

Turning one-sided illumination into two-sided illumination by target-enclosing interferometric redatuming

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We present a novel method to transform seismic transmission responses with sources at the surface and receivers above and below a selected target zone in the subsurface into virtual data with sources and receivers located at the initial receiver locations. The method is based on inverting a series of multidimensional equations of the convolution- and the correlation-type. The required input data can be computed from surface seismic data with a new iterative scheme that is currently being developed. The output data contains virtual sources that illuminate the target not only from above (as in the original data), but also from below, facilitating the needs of seismic imaging and inversion in an optimal way. The method is nonlinear in the sense that all internal multiples are correctly accounted for and true amplitude in the sense that the virtual sources are forced to inherit uniform radiation patterns even though the overburden is strongly heterogeneous.

Introduction

When seismic data are available with sources at the surface and receivers at a particular level Λ_x in the subsurface, sources can be redatumed to this level with interferometric redatuming (IR) by multidimensional deconvolution (Wapenaar et al., 2008; van der Neut, 2012). Unlike conventional redatuming methods (Berryhill, 1984), IR does address both primary and multiple reflections and provides a response as if the medium above Λ_x were homogeneous. Recently, it has been shown that Green's functions to any level Λ_x can be retrieved from surface seismic data, using a novel iterative scheme (Broggini et al., 2012; Wapenaar et al., 2012). With IR, these Green's functions can be transformed into virtual responses as if sources and receivers were both located at Λ_x . These virtual responses can be used for accurate imaging and characterization below this level, since all internal multiple reflections from heterogeneities above this level have been effectively eliminated from the data.

If we are able to generate full Green's functions from the surface to any level in the subsurface, we might go beyond conventional IR. In this abstract we introduce a new alternative redatuming scheme that we refer to as target-enclosing interferometric redatuming (TEIR). In TEIR, we transform Green's functions at two subsurface levels Λ_1 and Λ_2 into virtual data as if sources and receivers were located at both Λ_1 and Λ_2 in a reference medium. This reference medium is identical to the physical medium in the volume V between Λ_1 and Λ_2 and homogeneous outside this volume. In conjunction with the scheme of Wapenaar et al. (2012), TEIR allows us to turn conventional surface seismic data with one-sided illumination into virtual data with two-sided illumination.

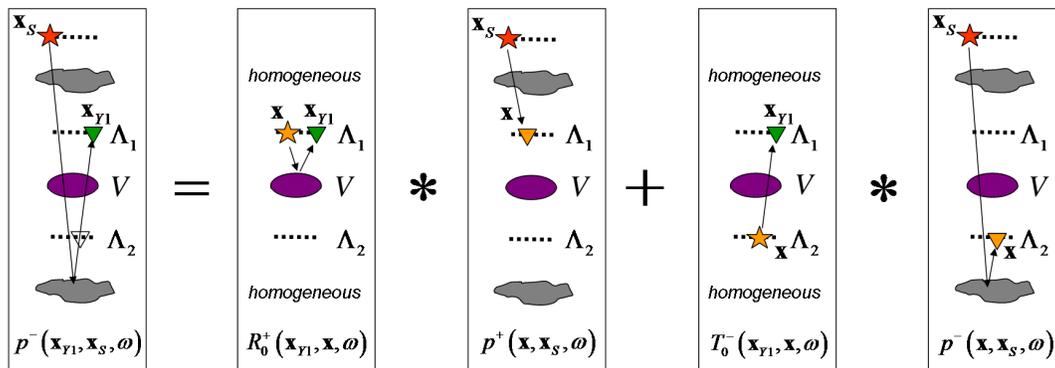


Figure 1 Illustration of the different terms in equation 1.

