Sampling and illumination aspects of seismic interferometry in horizontally layered media

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Summary

Seismic Green's function retrieval or seismic interferometry (SI) refers to the principle of generating new seismic responses by crosscorrelating seismic observations at different receiver locations. We consider retrieving a reflection response between receivers at an (approximately) horizontally layered medium. Only transmission responses due to sources that are, in a Fresnel sense, inline with the receivers are needed as an input for the SI relation. The sampling criterion for the sources is much more relaxed than Nyquist. Sources at the edges of the source distribution will cause distortion of the retrieved reflections or even spurious events. Based on a tau-p transform of the transmission responses, a filter can be designed to remove kinematically wrong events from the retrieved results.

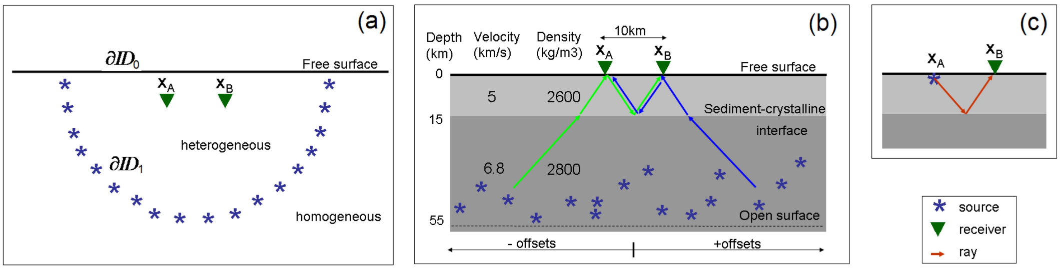

Figure 1: (a) General SI configuration for a medium bounded on one side by a free surface; (b) a layered medium with natural sources in the subsurface, which transmission responses are recorded by the receivers on the surface; (c) a retrieved reflection between two receiver positions.

Introduction

Seismic Green's function retrieval or seismic interferometry (SI) refers to the principle of generating new seismic responses by crosscorrelating seismic observations at different receiver locations. SI can be applied independently of the scale and wavetype under consideration. SI consists of a surface integration of correlations over source positions. The part of the Green's function that can be retrieved depends on the size of this integration surface (the illumination aperture). The quality of the retrieved Green's function is dependent on the sampling of this surface (source sampling). In this abstract the influence of both sampling and illumination is studied.

Theory

For an acoustic lossless medium with a free surface (Fig. 1(a)), Wapenaar and Fokkema (2006) derived an exact SI relation and a modified one which is easier for practical applications:

$$\frac{2}{\rho v}\int_{\partial\mathbb{D}_1} G(\mathbf{x}_\mathcal{A},\mathbf{x},t) * G(\mathbf{x}_\mathcal{B},\mathbf{x},-t)d^2\mathbf{x} \approx G(\mathbf{x}_\mathcal{A},\mathbf{x}_\mathcal{B},-t) + G(\mathbf{x}_\mathcal{A},\mathbf{x}_\mathcal{B},t), \tag{1}$$

where $G(\mathbf{x}_\mathcal{A},\mathbf{x}_\mathcal{B},t)$ denotes the Green's function observed at $\mathbf{x}_\mathcal{A}$ due to an impulsive volume injection-rate density source at $\mathbf{x}_\mathcal{B}$, v and ρ are the the P-wave velocity and density of the layer with the sources, respectively, and $*$ denotes convolution. $\partial\mathbb{D}_1$ is a surface of sources, which in 2D is a line of sources as denoted in Fig. 1(a). Also heterogeneities outside $\partial\mathbb{D}_1$ are treated correctly if the source distribution along $\partial\mathbb{D}_1$ is irregular (in the direction normal to $\partial\mathbb{D}_1$). Equation 1 could be applied to retrieve a high-frequency reflection responses between two receiver positions (Fig. 1(c)) using transmission responses due to local seismic sources (Fig. 1(b)). On the other end of the spectrum, the same equation could be applied on transmissions due to earthquakes at teleseismic distances, which are recorded by a coarsely sampled array of receivers. Doing so, the reflection response of the crust and upper mantle can be retrieved.

Since a lossless medium is assumed for equation 1, the transmission responses need to be corrected for inelastic losses before crosscorrelation (Draganov *et al.*, 2008).

