M}$ we may write

$$\mathbf{M} = \begin{pmatrix} \varrho_H & 0 & 0 \\ 0 & \varrho_H & 0 \\ 0 & 0 & \varrho_V \end{pmatrix} \quad . \tag{21}$$

If we choose a homogeneous medium we finally obtain

$$c_H^2\{\partial_x^2 P + \partial_y^2 P\} + c_V^2 \partial_z^2 P + \omega^2 P = 0 \quad , \tag{22}$$

where

$$c_H^2 = (\varkappa\varrho_H)^{-1} \tag{23}$$

and

$$c_V^2 = (\varkappa\varrho_V)^{-1} \quad , \tag{24}$$

c_H and c_V being the (frequency-dependent) horizontal and vertical phase velocities, respectively. Note that the anisotropy is elliptical and that the inverse Fourier transforms of c_H^{-2} and c_V^{-2} are causal.

In the following we consider 2-D wave propagation and we transform equation (22) to the rayparameter-frequency domain. To this end we replace ∂_x^2 by $-\omega^2 p^2$, yielding

$$\partial_z^2 \tilde{P} = -(\omega^2/c_V^2)(1 - p^2 c_H^2)\tilde{P} \quad , \tag{25}$$

or

$$\partial_z \tilde{P} = \pm j(\omega/c_V)(1 - p^2 c_H^2)^{1/2} \tilde{P} \quad . \tag{26}$$

The solution for downward propagation reads

$$\tilde{P}^+(z_m;p,\omega) = \tilde{W}^+(z_m,z_0;p,\omega)\ \tilde{P}^+(z_0;p,\omega) \quad , \tag{27}$$

where

$$\tilde{W}^+(z_m,z_0;p,\omega) = \exp\{-j(\omega/c_V)(1 - p^2 c_H^2)^{1/2} \Delta z\} \quad , \tag{28}$$

with $\Delta z = z_m - z_0$.

SERIES EXPANSIONS OF THE TRANSMISSION RESPONSES

In this section we present series expansions of the transmission responses of a finely layered medium and of a homogeneous anisotropic medium with losses. By matching the coefficients in both series expansions, the extended macro-model parameters are found.

The transmission response of a finely layered medium

Consider the generalized primary extrapolation operator $\tilde{W}_g^+ = \tilde{W}_p^+ \tilde{C}$, with $\tilde{W}_p^+$ defined in equation (13) and $\tilde{C}$ in equation (16). For small p we may write

$$\tilde{W}_g^+(z_m,z_0;p,\omega) \approx \exp[\{-j\omega\Delta z(<1/c> + A(\omega)/j\omega)\}\ p^0$$
$$+ (1/2)\{j\omega\Delta z<c>(1 + (\alpha - n)A(\omega)/<1/c>j\omega)\}\ p^2] \quad . \tag{29}$$

The transmission response of a homogeneous anisotropic medium with losses

Consider the extrapolation operator $\tilde{W}^+$ in a homogeneous anisotropic medium with losses, as defined in equation (28). For small p we may write

$$\tilde{W}^+(z_m,z_0;p,\omega) \approx \exp[-j\omega\Delta z(1/c_V)p^0 + (1/2)j\omega\Delta z(c_H^2/c_V)p^2] \quad . \tag{30}$$

Matching the coefficients

By matching the coefficients for p^0 and p^2 in equations (29) and (30) we obtain the extended macro model parameters.

- Coefficients for p^0:

$$c_V^{-1} = \langle 1/c \rangle + A(\omega)/j\omega \quad . \tag{31}$$

- Coefficients for p^2:

$$c_H^2 = \langle c \rangle \, c_V[1 + (\alpha - n)A(\omega)/\langle 1/c \rangle j\omega] \quad . \tag{32}$$

Hence, the transmission response of a finely layered medium can be mimicked by modeling the transmission response of a homogeneous anisotropic lossy medium, in which the complex medium parameters c_V and c_H are given by equations (31) and (32), respectively. In the following we refer to c_V and c_H as the parameters of the *extended macro model* (note that for the situation studied so far the extended macro model consists of one homogeneous layer only).

