Autofocusing for retrieving the Green's function in the presence of a free surface.

Satyan, Singh[1], Roel, Snieder[1], Jyoti Behura[1], Joost, van Der Neut[2], Kees, Wapenaar[2], Evert, Slob[2]
1. Colorado School of Mines
2. Delft University of Technology

Summary

Recent work on autofocusing with the Marchenko equation has shown how the Green's function for a virtual source in the subsurface can be obtained from reflection data. The response to the virtual source is the Green's function from the location of the virtual source to the surface. The Green's function is retrieved using only the reflection response of the medium and an estimate of the first arrival at the surface from the virtual source. Current techniques, however, only include primaries and internal multiples. Therefore, all surface-related multiples must be removed from the reflection response prior to the Green's function retrieval. Here, we present a new scheme that includes primaries, internal multiples, and free-surface multiples. In other words, we retrieve the Green's function in the presence of the free surface. The information needed for the retrieval are the reflection response at the acquisition surface and an estimate of the first arrival at the surface from the virtual source. The reflection response, in this case, includes the free-surface multiples; this makes it possible to include these multiples in the imaging operator and it obviates the need for surface-related multiple elimination.

Introduction

To focus a wavefield at a point in a medium only requires surface reflection data and an estimate of the first arriving wave at the surface from a point source at the focusing location (Broggini et al. [2012], Broggini and Snieder [2012], and Wapenaar et al. [2013]). Unlike in seismic interferometry (Bakulin and Calvert [2006]), no receivers are required at the desired focusing location, i.e. the virtual source location. Significantly, the detailed medium parameters need not be known to focus the wavefield. However the travel-time of the direct-arrival of the virtual source to the surface is required. To obtain this travel time one only needs a macro-model of the velocity.

The focusing scheme of Broggini et al. [2012], Broggini and Snieder [2012], and Wapenaar et al. [2013] is an extension of the algorithm of Rose [2002a,b] who shows an iterative scheme that solves the Marchenko equation for wavefield focusing in one dimension. The focused events in the wavefield for the virtual source consist of primaries and internal multiples (Wapenaar et al. [2013]) but not free-surface multiples. Importantly, Rose [2002a,b] derived the focusing method (auto-focusing) for single-sided illumination with sources and receivers on one side of the medium, similar to current geophysical acquisition methods.

The algorithm of Broggini et al. [2012] requires the removal of free-surface multiples from the reflection response of the medium to retrieve the Green's function by autofocusing. The removal of the free-surface multiples can be achieved by Surface Related Multiple Elimination (SRME) (Verschuur et al. [1992]).

In this paper, we modify the focusing algorithm of Rose [2002b], Broggini et al. [2012] and Wapenaar et al. [2013] to focus not only primaries and internal multiples but also the free-surface multiples. We achieve such focusing using reflected waves in the presence of a free surface and an estimate of the first arrival from the focus location to the surface. Notably, our proposed auto-focusing scheme obviates the need for SRME. In addition, we show 1D examples of the retrieved Green's function in a 9 layer model in comparison to the model Green's function.

Theory

The theory of focusing the wavefield without a free surface, i.e. retrieving the Green's function G_0, is discussed by Rose [2002b], Broggini et al. [2012], and Wapenaar et al. [2013]. In our notation, any wavefield quantity with a subscript 0 (e.g R_0) signifies that no free-surface multiples are present. In the focusing scheme of Broggini et al. [2012], and Wapenaar et al. [2013] they remove the free-surface multiples from the reflection response R (by SRME) to get R_0 and then compute G_0, the Green's function in the absence of the free surface.

We generalize the formulation of Wapenaar et al. [2013] to include free-surface multiples. In our case, the reflections from the free surface are included in the focusing scheme similar to the treatment by Wapenaar et al. [2004] of free-surface multiples; hence no SRME is required.

We begin by defining our spatial vector field by its horizontal coordinates and depth coordinates, for instance, $\mathbf{x_0} = (\mathbf{x_H}, x_{3,0})$, where $\mathbf{x_H}$ are the horizontal coordinates at a depth $x_{3,0}$. We define a solution for the waves that focus at a point in a medium, called the focusing solutions. Wapenaar et al. [2013] define two focusing solutions; f_1 and f_2. The f_1 solution involves waves that focus at $\mathbf{x_i'}$ at a defined depth level (∂D_i) for incoming and outgoing waves at the acquisition surface (∂D_0) at $\mathbf{x_0}$. The solution f_2 is somewhat the opposite of f_1 as it is a solution for waves that focus just above ∂D_0 at $\mathbf{x_0''}$ for incoming and outgoing waves at ∂D_i. The focusing solutions exist in a reference medium that has the same material properties as the actual inhomogeneous medium between ∂D_0 and ∂D_i and that is homogeneous above ∂D_0 and reflection-free below ∂D_i. Therefore, the boundary conditions on ∂D_0 and ∂D_i in the reference medium, where the focusing solution exist, are reflection free. Note that this boundary condition need not be the same as the actual medium. The focusing solutions can be separated into up-going and down-going waves; the first focusing solution in the frequency domain reads (Wapenaar et al. [2013])

