Introduction

The synthesis of virtual wavefields makes it possible to reconstruct virtual sources (or receivers) and reflectors at the location of the receivers (sources). The approach enables geophysicists to obtain wavefields from redatumed sources (or at redatumed receivers), with the advantage that the medium properties are not needed to reconstruct these new signals for exploration and passive seismic purposes (e.g., Bakulin and Calvert, 2006; Wapenaar et al, 2008; Schuster, 2009). Seismic interferometry (SI) is well known and typically based on crosscorrelation (or deconvolution) algorithms, which, under complete illumination (observation) conditions, make it possible to reconstruct the Green's function of the medium inside a volume encompassed by a representation surface. In particular applications, when one receiver is outside the representation boundary, SI is performed also by crossconvolution (e.g., Slob and Wapenaar, 2007). Recently, a new approach using the crossconvolution method was proposed by Poletto (e.g., Poletto and Farina, 2008a) to synthesize inside the bounded volume the wavefields created by a virtual reflector (VR) at the boundary. The Kirchhoff-Helmholtz integral representation of a virtual reflector in acoustic media is formulated by Poletto and Wapenaar (2009). The VR method finds useful applications for seismic exploration in combination with SI by crosscorrelation (Poletto and Farina, 2008b, 2009). Their analysis is applied to marine and borehole data, where the virtual wavefields obtained with the same source-receiver geometry are combined by suitable weights including the reflection coefficient at the boundary. For ideal zero-phase source wavelets, these weights change depending on the boundary reflection coefficient, integral representation and propagation Green's function properties. We may observe that this approach can be seen in a wider context than only for interferometry. For example it is similar to calculate simultaneous forward and backward propagation of marine multiples (Wiggins, 1988) or to describe forward and backward Kirchhoff-Helmholtz (KH) extrapolation of downgoing and upgoing wavefields (Wapenaar, 1993).

In this work we analyze the theoretical aspects in the joint representation of the VR and SI wavefields with the same source and receiver configurations. Here and in the following, we use simply SI to intend SI by crosscorrelation. We present the formulation of the Kirchhoff-Helmholtz representation integrals used in the combination of the SI and VR wavefields in arbitrary acoustic media. The analysis calculates the VR and SI combination terms and coefficients for wavefield representations by receivers surrounding two sources in 2D and 3D (thus, without loss of generality for reciprocity, performing the synthesis of a virtual receiver at one of the sources).

Theory (VR)

Assume an arbitrary, inhomogeneous acoustic medium in a volume encompassed by a bounding surface S_o. The representation surface S_o is the observation surface where the receivers are used in all its points. Let A and B be two point sources included in the bounded volume. We assume ideal zero-phase unit source signals, so that in the Fourier frequency domain, where ω is the angular frequency, we can approximate $S(\omega)S(\omega) \cong S(\omega)S^*(\omega) \cong 1$, where '*' denotes complex conjugate, and neglect the source signature in the signal (VR) crossconvolutions and (SI) crosscorrelations. The scalar functions $U(A,B,\omega)$ and $G(B,A,\omega)$ represent the propagating wavefield and Green's function from B to A and from A to B, respectively. Where not necessary, the dependence on ω is omitted.

The VR wavefield between B and A can be expressed by the KH integral representation on S_o (Poletto and Wapenaar, 2009). Using the reciprocity theorem of convolution type we have

$$U(A,B) - G(B,A) = \frac{1}{4\pi}\int_{S_o} dS_o \left[G(r_o,A)\frac{\partial}{\partial n}U(r_o,B) - U(r_o,B)\frac{\partial}{\partial n}G(r_o,A) \right], \qquad (1)$$

where r_o is the integration point on S_o and n is the outward normal to the bounding surface (Fig. 1a).