In equation (31), the first term on the right-hand side accounts for the primary travel time and the second term for the dispersion. A similar remark applies to equation (32). It should be noted that when a well log is not available, the error in the primary travel time is of the same order of magnitude as the apparent additional time shift due to the dispersion. However, this does not undermine the usefulness of the extended macro model for true amplitude migration. The aforementioned travel-time error may lead to small positioning errors in the migrated result (similarly to when a conventional macro model would be used), but it will not affect the amplitude. Hence, the main reason for using the extended macro model in migration is to improve on the (angle-dependent) amplitude of the imaged reflectors rather than on their positioning.

EXAMPLES

In this section we illustrate the theory with a number of examples. We consider a horizontally layered medium consisting of 15,000 layers each with a thickness of 10 cm. The statistics of the fine-layering are described by fractal Brownian motion (Walden and Hosken, 1985; Herrmann, 1991). The average velocity $\langle c \rangle$ equals 2500 m/s and the density is taken constant. The standard deviation for the velocity is 413 m/s.

All results are presented in the rayparameter-time (p,τ) domain for a number of p-values, corresponding to plane wave propagation angles, ranging from normal incidence to 52 degrees (for clarity, in the figures the p-axis is labeled with ϕ_{eff}, defined as $\phi_{eff} = \arccos\sqrt{(1 - c_{eff}^2 p^2)}$, see equation 13b). We consider band-limited impulse responses; the time behaviour and spectrum of the zero-phase band-limited impulse (i.e., the wavelet and its spectrum) are shown in Figs. 2 and 3, respectively.

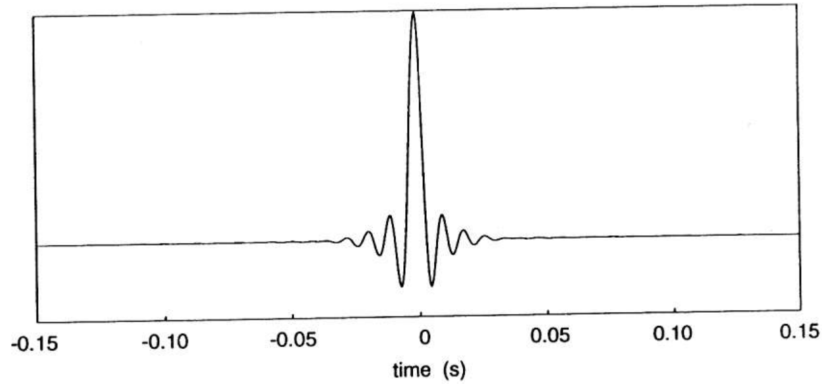

Fig. 2. Band-limited impulse.

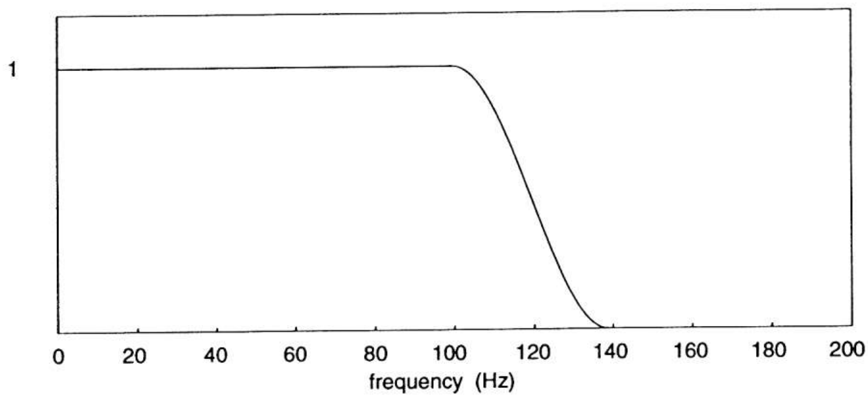

Fig. 3. Spectrum of the impulse in Fig. 2.

