

# Wavefield decomposition of field data, using a shallow horizontal downhole sensor array and a free-surface constraint

Niels Grobbe\*, Joost van der Neut, Carlos Almagro Vidal, Guy Drijkoningen and Kees Wapenaar Delft University of Technology, Department of Geoscience & Engineering, the Netherlands

# Summary

Separation of recorded wavefields into downgoing and upgoing constituents is a technique that is used in many geophysical methods. The conventional, multi-component (MC) wavefield decomposition scheme makes use of different recorded wavefield components. In recent years, land acquisition designs have emerged that make use of shallow horizontal downhole sensor arrays. Inspired by marine acquisition designs that make use of recordings at multiple depth levels for wavefield decomposition, we have recently developed a multi-depth level (MDL) wavefield decomposition scheme for land acquisition. Exploiting the underlying theory of this scheme, we now consider conventional, multi-component (MC) decomposition as an inverse problem, which we try to constrain in a better way. We have overdetermined the inverse problem by adding an MDL equation that exploits the Dirichlet free-surface boundary condition. To investigate the successfulness of this approach, we have applied both MC and combined MC-MDL decomposition to a real land dataset acquired in Annerveen, the Netherlands. Comparison of the results of overdetermined MC-MDL decomposition with the results of MC wavefield decomposition, clearly shows improvements in the obtained one-way wavefields, especially for the downgoing fields.



#### Introduction

Decomposed wavefields form the basis for various surface-related multiple elimination and deghosting procedures (e.g. Frijlink et al. (2011)) and for depth imaging using primary and multiple reflections (e.g. Muijs et al. (2007)). Novel methodologies that make use of downhole sensors, such as the virtual source method (Bakulin and Calvin, 2006), rely on decomposing the seismic wavefield at depth. However, applying wavefield decomposition to a real data set is often quite challenging. The conventional, multi-component (MC) wavefield decomposition scheme makes use of different recorded wavefield components, for example both pressure (P) and vertical component particle velocity ( $V_z$ ) data (e.g. (Day et al., 2013)). In practice, not all wavefield quantities required for the multi-component (MC) wavefield decomposition might be available. In addition, recordings can be obscured by different sensor characteristics, requiring calibration (Schalkwijk et al., 2003). In recent years, we can notice an emerging acquisition design in industry which makes use of downhole sensor arrays (e.g. Bakulin et al. (2012)). Inspired by marine acquisition designs that make use of recordings at multiple depth levels for successful wavefield decomposition (e.g. Moldoveanu et al. (2007)), we have developed a multi-depth level (MDL) wavefield decomposition scheme for land acquisition (Grobbe et al., 2013). We now interpret MC wavefield decomposition as an inverse problem. We will here investigate whether we can use the underlying MDL decomposition equations as an additional inversion constraint for the MC decomposition (MC-MDL), thereby combining the best of both worlds. We perform MC decomposition and MC-MDL decomposition on a real land dataset acquired in Annerveen, the Netherlands.

#### Theory

In the MC wavefield decomposition schemes, the downgoing and upgoing flux-normalized one-way wavefields, denoted by  $\tilde{p}^+$  and  $\tilde{p}^-$ , respectively, can be obtained by left-multiplying the two-way wavefield vector  $\tilde{q}$  with the inverse of the composition matrix  $\tilde{L}$ , thereby inverting the forward problem

$$\begin{pmatrix} \tilde{\mathbf{q}}_1 \\ \tilde{\mathbf{q}}_2 \end{pmatrix} = \begin{pmatrix} \tilde{\mathbf{L}}_1^+ & \tilde{\mathbf{L}}_1^- \\ \tilde{\mathbf{L}}_2^+ & \tilde{\mathbf{L}}_2^- \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{p}}^+ \\ \tilde{\mathbf{p}}^- \end{pmatrix}.$$
 (1)