$$f_1(\mathbf{x},\mathbf{x_i'},\omega) = f_1^+(\mathbf{x},\mathbf{x_i'},\omega) + f_1^-(\mathbf{x},\mathbf{x_i'},\omega), \tag{1}$$

while the second focusing solution reads

$$f_2(\mathbf{x},\mathbf{x_0''},\omega) = f_2^+(\mathbf{x},\mathbf{x_0''},\omega) + f_2^-(\mathbf{x},\mathbf{x_0''},\omega). \tag{2}$$

We find relationships between the focusing solutions by separating these solutions into one-way wavefields (Wapenaar et al. [2013]) and applying these wavefields to reciprocity theorems, Wapenaar and Grimbergen [1996]. For instance, in the frequency domain, the up-going wavefield of f_1 at ∂D_0 is $f_1^-(\mathbf{x_0},\mathbf{x_i'},\omega)$ while the down-doing wavefield is $f_1^+(\mathbf{x_0},\mathbf{x_i'},\omega)$. At or just below ∂D_i, the up-going wavefield of f_1 is 0 since in the reference medium below ∂D_i is homogeneous, while the down-going wavefield is $f_1^+(\mathbf{x_i},\mathbf{x_i'},\omega) = \delta(\mathbf{x_H} - \mathbf{x_H'})$. The solution f_2 is separated into one-way wavefields using similar reasoning, (more details of the relationships between these solutions are given in Wapenaar et al. [2013]). The relationship between the focusing solutions are (Wapenaar et al. [2013]):

$$f_1^+(\mathbf{x_0''},\mathbf{x_i'},\omega) = f_2^-(\mathbf{x_i'},\mathbf{x_0''},\omega), \tag{3}$$

and

$$-f_1^-(\mathbf{x_0''},\mathbf{x_i'},\omega)^* = f_2^+(\mathbf{x_i'},\mathbf{x_0''},\omega). \tag{4}$$

The wavefields in our actual medium can also be separated into one-way wavefields at the different depth levels, i.e. ∂D_0 and ∂D_i, as shown in Figure 1. Note, that the additional one-way wavefields that are added to the actual medium, in our case, in the presence of the free surface in comparison to without the free surface are the reflected waves from the free surface rR. In Figure 1, rR are the reflected waves from the free surface, where r is the reflection coefficient of the free surface and R are the recorded reflected waves from the subsurface. Consequently, in our case, the Green's functions at the different depth levels all include reflected waves from the free surface.

We use the convolution and cross-correlation reciprocity theorems to find relationships for the one-way wavefields of f_1 and the wavefields in the actual medium:

$$G^{-,+}(\mathbf{x_i'},\mathbf{x_0''},\omega) = \int_{\partial D_0} [f_1^+(\mathbf{x_0},\mathbf{x_i'},\omega) - r f_1^-(\mathbf{x_0},\mathbf{x_i'},\omega)] \; R(\mathbf{x_0},\mathbf{x_0''},\omega) d\mathbf{x_0} \; - f_1^-(\mathbf{x_0''},\mathbf{x_i'},\omega), \tag{5}$$

and

$$G^{+,+}(\mathbf{x_i'},\mathbf{x_0''},\omega) = \int_{\partial D_0} -[f_1^-(\mathbf{x_0},\mathbf{x_i'},\omega) - r f_1^+(\mathbf{x_0},\mathbf{x_i'},\omega)]^* \; R(\mathbf{x_0},\mathbf{x_0''},\omega) d\mathbf{x_0} \; + f_1^+(\mathbf{x_0''},\mathbf{x_i'},\omega)^*, \tag{6}$$

where * represents the complex conjugate, and 'r' is the reflection coefficient of the free surface. R is flux normalized so that the one-way reciprocity equations (Wapenaar and Grimbergen [1996]) holds, it follows that $r = -1$. Note the up-going Green's function ($G^{-,+}$) in the actual inhomogeneous medium at ∂D_0 is the reflection response R for a downward radiating source.