As a first example we compare the exact transmission response, obtained by numerical modeling in the finely layered medium, with the transmission response of a conventional macro model (here: a homogeneous isotropic lossless acoustic medium), that describes the finely layered medium as accurately as possible, see Fig. 4. Equation (13) is not suited for modeling the conventional macro-model response, because it contains two different velocities ($<1/c>$ and $<c>$). Therefore the macro-model response was modeled with the following operator

$$\tilde{W}_m^+ = (z_m, z_0; p, \omega) = \exp\{-j\omega(<1/c>^2 - p^2)^{1/2}\Delta z\} \quad . \tag{33}$$

Note that for vertical propagation ($p = 0$) this operator is identical to the primary extrapolation operator, given by equation (2) or (13).

From Fig. 4 it becomes clear that the conventional macro-model solution is a poor estimation of the transmission response in a finely layered medium. The dispersive attenuation effects are neglected altogether and, particularly at higher angles, the travel times do not match very well.

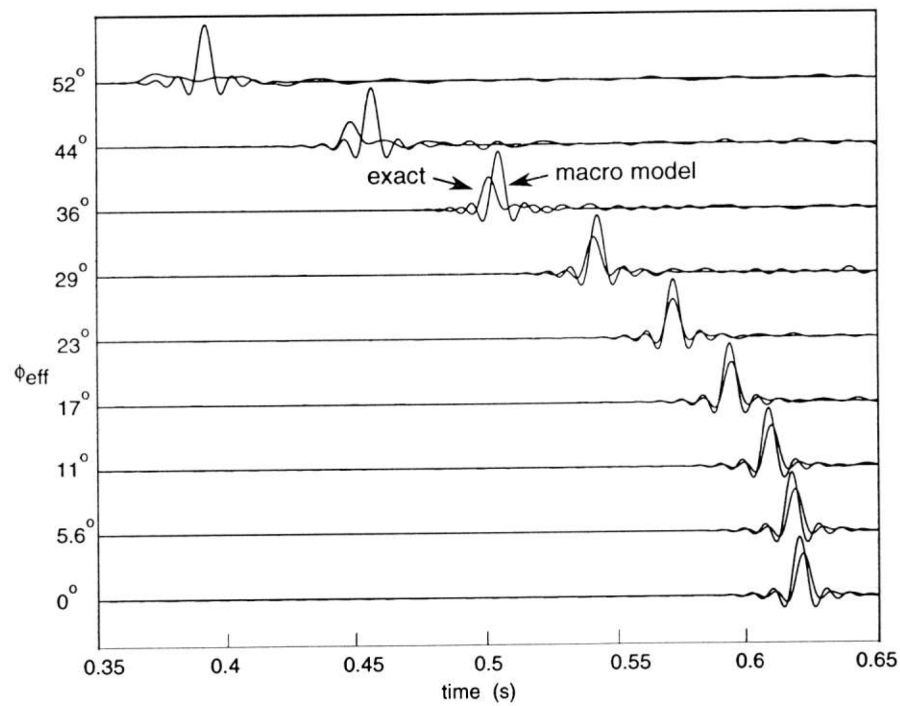

Fig. 4. Transmission responses in the rayparameter-time domain of a finely layered medium: *exact* versus *macro-model* response.

In Fig. 5 we compare the exact response with a particular version of the generalized primary operator $\tilde{W}_g^+ = \tilde{W}_p^+\tilde{C}$, with $\tilde{W}_p^+$ defined by equation (10) and $\tilde{C}$ defined in equation (16), with n = 4. Hence, $\tilde{W}_p^+$ describes the exact primary operator in the *true* (i.e., finely layered) medium and the correction operator $\tilde{C}$ is the O'Doherty-Anstey solution, modified for a fractal medium. In fact this is the approach followed by Herrmann and Wapenaar (1992). From Fig. 5 we observe that the generalized primary response matches the exact response very well, up to high propagation angles.

The accuracy obtained with the generalized primary in Fig. 5 is fully satisfactory. However, the aim of this paper was to propose an extended macro model, the response of which accurately matches the exact response. The result in Fig. 5 cannot be seen as a macro-model response, in particular because the primary operator $\tilde{W}_p^+$ was defined in the true medium.