Theory

Wapenaar et al. (2008) presented a forward model for IR, starting with a reciprocity theorem of the convolution type for flux-normalized one-way wavefields in the frequency-space domain. In this model, the upgoing (superscript $-$) field $p^-(\mathbf{x}_{y1}, \mathbf{x}_s, \omega)$ at receiver \mathbf{x}_{y1} at level Λ_1 in the subsurface is expressed in terms of the downgoing (superscript $+$) field $p^+(\mathbf{x}, \mathbf{x}_s, \omega)$ at the same level and the upgoing field at a deeper level Λ_2 , according to

$$p^-(\mathbf{x}_{y1}, \mathbf{x}_s, \omega) = \int_{\Lambda_1} R_0^+(\mathbf{x}_{y1}, \mathbf{x}, \omega) p^+(\mathbf{x}, \mathbf{x}_s, \omega) d^2\mathbf{x} + \int_{\Lambda_2} T_0^-(\mathbf{x}_{y1}, \mathbf{x}, \omega) p^-(\mathbf{x}, \mathbf{x}_s, \omega) d^2\mathbf{x}. \quad (1)$$

In this representation, that is schematically illustrated in Figure 1, \mathbf{x}_s is the source location at the surface and ω is the angular frequency. The field R_0^+ is the reflection response at level Λ_1 in the reference medium (indicated by the subscript 0) and T_0^- is the transmission response from level Λ_2 to Λ_1 in the same medium. By choosing level Λ_2 infinitely deep in the subsurface and assuming the medium below this level to be homogeneous, it can be shown that the second integral vanishes (Wapenaar et al., 2008). The remaining integral can be inverted for R_0^+ , being the essence of conventional IR (Van der Neut, 2012). However, by having chosen the lower level infinitely deep, the retrieved response contains reflections from the entire medium below the upper level.

In TEIR we make a different choice by selecting Λ_2 below a target zone in the subsurface (and Λ_1 above this zone). As a consequence, we have a single equation with two unknowns, leading to an ill-posed inverse problem. To constrain this problem, we introduce a second forward model. Starting with a reciprocity theorem of the correlation type and repeating the steps in the derivation of Wapenaar et al. (2008), we find

$$p^+(\mathbf{x}_{Y1}, \mathbf{x}_s, \omega) = \int_{\Lambda_1} (R_0^+(\mathbf{x}_{Y1}, \mathbf{x}, \omega))^* p^-(\mathbf{x}, \mathbf{x}_s, \omega) d^2\mathbf{x} + \int_{\Lambda_2} (T_0^-(\mathbf{x}_{Y1}, \mathbf{x}, \omega))^* p^+(\mathbf{x}, \mathbf{x}_s, \omega) d^2\mathbf{x}, \quad (2)$$

where superscript * denotes complex conjugation. Equation 1 describes the field that leaves V at the upper level in terms of all fields that enter V , whereas equation 2 describes the field that enters V at the upper level in terms of all fields that leave V . Two similar expressions can be derived for the fields that leave and enter V at the lower level:

$$p^+(\mathbf{x}_{Y2}, \mathbf{x}_s, \omega) = \int_{\Lambda_1} T_0^+(\mathbf{x}_{Y2}, \mathbf{x}, \omega) p^+(\mathbf{x}, \mathbf{x}_s, \omega) d^2\mathbf{x} + \int_{\Lambda_2} R_0^-(\mathbf{x}_{Y2}, \mathbf{x}, \omega) p^-(\mathbf{x}, \mathbf{x}_s, \omega) d^2\mathbf{x}, \quad (3)$$

$$p^-(\mathbf{x}_{Y2}, \mathbf{x}_s, \omega) = \int_{\Lambda_1} (T_0^+(\mathbf{x}_{Y2}, \mathbf{x}, \omega))^* p^-(\mathbf{x}, \mathbf{x}_s, \omega) d^2\mathbf{x} + \int_{\Lambda_2} (R_0^-(\mathbf{x}_{Y2}, \mathbf{x}, \omega))^* p^+(\mathbf{x}, \mathbf{x}_s, \omega) d^2\mathbf{x}, \quad (4)$$