(a) **(b)**
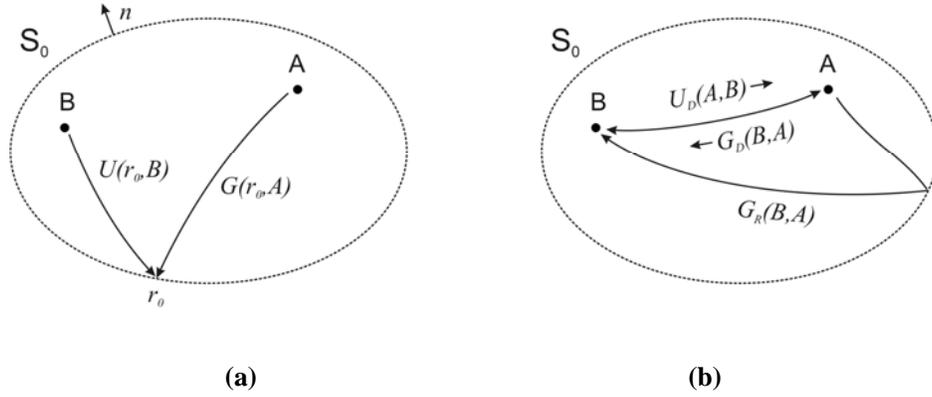
Figure 1 a) Kirchhoff-Helmholtz representation concept. b) Direct and reflected waves. U is the direct free-space solution from B to A. G is the Green's function from A to B with direct (G_D) and reflected from the boundary (G_R) wavefields.

The VR representation is obtained by assuming different boundary conditions for U and G on S_o. Assume a rigid boundary (reflection coefficient $R=+1$) with the Neumann boundary condition on S_o

$$\frac{\partial G(r_o, A)}{\partial n} = 0, \tag{2}$$

for $G(r_o, A)$ but not for the free-space solution $U(r_o, B)$. Using Eq. (2) and the approximation $\partial/\partial n \cong -i\omega/c$ where $i = \sqrt{-1}$ and c is the acoustic velocity, we obtain

$$U(A,B,\omega) - G(B,A,\omega) \cong \frac{-i\omega}{4\pi c} \int_{S_o} dS_o G(r_o, A, \omega) U(r_o, B, \omega). \tag{3}$$

With the notation of Fig. 1b, where $U(A,B)=U_D(A,B)$ by definition, $G=G_D+G_R$, using reciprocity and the equivalence of the scalar functions $G_D(B,A)=U_D(A,B)$, we can express the VR signal as

$$G_R(A,B,\omega) \cong \frac{i\omega}{2\pi c} \int_{S_o} dS_o U(r_o, A, \omega) U(r_o, B, \omega). \tag{4}$$

In Eq. (4) we have used the approximation that the Green's function $G(r_o, A)$ is represented by two times the propagating function $U(r_o, A)$ measured on S_o, and Eq. (4) is strictly valid when the two terms of the integrand obey different boundary conditions on S_o.

Theory (SI)

We use SI in the reciprocal sense with respect to the conventional one, adopting the same source receiver configuration of VR. To derive the joint formulation we modify the reciprocity equation of crosscorrelation type (Wapenaar and Fokkema, 2006) as

$$U(A,B) - G*(B,A) = \frac{1}{4\pi} \int_{S_o} dS_o \left[G*(r_o, A)\frac{\partial}{\partial n} U(r_o, B) - U(r_o, B)\frac{\partial}{\partial n} G*(r_o, A) \right], \tag{5}$$

where we assume unit mass density. For convenience, we take the complex conjugate of Eq. (5) and use the boundary condition for G on S_o, which gives

$$U*(A,B) - G(B,A) = \frac{1}{4\pi} \int_{S_o} dS_o G(r_o,A) \frac{\partial}{\partial n} U*(r_o,B).$$ (6)

Using the same reasoning of the VR representation we obtain the crosscorrelation SI formulation as

$$-U*(B,A,\omega) + G(A,B,\omega) \cong \frac{-i\omega}{2\pi c} \int_{S_o} dS_o U(r_o,A,\omega) U*(r_o,B,\omega),$$ (7)

where $G=G_D+G_R$, and where we have used the approximation $\partial/\partial n \cong i\omega/c$ for the gradient of the complex conjugate.

Signal combination

The causal part of the representation Eq. (7) is similar to Eq. (4), with opposite sign. This difference in the virtual signal polarity is caused by the conjugate spatial derivative, and corresponds to different stationary-phase effects for (elliptic) VR and (hyperbolic) SI conditions (Poletto and Farina, 2009). The analysis shows that the upward and downward convexity, in the stationary-phase diagrams of the crossconvolved and crosscorrelated traces of a 2D (3D) model before stacking, causes opposite phases in the wavelets of the stacked traces. Combination of the causal Eq. (4) and of the causal part of Eq. (7) (i.e., neglecting $U*$ in the left-hand side term) performs the subtraction of the virtual reflection G_R and provides the representation of the direct signal as

$$G_D(A,B,\omega) \cong \frac{-i\omega}{2\pi c} \int_{S_o} dS_o U(r_o,A,\omega) \left[U(r_o,B,\omega) + U*(r_o,B,\omega) \right].$$ (8)