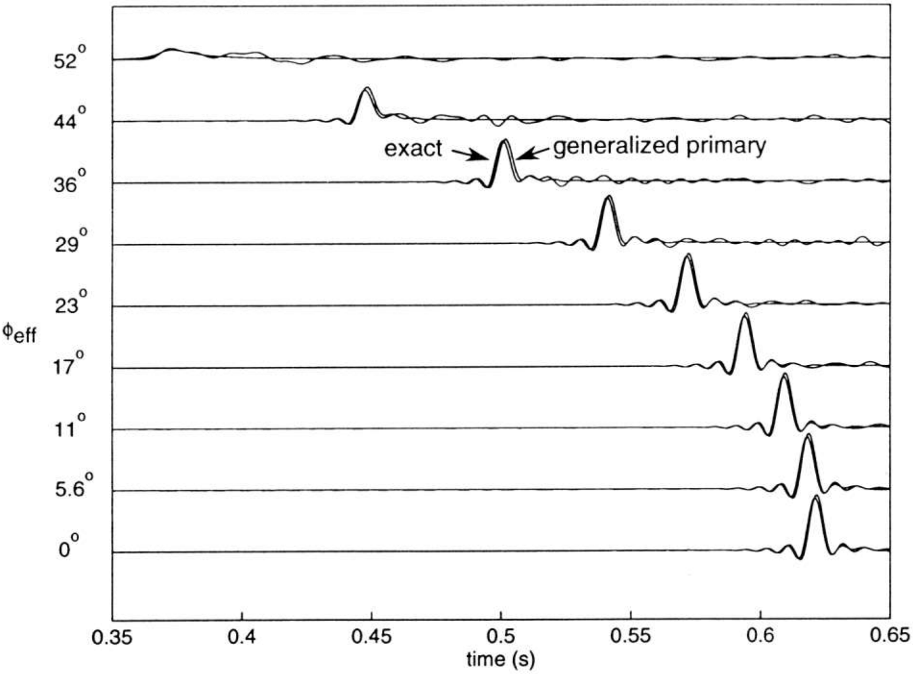

Fig. 5. *Exact* versus *generalized primary* response.

In Fig. 6 we compare the exact response with our extended macro-model response, given by $\tilde{W}^+$, as defined in equation (28), with c_V and c_H defined in equations (31) and (32). Note that this extrapolation operator is defined in a *homogeneous* anisotropic lossy macro model with complex medium parameters. From Fig. 6 we observe that the match is quite accurate, up to a propagation angle of 30 degrees. Beyond this angle, the travel times in particular become less accurate. The dispersive attenuation effects, however, are accurate up to higher angles and are almost just as good as in the generalized primary solution in the previous example. This is more clearly illustrated in Fig. 7a, where we compare the generalized primary response with the extended macro-model response, after a removal of the primary operator. In Fig. 7b the extended macro-model response is compared with the exact response, again after a removal of the primary operator. We observe that the match is quite good up to a propagation angle of 45 degrees. This example suggests that if we want to improve the extended macro model, we should primarily be concerned with finding a better approximation of the travel times.