where R_0^- is the reflection response from level Λ_2 to level Λ_2 (illuminating the target from below) and T_0^+ is the transmission response from level Λ_1 to Λ_2 . We discretize equations 1, 3 and the complex conjugate of equations 2 and 4, using the notation of Berkhout et al. (2012). In this notation, row i and column j contain data from receiver i and source j . All four equations can be cast into a single matrix inverse problem, which can be inverted for \mathbf{R}_0^+ , \mathbf{T}_0^- , \mathbf{R}_0^- and \mathbf{T}_0^+ , being discrete representations of R_0^+ , T_0^- , R_0^- and T_0^+ , respectively:

$$\begin{pmatrix} \mathbf{p}_1^- & \mathbf{p}_1^{+*} \\ \mathbf{p}_2^+ & \mathbf{p}_2^{-*} \end{pmatrix} = \begin{pmatrix} \mathbf{R}_0^+ & \mathbf{T}_0^- \\ \mathbf{T}_0^+ & \mathbf{R}_0^- \end{pmatrix} \begin{pmatrix} \mathbf{p}_1^+ & \mathbf{p}_1^{-*} \\ \mathbf{p}_2^- & \mathbf{p}_2^{+*} \end{pmatrix}, \quad (5)$$

where \mathbf{p}_1^\pm and \mathbf{p}_2^\pm are the up- and downgoing fields at levels Λ_1 and Λ_2 , respectively. By solving this problem through (least-squares) inversion, we enforce that all virtual sources have uniform radiation patterns. To retrieve the fields R_0^- (illuminating the target from below) and T_0^+ , it is required that sufficient multiple scattering is generated below Λ_2 . Therefore, the inverse problem can only be solved if the medium is sufficiently heterogeneous below this array.

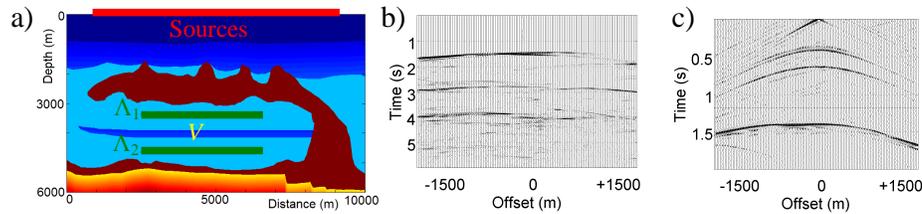


Figure 2 a) Velocity model. Sources are located at the red array at the surface. Receivers are located at the green arrays Λ_1 and Λ_2 . b) Downgoing field from a source at Λ_1 . c) Retrieved record R_0^+ by conventional IR for a virtual source in the center of Λ_1 .

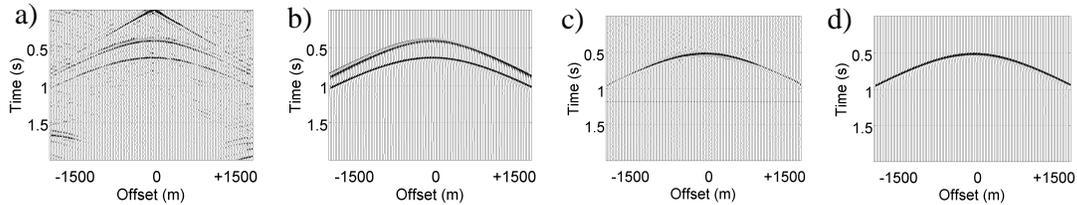


Figure 3 a) Retrieved record R_0^+ by TEIR for a virtual source in the center of Λ_1 with receivers at Λ_1 versus b) its directly modelled equivalent. c) Retrieved record \hat{T}_0^- by TEIR for a virtual source in the center of Λ_2 with receivers at Λ_1 versus d) its directly modelled equivalent.