Here, the + sign indicates downgoing wavefields, the – sign indicates upgoing wavefields and the tilde sign indicates that we are working in the horizontal wavenumber-frequency domain. Here,  $\tilde{\mathbf{L}}_1^{\pm}$  and  $\tilde{\mathbf{L}}_2^{\pm}$  represent submatrices of the energy flux-normalized composition matrix  $\tilde{\mathbf{L}}$  that depend on the medium properties at the receiver level (e.g. Wapenaar (1998)). In principle, any normalization of the composition matrix will work. Further,  $\tilde{\mathbf{q}}_1$  and  $\tilde{\mathbf{q}}_2$  represent subvectors of the two-way field quantity vector  $\tilde{\mathbf{q}}$ . Equation 1 holds for all physical wave phenomena, like for example acoustic, elastodynamic and seismoelectric wavefields. As can be observed in (1), in order to be able to perform the up/down decomposition correctly, all two-way wavefield components of  $\tilde{\mathbf{q}}$  must have been recorded. In practice, not all of these field quantities might be available, or they might be obscured by different sensor characteristics (Schalkwijk et al., 2003). In the MDL decomposition scheme, the decomposition problem is treated slightly different (Moldoveanu et al. (2007), Grobbe et al. (2013)). We write the decomposed downgoing and upgoing flux-normalized wavefields at one depth level in terms of the other, respectively:

$$\tilde{\mathbf{p}}_{A}^{+} = \tilde{\mathbf{F}}^{+}(z_{A}, z_{B})\tilde{\mathbf{p}}_{B}^{+}; \qquad \tilde{\mathbf{p}}_{A}^{-} = \tilde{\mathbf{W}}^{-}(z_{A}, z_{B})\tilde{\mathbf{p}}_{B}^{-},$$
(2)

where,  $z_A < z_B$  and z increases with depth. The inverse wavefield extrapolation operator  $\tilde{\mathbf{F}}^+(z_A, z_B)$  in equation (2) is closely related to the forward propagator  $\tilde{\mathbf{W}}^-(z_A, z_B)$  as (Wapenaar, 1998):

$$\tilde{\mathbf{F}}^+(z_A, z_B) \approx (\tilde{\mathbf{W}}^-(z_A, z_B))^*.$$
(3)

Here, the asterix (\*) denotes complex conjugation. The approximation sign is applied because this equation is not valid for the evanescent wavefield. The forward wavefield extrapolation operator  $\tilde{\mathbf{W}}^{-}(z_A, z_B)$ , extrapolates the upgoing (-) wavefield forward, from depth level  $z_B$  to depth level  $z_A$ . The inverse wavefield extrapolation operator  $\tilde{\mathbf{F}}^{+}(z_A, z_B)$ , extrapolates the downgoing wavefield (+) backward from depth



level  $z_B$  to depth level  $z_A$ . Using these wavefield extrapolation operators, we can express the one-way wavefields at one level in terms of the observed fields at multiple levels, which forms the basis of the MDL decomposition scheme (Moldoveanu et al. (2007), Grobbe et al. (2013)). Let us now look at the decomposition problem as an inverse problem. Starting from equation 1, we try to improve the decomposition with an additional inversion constraint: the free-surface condition from the MDL decomposition scheme, where depth level  $z_A$  coincides with the free-surface. This corresponds to the Annerveen acquisition geometry, where  $z_A = 0$  m and  $z_B = 50$  m. This leads to the following overdetermined inverse problem:

$$\begin{pmatrix} \tilde{\mathbf{q}}_{1,B} \\ \tilde{\mathbf{q}}_{2,B} \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} \tilde{\mathbf{L}}_{1+B}^{+} & \tilde{\mathbf{L}}_{1-B}^{-} \\ \tilde{\mathbf{L}}_{2,B}^{+} & \tilde{\mathbf{L}}_{2-B}^{-} \\ \tilde{\mathbf{L}}_{1+B}^{+} \tilde{\mathbf{F}}^{+} & \tilde{\mathbf{L}}_{1-B}^{-} \tilde{\mathbf{W}}^{-} \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{p}}_{B}^{+} \\ \tilde{\mathbf{p}}_{B}^{-} \end{pmatrix}.$$
(4)