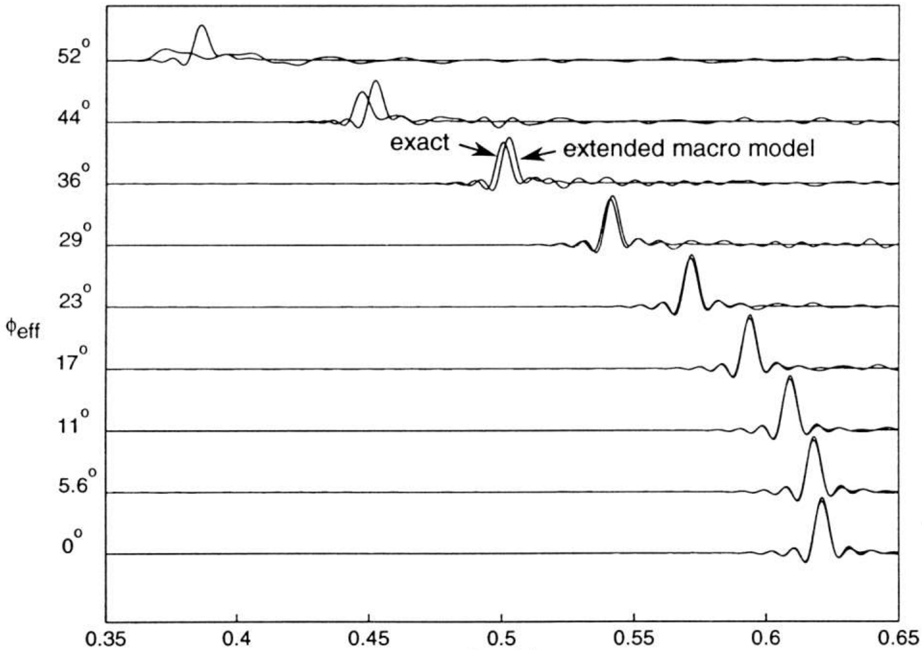

Fig. 6. *Exact* versus *extended macro-model* response.

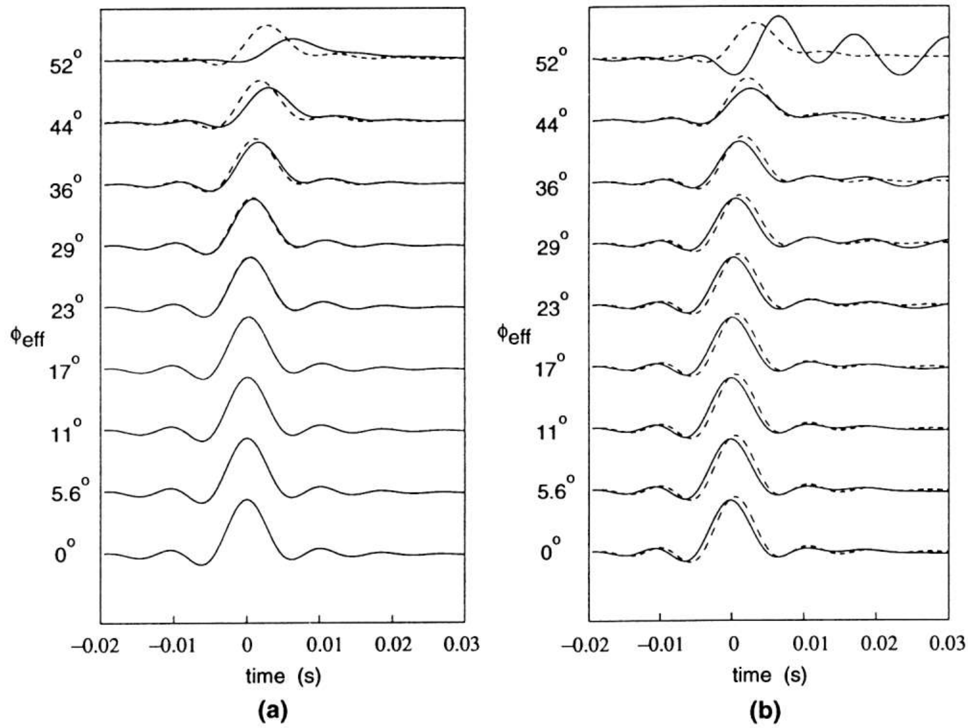

Fig. 7. Transmission responses in the rayparameter-time domain of the finely layered medium after a removal of the primary operator. (a) *generalized primary* (solid) versus *extended macro-model* response (dotted). (b) *exact* (solid) versus *extended macro-model* response (dotted).

Finally, in Figs. 8 and 9 we compare, for two angles of incidence, the exact response with the conventional macro-model response and with the extended macro-model response. From these figures it becomes clear that the extended macro-model response is significantly more accurate than the conventional macro-model response.