One of the simplifications of equation 1 with respect to the exact representation (Wapenaar and Fokkema, 2006) is that a dipole response is approximated by neglecting a $|cos\alpha(\mathbf{x})|$ term, where $\alpha(\mathbf{x})$ is the local angle between the pertinent ray and the normal to $\partial\mathbb{D}_1$. When the sources are regularly sampled at a fixed depth, as in Fig. 1(b), neglecting this term results in a retrieved reflection response that is higher in relative amplitude at large offsets than at short offsets.

In this abstract we consider the application of SI to horizontally layered media. For these media, the direct wave is the arrival with the highest horizontal slowness p_x in the transmission response. There are no scatterers (Huygens sources) adding higher inline or crossline horizontal slownesses to the transmission responses. Therefore, the source surface $\partial\mathbb{D}_1$ (Fig. 1(a)) needs to be sufficiently covered with actual sources. It suffices to have either only sources at negative

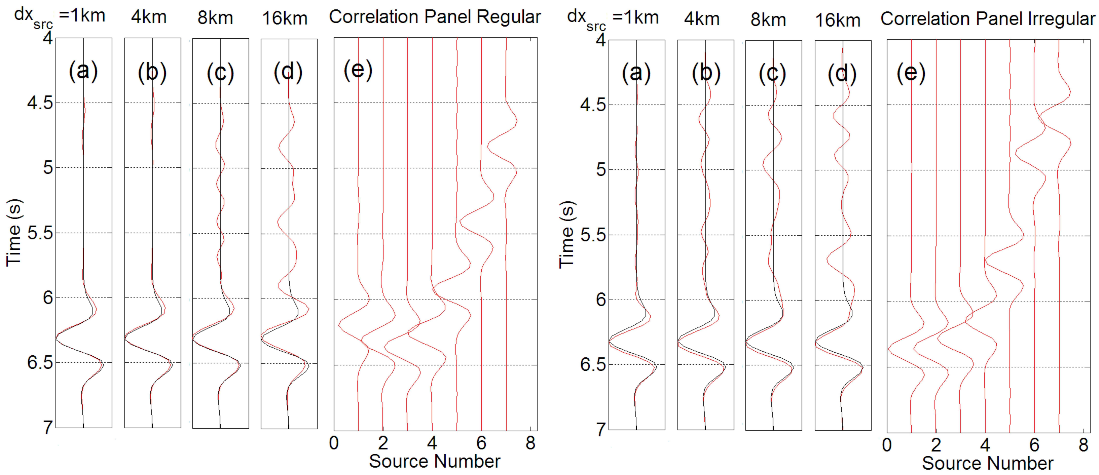

Figure 2: A retrieved reflection response (red) and a reference response (black) for a source at $\mathbf{x}_{\mathcal{A}}$ and a receiver at $\mathbf{x}_{\mathcal{B}}$ as in (Fig. 1(b)), for an (average) source spacing of 1 (a), 4 (b), 8 (c) and 16 (d) km. A correlation panel for a source spacing of 16 km (e). The left 5 figures are for a regular source distribution, the right 5 figures are for a random source distribution.

offsets or at positive offsets (as defined in Fig. 1(b)). The reflection in Fig. 1(c) can be retrieved with sources contributing to the Fresnel zone around the green ray in Fig. 1(b); in this case, the retrieval is in $G(\mathbf{x}_{\mathcal{A}}, \mathbf{x}_{\mathcal{B}}, t)$. Or the reflection can be retrieved with sources contributing to the Fresnel zone around the blue ray in Fig. 1(b); in this case the retrieval is in $G(\mathbf{x}_{\mathcal{A}}, \mathbf{x}_{\mathcal{B}}, -t)$.

Sampling criterion for 2D horizontally layered media

First we consider the sampling criterion for regularly sampled sources on a fixed depth. We place sources with $f_{max} = 5Hz$ in a crustal-scale model (Fig. 1(b)), between -60 and 40 km offset. The spatial Nyquist criterion for this configuration is $dx_{src} < dx_{NQ} = \frac{v_{min}}{2f_{max}} = \frac{5}{2 \cdot 5} = 0.5$km. The source sampling required to retrieve body waves, as in our example, is less strict: $dx_{src} < dx_{SI} = \frac{v_{x,min}}{2 \cdot 2f_{peak}} = \frac{60}{2 \cdot 2 \cdot 2} = 7.5$km, where $v_{x,min}$ $(= \frac{1}{p_{x,max}})$ is the lowest apparent velocity along the array in the data before crosscorrelation, which is due to a source at the largest offset. A factor of 2 is included since events can become twice as steep due to crosscorrelation. f_{max} is relaxed to f_{peak}.