For our real data example, we will consider scalar versions of equations 1, referred to as MC decomposition, and 4, referred to as MC-MDL decomposition. Here,  $\tilde{q}_1 = \tilde{P}$ , the acoustic pressure field, and  $\tilde{q}_2 = \tilde{V}_z$ , the vertical component of the particle velocity. The flux-normalized scalars  $\tilde{L}_1$  and  $\tilde{L}_2$ , as well as the scalar wavefield extrapolation operators  $\tilde{W}^-$  and  $\tilde{F}^+$ , are taken as defined in Wapenaar (1998). As can be observed, the added row in equation 4 overdetermines the inverse problem, but does not require additionally recorded fields. The added equation makes use of the Dirichlet free-surface boundary condition, where the pressure at the free-surface ( $z_A$ ) equals zero. We will now investigate if this overdetermined inverse problem improves the decomposition results of an dynamite-source data set acquired at Annerveen, a village in the North of the Netherlands. The inverse problem will be solved in the leastsquares sense. Other approaches, like sparsity promotion (Van der Neut and Herrmann, 2012) could also be considered. The results will be presented for a selected shot-record of the dataset, but can be obtained for each shot individually.

### **Results and Discussion**

The data have been acquired on land in Annerveen, located in the North of the Netherlands. One receiver array consisting of 96 receivers with a spacing of 11.75 meters was buried at 50 meters depth. In addition, 144 shots were carried out at 4 meters depth with a source spacing of 11.75 meters, alternating positions with respect to the receiver positions. The receivers have registered both the pressure and vertical component particle velocity fields. Several initial data processing steps need to be performed, before we carry out the wavefield decomposition. We use standard filtering techniques to filter out the surface-waves as good as possible. In addition, a few dead traces are removed. Since we are dealing with a pseudo-2D data set, we correct the amplitudes with the square root of time. In addition, the data show quite a variety in amplitudes for different shots. Therefore, we carry out a shot normalization, where we normalize the shotgathers with the power of each shot. Since the MC-MDL decomposition scheme assumes depth level  $z_A$  to be coinciding with the free-surface of the Earth, and depth level  $z_B$  corresponding to the receiver level at 50 m depth, one can directly notice that our source in this configuration is located between the two depth levels. The theory does not account for this configuration (Grobbe et al., 2013). However, by removing the incident fields from the data set (i.e. direct field and direct source ghost), the MC-MDL decomposition can still be applied to the remaining reflected data set. If desirable, we can treat the incident fields independently (for example in MDD applications). We remove the direct field by applying a time gate, which has been selected by visual inspection (Figure 1). We carry out the visual inspection looking at an average over 10 clean shots, arranged according to offset. The underlying assumption of this approach is that the Earth is horizontally layered over the distance of these 10 shots, which is a reasonable assumption considering the area of interest.

The crucial parameter for our acoustic case MC-MDL decomposition that needs to be determined, is the P-wave velocity in the layer between depth levels  $z_A$  and  $z_B$ . The P-wave velocity determines, via the vertical wavenumber  $k_z$ , the forward and inverse extrapolation operators  $\tilde{W}^-$  and  $\tilde{F}^+$ , respectively. Furthermore, the P-wave velocity is important in the composition matrix  $\tilde{L}$  (Wapenaar, 1998). Here, we determine the P-wave velocity by looking at the arrival time difference between a strong upgoing





**Figure 1** (a) Pressure data average over 10 shots, offset arranged. The black line represents the start of the Hanning taper (length 10, downwards), separating incident fields from reflected fields. The arrows indicate the events used for the P-wave velocity estimate. (b) Same as (a), but now for  $V_z$ . (c) Shot-record of two-way reflected P data. The black box represents the window for calibrating P and  $V_z$  and also the window for the upgoing event free-surface calibration. The dark green box indicates the selected downgoing event for the free-surface calibration. **d)** Same as (c), but now for  $V_z$ .

