Virtual plane-wave retrieval and imaging via Marchenko redatuming

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Summary

Marchenko redatuming is a novel scheme used to retrieve up- and down-going Green's functions in an unknown medium. Marchenko equations are based on reciprocity theorems and are derived on the assumption of the existence of so called focusing functions, i.e. functions which exhibit time-space focusing properties once injected in the subsurface. In contrast to interferometry but similarly to standard migration methods, Marchenko redatuming only requires an estimate of the direct wave from the virtual source (or to the virtual receiver), illumination from only one side of the medium, and no physical sources (or receivers) inside the medium. In this contribution we consider a different time-focusing condition within the frame of Marchenko redatuming and show how this can lead to the retrieval of virtual plane-wave-responses, thus allowing multiple-free imaging using only a 1 dimensional sampling of the targeted model. The potential of the new method is demonstrated on a 2D synthetic model.

Introduction

Marchenko redatuming estimates Green's functions between the earth's surface and arbitrary locations in the subsurface (Broggini et al. (2012b) ; Wapenaar et al. (2014)). These Green's functions are evaluated using reciprocity theorems involving so called 'focusing functions', i.e. wavefields which achieve a time-space focus in the subsurface. Redatumed Green's functions can be used to provide multiple-free images directly (Broggini et al. (2014)). However, this approach requires as many virtual sources as there are image points in the subsurface and its cost is a linear function of the size of the area to be imaged.
Despite its requirements on the quality of the reflection response, the Marchenko scheme has already been successfully applied to field data (Ravasi et al. (2016); Staring et al. (2017)), and recent advances have shown how the requirements above can be considerably relaxed (Ravasi (2017)).
Here we show how focusing functions associated with virtual plane-wave-responses can be derived by solving a modified Marchenko equation. The virtual plane-wave-responses can be used to efficiently image the subsurface involving only a fraction of virtual-responses as compared to standard Marchenko methods, combining the areal-source methods for primaries (Rietveld et al. (1992)) and the extended virtual-source Marchenko method addressed by Broggini et al. (2012a).

Method and Theory

Reciprocity theorems for one-way flux-normalized wavefields relate up- and down-going wavefield components of two states A and B evaluated at two depths. Convolution and cross-correlation reciprocity theorems can be expressed in the time domain as follows (Wapenaar and Grimbergen (1996)):

$$\int_{\Lambda_a} d^2\mathbf{x}\{p_A^+ * p_B^- - p_A^- * p_B^+\} = \int_{\Lambda_f} d^2\mathbf{x}\{p_A^+ * p_B^- - p_A^- * p_B^+\}, \quad (1)$$

$$\int_{\Lambda_a} d^2\mathbf{x}\{p_A^+ \star p_B^+ - p_A^- \star p_B^-\} = \int_{\Lambda_f} d^2\mathbf{x}\{p_A^+ \star p_B^+ - p_A^- \star p_B^-\}, \quad (2)$$

where superscripts $+$ and $-$ indicate down- and up-going constituents, Λ_a and Λ_f stand for two arbitrary depth levels and $*$ and $\star$ correspond to convolution and cross-correlation, respectively. Equations (1) and (2) assume that the medium parameters are identical for both states in the volume circumscribed by Λ_a and Λ_f, and that no sources exist between these depth levels. Moreover, while (1) is valid for lossy media, (2) requires the medium to be lossless between the levels Λ_a and Λ_f and neglects evanescent waves.