The two-way Green's function is obtained by adding equations 5 and 6 as well as using equations 1, 2 and the relationship between f_1 and f_2 (equation 3 and 4):

$$G(\mathbf{x_i'},\mathbf{x_0''},\omega) = \int_{\partial D_0} [f_2(\mathbf{x_i'},\mathbf{x_0},\omega) + r f_2(\mathbf{x_i'},\mathbf{x_0},\omega)^*] \; R(\mathbf{x_0},\mathbf{x_0''},\omega) d\mathbf{x_0} \; + f_2(\mathbf{x_i'},\mathbf{x_0''},\omega)^*. \tag{7}$$

We retrieve G the same way we retrieve G_0 as discussed in Wapenaar et al. [2013], except we use equation 7 instead of equation 8 for the Green's function equation.

$$G_0(\mathbf{x_i'},\mathbf{x_0''},\omega) = f_2(\mathbf{x_i'},\mathbf{x_0''},\omega)^* + \int_{\partial D_0} f_2(\mathbf{x_i'},\mathbf{x_0},\omega) R_0(\mathbf{x_0},\mathbf{x_0''},\omega) d\mathbf{x_0}. \tag{8}$$

Equation 8 is the expression to retrieve the Green's function which includes primaries and internal multiples but not free-surface multiples.

Importantly, equation 7 simplifies to equation 8 in the limiting case when $r \to 0$ since we will no longer have reflections from the free surface.

Numerical Examples

We consider a 1D model that has a high impedance layer generic to salt models as shown in Figure 2(a). Receivers were placed $5m$ below the surface to record the reflected waves. In 1D to retreive the Green's function, it is sufficient to use the travel time of the first arriving wave from the virtual source to the surface. However, in 3D media, a smooth version of the slowness (1/velocity) is needed to get an estimate of the direct arriving wave from the virtual source to the surface. This estimate can be obtained using finite-difference modeling of the waveforms.

This Green's function G is arbitrarily scaled to its maximum amplitude (Figure 2(b)), it is the response at the surface, ∂D_0, to the virtual source (located at 2.25 km [red dot, Figure 2(a)]). We also model the Green's function using finite differences to ensure that the Green's function retrieved from our autofocusing algorithm is accurate, we superimposed this result on Figure 2(b). Figure 2(b) is zoomed in to better illustrate the model and retrieved Green's function, for this reason the first arrival at time 0.85s is clipped. The difference between the modeled and the retrieved Green's function is negligible relative to the average amplitude of the Green's function, as seen in Figure 2(b), and can be attributed to numerical errors.

Conclusion

We extended the retrieval of the Green's function to include the presence of a free surface. This method recovers internal multiples, and now also free-surface multiples. Signicantly, our proposed method does not require any surface-related multiple removal of the reflection response. In addition, we need an estimate of the first arrival at the surface from the virtual source in the subsurface. To obtain the first arrival, we only need a macro model of the velocity, but the small scale details of the velocity and density need not be known.

Acknowledgments

This work was funded by the sponsor companies of the Consortium Project on Seismic Inverse Methods for Complex Structures. We are also grateful to Diane Witters for her help in preparing this abstract.

References

Bakulin, A. and Calvert, R. [2006] The virtual source method: Theory and case study. *Geophysics*, **71**(4), SI139–SI150.

Broggini, F. and Snieder, R. [2012] Connection of scattering principles: a visual and mathematical tour. *European Journal of Physics*, **33**(3), 593.

Broggini, F., Snieder, R. and Wapenaar, K. [2012] Focusing the wavefield inside an unknown 1D medium: Beyond seismic interferometry. *Geophysics*, **77**(5).

Rose, J. [2002a] Time reversal, focusing and exact inverse scattering. *Imaging of complex media with acoustic and seismic waves*, 97–106.

Rose, J. [2002b] 'Single-sided'autofocusing of sound in layered materials. *Inverse problems*, **18**(6), 1923.

Verschuur, D., Berkhout, A. and Wapenaar, C. [1992] Adaptive surface-related multiple elimination. *Geophysics*, **57**(9), 1166–1177, ISSN 0016-8033, doi:10.1190/1.1443330.

Wapenaar, C. and Grimbergen, J. [1996] Reciprocity theorems for one-way wavefields. *Geophysical Journal International*, **127**(1), 169–177.

Wapenaar, K., Broggini, F., Slob, E. and Snieder, R. [2013] Three-dimensional single-sided marchenko inverse scattering, data-driven focusing, green's function retrieval, and their mutual relations. *Phys. Rev. Lett.*, **110**, 084301, doi:10.1103/PhysRevLett.110.084301.

Wapenaar, K., Thorbecke, J. and Draganov, D. [2004] Relations between reflection and transmission responses of three-dimensional inhomogeneous media. *Geophysical Journal International*, **156**(2), 179–194, doi:10.1111/j.1365-246X.2003.02152.x.