Fig. 2(left) shows the retrieved reflection for a source spacing of successively 1 (a), 4 (b), 8 (c) and 16 (d) km and the correlation panel (the integrand of equation 1) for a source spacing of 16 km (e). Each trace in the correlation panel corresponds to the contribution from one source. The sources that are at or near the stationary point of the event in the correlation panel (which is denoted corr-event in the following) contribute to the retrieval of the reflection. In Fig. 2(left)(e) it can be seen that the amplitude of the side lobes of the corr-event do not interfere completely destructively, which results in the non-physical energy in the stack (d), which is called correlation noise. The same, but less severe, is the case for (c). When the source sampling satisfies dx_{SI}, as in (a) and (b), this correlation noise does not appear.

Fig. 2(right) depicts the retrieved reflections for a source location which is random between -60 and 40 km offset and between 45 and 65 km depth. For this random distribution of sources correlation noise appears between an average source spacing $(\widetilde{dx}_{src})$ of 1 and 4 km and becomes more severe with larger spacings.

For both the regular as well as the random source distribution, the reflection is still retrieved correctly for an (average) spacing of 16 km. The correlation noise can largely be removed by applying a median filter, using the fact that the move-out of the correlation noise is either random or larger than the move-out of actual reflections. In Draganov *et al.* (2004) it was shown that source spacings > dx_{SI} are still sufficient when the retrieved reflections are used to make an image. That is, because the correlation noise has a non-hyperbolic move-out, these amplitudes

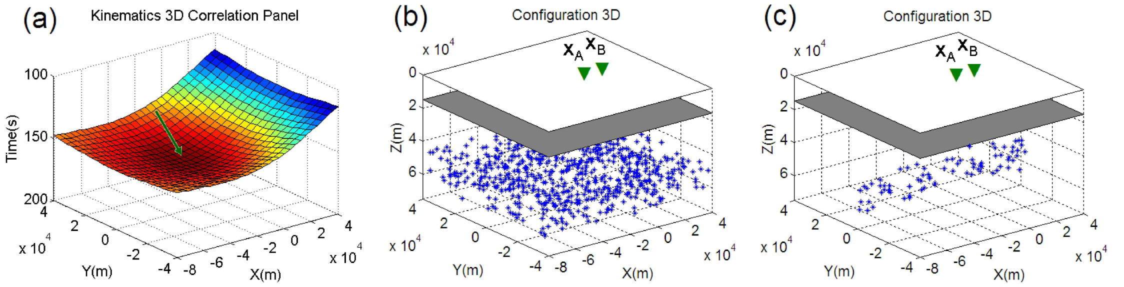

Figure 3: (a) The kinematics of the corr-event for retrieving a reflection, (b) the model with an irregular source distribution and (c) the model with the relevant part of this distribution.

are largely neglected in a migration algorithm. Furthermore, in migration many retrieved shot records are used to image the same reflector and therewith random noise is eliminated.

If the source sampling is not sufficient, dx_{SI} can be increased by low-pass filtering the transmission responses (reduce f_{peak}) or reject large-offset and or near-surface sources (increase $v_{x,min}$).

Sampling and illumination in a 3D horizontally layered medium

In the previous section we considered the application of SI in 2D for which we needed a line of sources. The SI relation 1 is valid though for 3D, in which case the integration is over a surface of sources. Fig. 3(a) depicts the kinematics of the integrand in 3D, at times relevant for the retrieval of the primary reflection of the layer at 15 km (Fig. 3(b)). The stationary phase ($\partial_y \partial_x I = 0$, where I stands for the integrand) and its Fresnel zone are indicated with the Green arrow and dark red surface, respectively. $\partial_x I = 0$ is a priori unknown since the depth of the reflector and the velocities in the subsurface are a priori unknown. Though $\partial_y I = 0$, is for a horizontally layered medium at y=0. Thus only sources in the Fresnel zone around y=0 contribute to the retrieval. Therefore, of all the sources in (b), only those depicted in (c) are relevant. Including sources that are at larger crossline offsets is of no harm, as long as the sampling of the sources in the crossline direction is sufficient. The crossline sampling dy_{SI} needed is similar to the inline sampling.