The corresponding time result is given by the causal part of the inverse Fourier transform of Eq. (8). This result can be generalized for arbitrary reflection coefficients, with a combination by suitable weights (filters) α and β, which can be expressed (as proposed by Poletto and Farina, 2008a) as

$$C(A,B,\omega) = \frac{-i\omega}{2\pi c} \int_{S_o} dS_o U(r_o,A,\omega) \left[\alpha U(r_o,B,\omega) + \beta U*(r_o,B,\omega) \right].$$ (9)

Equation (9) corresponds to the trace-stacking equations (Poletto and Farina, 2009), and can be used for selected events: in the VR and SI approximation for the boundary conditions, with appropriate coverage, taking into account the phase in the integral representation and wavefield propagation. Examples are given in the literature for acoustic cavities, and seismic marine data (Poletto and Farina, 2008b, 2009). In this work, we present the application with synthetic acoustic examples in uniform media, where we analyze the differences in the propagation- and integration-phase effects for the joint virtual signal representation.

Example (2D model)

We use the 2D acoustic model of Fig. 2a. The data are modeled both without and with top and bottom contrast layers (dark grey) with the same source/receiver geometry. The background medium velocity is 2000 m/s, the contrast layer velocity is 20 000 m/s. This contrast simulates a rigid boundary with reflection coefficient $R = 0.8182$. Grid mesh dimensions are $\Delta x = \Delta z = 2$ m. Receivers are located at the top and bottom interfaces (namely, S_o). The source signal in A and B is a zero-phase 40-Hz peak-frequency Ricker's wavelet. Time propagation is 4 s. Output sampling rate 1 ms.

Figure 2b shows the combination of the SI and VR signals obtained by trace stacking over the top and the bottom recording lines (surface S_o) in the 2D model with the contrast medium. The VR trace (second trace of the panel) is plotted with reversed polarity (due to propagation and integration phase effects), and scaled by the boundary reflection coefficient before the combination with SI (first trace of the panel). The last trace is the result of the virtual-reflection subtraction.

| (a) | (b) |
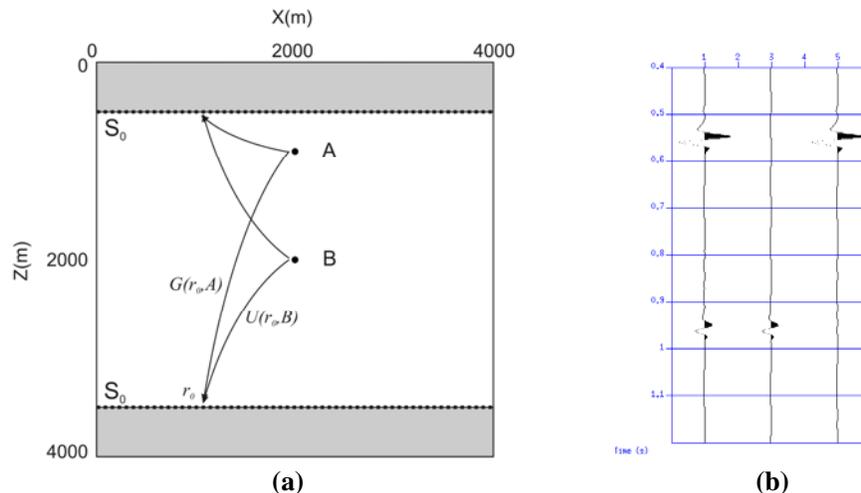
Figure 2 *(a) 2D acoustic model used with the contrast medium (top and bottom layers). Receivers are used at the top and bottom boundaries, sources in A and B. The signals from the sources in A and B and recorded at the receiver lines are crossconvolved and crosscorrelated. (b) The traces represent a time window of the virtual signals (selected events). From left to right we have: SI, VR, and combination, after scaling the VR signal by the reflection coefficient R=0.8182. The first event is the direct arrival from A to B. The second event is the virtual signal from A to the top boundary and then reflected to B. The reflection event is subtracted in the last trace, result of the combination.*

Conclusions

This work presents the theoretical Kirchhoff-Helmholtz formulation for the joint (i.e., using the same configuration) representation of VR and SI by crosscorrelation in arbitrary inhomogeneous acoustic media. The combination equations can be used for selected seismic events to represent both the VR and SI wavefields in the approximation of having different conditions at the boundary for the composed scalar functions. The analysis shows that the combination weights (phases) depend on the properties of the propagating functions and representation integrals.

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