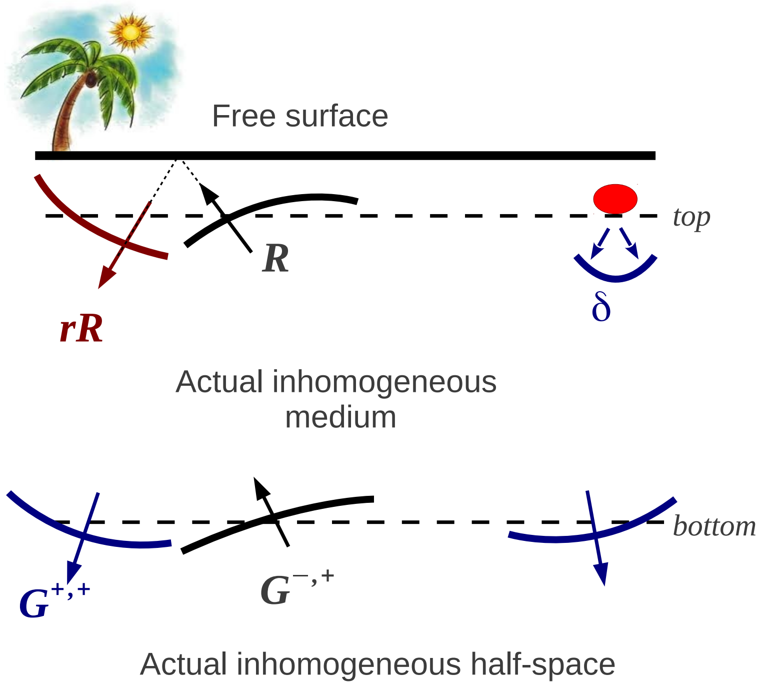

Figure 1 *Green's functions in the actual inhomogeneous medium in the presence of a free surface.*

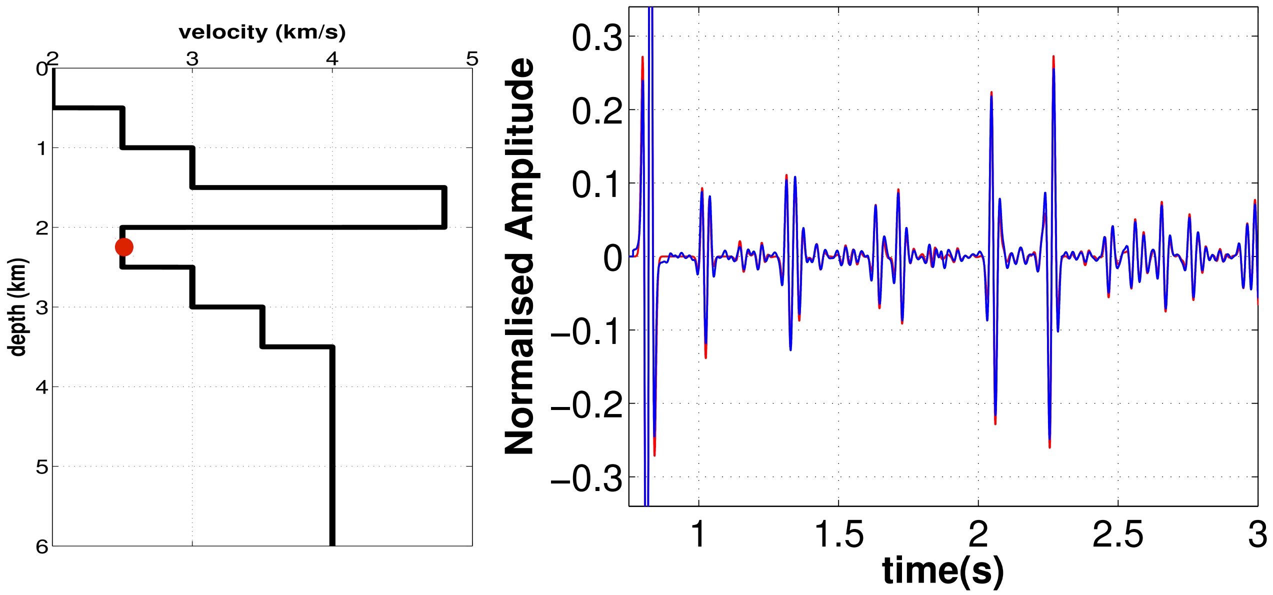

Figure 2 *(a) Velocity model with dot indicating the position of the virtual source, (b) Retrieved Green's function (blue) and model Green's function (red) with normalized amplitudes.*