reflection and its receiver side ghost. To identify these two events, we make use of the two individual pressure and particle velocity datasets, and exploit our knowledge about polarity reversal of registered events. Effectively, this means that P and  $V_{z}$  have opposite polarity for the first upgoing reflection, but identical polarities for the later receiver side ghost. This can be clearly observed in Figure 1, indicated by the two arrows. Based on the zero-offset time difference between those two events and knowing the propagation pathlength ( $2 \times 50 = 100 \text{ m}$ ), the P-wave velocity can be estimated. Our best estimate of the P-wave velocity is  $c_P = 1639 m/s$ . Exact knowledge of the density is not required, since it is just a scalar that occurs in each element of the composition matrix. To precondition the inversion, we scale composition matrix element  $\tilde{L}_2$  with the impedance, resulting in a better-posed inverse problem. We start with the MC wavefield decomposition, according to equation 1. Since both the pressure and particle velocity data are involved simultaneously in MC decomposition schemes, we want to make sure that the sensors are correctly calibrated. We therefore focus on a clear event in the two-way recorded dataset and select a data window around this upgoing event. We select the top window (black box), as indicated in Figures 1c and 1d. We use a least-squares minimization subtraction algorithm to find the correct scaling factor between the pressure and particle velocity data and scale the data accordingly, such that the energy of this upgoing event is minimized in the downgoing gather. We now carry out the MC decomposition, resulting in the decomposed flux-normalized one-way wavefields shown in Figures 2a and 2b. Next, we focus on the MC-MDL decomposition. Looking at row 3 of equation 4, we observe that the following relation must hold at the free-surface

$$\tilde{L}_{1}^{+}\tilde{F}^{+}\tilde{p}_{B}^{+} = -\tilde{L}_{1}^{-}\tilde{W}^{-}\tilde{p}_{B}^{-}.$$
(5)

This equation also holds for an individual event. We enforce equation 5 to hold by selecting a certain upgoing event and its corresponding downgoing event, indicated with the two boxes in Figures 1c and 1d. The term  $\tilde{L}_1^+ \tilde{p}_B^+$  then corresponds to the selected downgoing event in the two-way pressure dataset, illustrated by the dark green boxes in Figures 1c and 1d, and  $\tilde{L}_1^- \tilde{p}_B^-$  to the selected upgoing event in the two-way pressure dataset, indicated by the black boxes in Figures 1c and 1d. We will propagate the two-way dataset, including the selected upgoing event, forward in time to the free-surface using  $\tilde{W}^-$ . Secondly, we will propagate the two-way dataset, including the selected upgoing event, backward in time to the free-surface. Here, equation 5 must hold. We now calibrate the two-way events at the free-surface with each other, using a least-squares minimization subtraction algorithm on the selected event. A similar minimization problem has been defined for the vertical component particle velocity field. Both minimization problems are solved for simultaneously and the calibration factor is applied to



the matrix element containing  $\tilde{F}^+$  (in equation 4). The overall weight of the bottom row in equation 4 can be further tuned according to preference. We are now all set to carry out the MC-MDL wavefield decomposition. The results of this overdetermined MC-MDL decomposition problem are shown in

![](_page_4_Figure_2.jpeg)

*Figure 2* (a) Upgoing, MC decomposed fields. (b) Downgoing, MC decomposed fields. (c) Upgoing, MC-MDL decomposed fields. (d) Downgoing, MC-MDL decomposed fields.

Figures 2c and 2d. What can be clearly observed is that by adding the extra constraint to the inversion, we have improved the decomposition results, especially for the downgoing fields. In addition, it can be observed that the MC decomposition result has vertical 'white' bands at certain offsets, corresponding to dead or noisy traces in the two-way recorded data. Our MC-MDL decomposition result does not show these 'white' bands so strongly. This is explainable due to the applied wavefield extrapolation operators in the wavenumber-frequency domain, implicitly yielding an interpolation between the traces.

## Conclusions

We have carried out a multi-component (MC) wavefield decomposition on a real land dataset. Considering decomposition as an inverse problem, we have shown that by adding an extra equation to the MC composition matrix, we can overdetermine the inverse problem. Since this equation makes use of the Dirichlet free-surface boundary condition, we do not require additionally recorded fields. Comparison of the results of this overdetermined MC-MDL decomposition scheme with the results of the conventional MC wavefield decomposition, clearly shows improvements in the obtained one-way flux-normalized wavefields, especially for the downgoing fields.

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