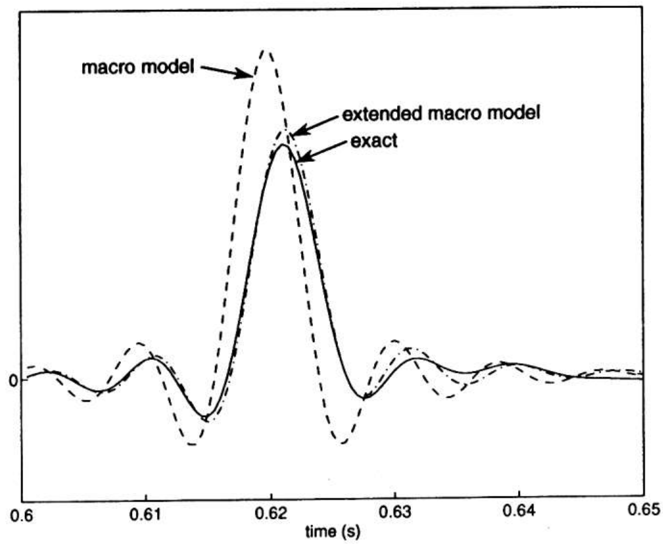

Fig. 8. Normal incidence transmission responses of the finely layered medium: *exact* versus *macro-model* versus *extended macro-model* response.

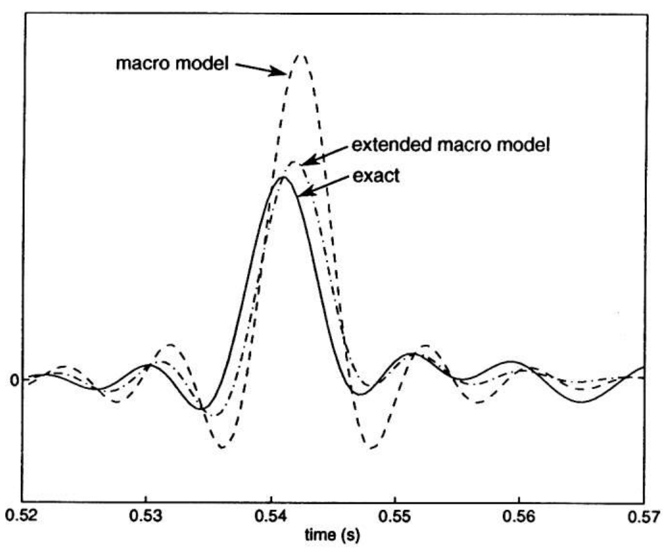

Fig. 9. Oblique incidence (29 degrees) transmission responses of the finely layered medium: *exact* versus *macro-model* versus *extended macro-model* response.

CONCLUSIONS

We have shown that it is possible to mimic fine-layering effects by the response of an extended macro model (so far represented by a homogeneous anisotropic lossy medium). The examples show that the travel times of the extended macro model response are accurate up to 30 degrees and that the dispersive attenuation is mimicked accurately up to 45 degrees. Although the results are less accurate than those obtained with the generalized primary operator, the extended macro model approach has some important advantages. The extended macro model approach is not restricted to plane-wave responses of horizontally layered media, and it can be easily generalized to structurally complex configurations, simply by assigning complex velocities c_V and c_H (equations 31 and 32) to each macro layer of a 2-D or 3-D 'conventional' macro model. A point-source response (Green's function) can be obtained by numerically solving wave equation (20) or (22) in each macro layer (for instance by ray tracing or finite difference modeling), imposing the appropriate boundary conditions at the layer interfaces. When in a certain area the fine-layering is tilted, this can be easily incorporated by applying coordinate rotation matrices (containing direction cosines) to the mass-density tensor $\mathbf{M}$, as defined in equation (21).

The main application of the extended macro model will lie in the numerical generation of (approximate) generalized primary operators for 2-D or 3-D true-amplitude migration in structurally complex media, containing significant fine-layering effects (Wapenaar and Herrmann, 1994).

The main points of current research concern:

- mimicking the fine-layering response more accurately by taking more terms into account in the series expansions of the operators and thus allowing a more general mathematical model for the anisotropy of the replacement medium (Slot, 1994),

- accounting for 3-D heterogeneities per macro layer.

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