Example

Synthetic data have been generated in the velocity model that is shown in Figure 2a. At the surface, 161 sources are located and 81 receivers are positioned at two arrays Λ_1 and Λ_2 . Strong salt reflections can be found below Λ_1 and above Λ_2 , providing rich illumination to constrain the inverse problem. Up- and downgoing wavefields are separated at both depth levels. In Figure 2b, we show the downgoing field of an arbitrary shot as observed at array Λ_1 for illustration. Aim is to redatum the sources to the receiver levels and to eliminate reflections outside the volume V enclosed by these levels. Conventional IR is applied with the data at the upper array only (van der Neut, 2012), yielding a response R_0^+ as if there were a virtual source at level Λ_1 , see Figure 2c. Besides the target reflections from V at ~ 0.5 s and ~ 0.7 s, we also observe a strong reflection at ~ 1.4 s, coming from the part of the salt body below Λ_2 . This event is retrieved because the second integral in equation 1 has not been evaluated by IR, leading to the reflection response of a reference medium that is homogeneous above Λ_1 but identical to the physical medium below Λ_2 .

Next, we apply the new algorithm TEIR. Input data are the decomposed data at both depth levels. Output data are R_0^+ , T_0^- , R_0^- and T_0^+ , obtained by least-squares inversion of equation 5. In Figure 3a, we show a shot from R_0^+ , which can be interpreted as if there were a virtual source and receivers at Λ_1 . Note the similarity with Figure 2c, except for some artefacts that are subject of current investigation. The reflector at ~ 1.4 s has been eliminated by TEIR, as we have defined the reference medium to be homogeneous below Λ_2 in the forward models. TEIR also provides T_0^- , having a virtual source at Λ_2 and receivers at Λ_1 . This response and its directly modelled equivalent are shown in Figure 3c-d.

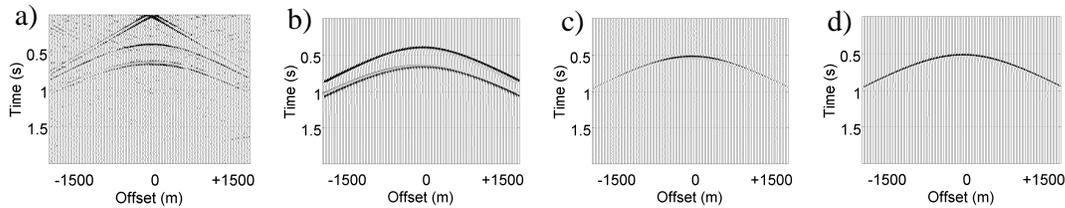


Figure 4 a) Retrieved record R_0^- by TEIR for a virtual source in the center of Λ_2 with receivers at Λ_2 versus b) its directly modelled equivalent. c) Retrieved record \hat{T}_0^+ by TEIR for a virtual source in the center of Λ_1 with receivers at Λ_2 versus d) its directly modelled equivalent.

Besides these responses, we have also retrieved R_0^- , which can be interpreted as if there were virtual sources and receivers at Λ_2 , illuminating the target zone from below. A retrieved record from this response and its directly modelled equivalent are shown in Figure 4a-b. Note that these responses are constructed solely from multiply scattered waves, providing the required illumination from below. Finally we find T_0^+ , having a virtual source at Λ_1 and receivers at Λ_2 . This response and its directly modelled equivalent are shown in Figure 4c-d. For all responses, we report a proper match, except that the amplitude-versus-offset characteristics are not properly retrieved (yet), which is subject of current investigation.

Discussion and conclusion

Ideally, we want to retrieve Green's functions to levels Λ_1 and Λ_2 from surface seismic data with the scheme of Wapenaar et al. (2012), followed by TEIR. Such an approach would allow us to turn conventional seismic data with one-sided illumination (from above only) into virtual data with two-sided illumination (both from above and below). The additional illumination from below, being constructed solely from multiply scattered waves, could complement the primary seismic illumination in shadow zones and provide additional input to constrain the inversion for rock and fluid properties. Note also that the data volume is reduced significantly by TEIR, since we have datumed the acquisition levels close to the target zone. We emphasize that the results we have shown in this abstract are preliminary and that more could probably be achieved with additional (reciprocity and sparsity) constraints and filters to make the inversion more robust, which is currently being investigated. The method should also still be tested in different environments with less favorable scattering and additional complications such as elastodynamic wave conversions, diffractions, intrinsic losses and noise.

References

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