For applications of SI with natural sources, where the source sampling is unlikely to be sufficient at all azimuths, the sources at larger crossline offsets than the Fresnel zone can be rejected, since they will only add correlation noise. When, e.g., the polarization of the direct wave is recorded or the wavefront is measured with a grid of receivers, the source direction can be estimated.

Illumination in a 2D horizontally layered medium

Again we consider the same subsurface model as in Fig. 1(b), but instead of two receivers we have an array of 41 receivers, as illustrated in Fig. 4(a). Our goal is to retrieve the reflection response as if there is a source at the middle receiver. This time we use a limited range of sources. The correlation panel and the stack for retrieving the reflection as if there was a source and a receiver at position 21, is given at Fig. 4(b). It can be seen that the retrieved event is not only due to stacking over the stationary phase (denoted with a green arrow), but also, because of the limited aperture, the edges of the corr-event give a too large contribution (denoted with circles). For this reason, the amplitude and phase of the retrieval are not perfect, but the timing is. The correlation panel and the stack for retrieving the reflection between receiver 21 and 41 is given at Fig. 4(c). It can be seen that the retrieval has not the right dynamics neither the right kinematics. This is so, because the stationary-phase contributors are not in the data, i.e., no transmission responses have been recorded from sources that would contribute to the Fresnel zone around the dashed blue ray. The spurious event in the stack is due to the edges of the corr-event (denoted with circles). The influence of the edges of the corr-events could be suppressed

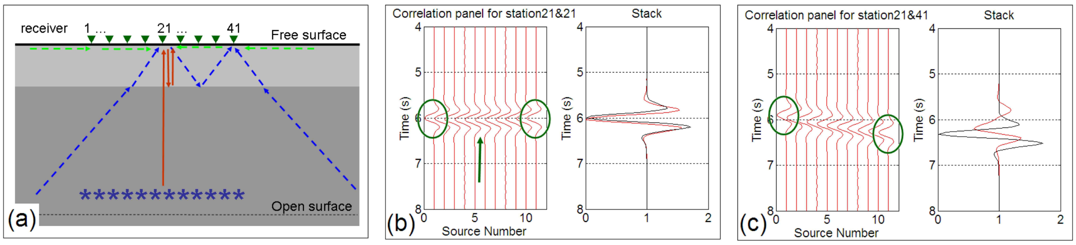

Figure 4: A configuration with 41 receivers and an insufficient aperture of sources (a), the correlation panel and stack for a retrieval between position 21 & 21 (b) and position 21 & 41 (c)

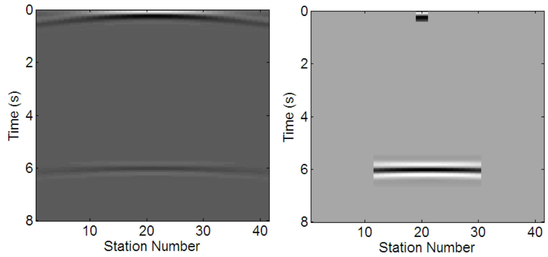

Figure 5: A retrieved reflection response as if there were a source at the middle receiver and receivers at all station position; before (left) and after (right) p_x filtering

with a taper (Mehta *et al.*, 2008).

The result after applying SI for a source at receiver 21 and receivers at all the other places is depicted at Fig. 5(left). To retrieve the direct wave, sources need to be available at or near the surface, contributing to the Fresnel zone around the dashed green rays in Fig. 4(a), which sources were not present. The first arrival in Fig. 5(left), which appears to be a direct wave, is in fact a spurious event due to the edges of the corr-events.

It is not possible to retrieve apparent slownesses p_x that are larger than the ones present in the data before crosscorrelation. To get a good indication of the initial illumination, which would be (partly) unknown when using natural sources, we bring the transmission response to the τ-p_x domain and stack the τ-p_x plots of the individual sources. Based on the slowness range in which data is available, a p_x bandpass filter is designed. Fig. 5(right) is the retrieval after filtering; only the retrievals with the right kinematics have been preserved.

Conclusions

We considered retrieving a reflection response between receivers at an (appr.) horizontally layered medium. Only transmission responses due to sources that are, in a Fresnel sense, inline with the receivers are needed. The sampling criterion for the sources is much more relaxed than Nyquist. Sources at the edges of the source distribution can cause distortion of the retrieved reflections or even lead to spurious events. Based on a τ-p_x transform of the transmission responses a filter is designed to remove kinematically wrong events from the retrieved results.

Acknowledgements

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