Following van der Neut et al. (2015) we will consider Λ_a and Λ_f to be the acquisition surface and a redatuming level, respectively. Moreover, we consider for state A a truncated medium identical to the physical medium above Λ_f and reflection-free below this level, while for state B we choose the physical medium. In standard space-time focusing, it is assumed that the down-going component of the focusing function, i.e. f_1^+, satisfies the following property along $\Lambda_f : f_1^+(\mathbf{x}, z_f; \mathbf{x}_F, z_f; t) = \delta(t)\delta(\mathbf{x} - \mathbf{x}_F)$, where $\mathbf{x}_F, z_f$ are the coordinates of focal point in the subsurface. For the time-focusing approach, we also assume the medium in state A to be truncated below Λ_f. However, differently from the standard space-time focusing approach, we define F_1^+ as satisfying the following time-focusing property along $\Lambda_f : \forall \mathbf{x} \in \Lambda_f, F_1^+(\mathbf{x}, z_f; z_f, t) = \delta(t)$, where z_f is the depth of the horizontal focal plane in the subsurface. For state B, following the standard approach, we place a point source for a downgoing wavefield at $\mathbf{x}_B$ just above the surface (van der Neut et al. (2015)). Substituting these definitions into equations (1) and (2) we get:

$$\mathbf{f_1^-} + \int_{\Lambda_f} d^2\mathbf{x}\mathbf{g}^- = \mathbf{R}\mathbf{f_1^+}; \quad \mathbf{f_1^+} - \int_{\Lambda_f} d^2\mathbf{x}\mathbf{g}^{+\star} = \mathbf{R}^\star\mathbf{f_1^-}, \quad (3)$$

where the superscript $\star$ indicates time-reversal. This set of equations differs from the system derived in standard Marchenko redatuming, where thanks to the space-time focusing properties of f_1^+ along Λ_f equation (3), reduces to:

$$\mathbf{f_1^-} + \mathbf{g}^- = \mathbf{R}\mathbf{f_1^+}; \quad \mathbf{f_1^+} - \mathbf{g}^{+\star} = \mathbf{R}^\star\mathbf{f_1^-}, \quad (4)$$

The underdetermined system in equation (4), which represents the basis for standard Marchenko re-

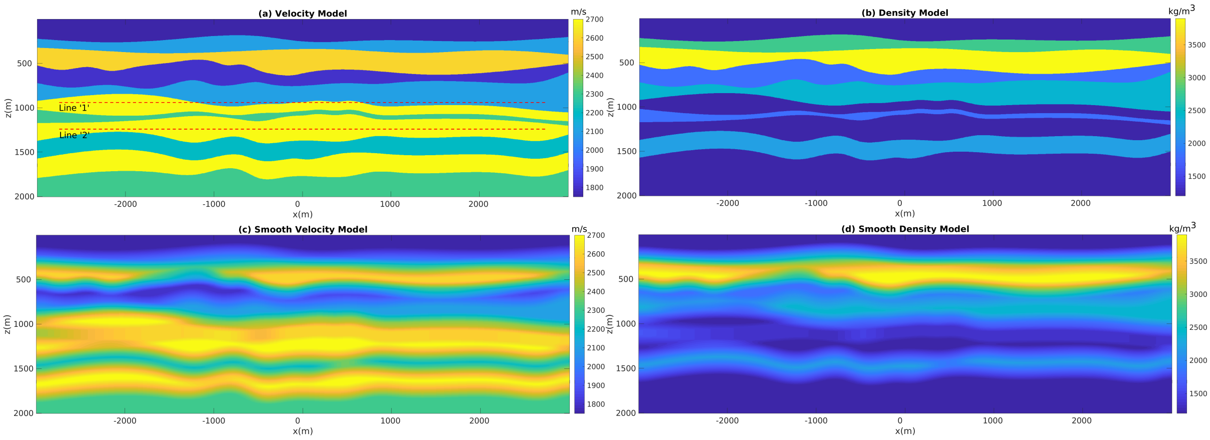

Figure 1 *(a) Velocity model of the synthetic model used in the numerical experiment. Dashed lines and stars represents subsurface planes and points for time and space-time focusing, respectively.(b) Density model of the synthetic model used in the numerical experiment. (c) and (d) Smooth Velocity and Density models used to provide input for Marchenko redatuming.*

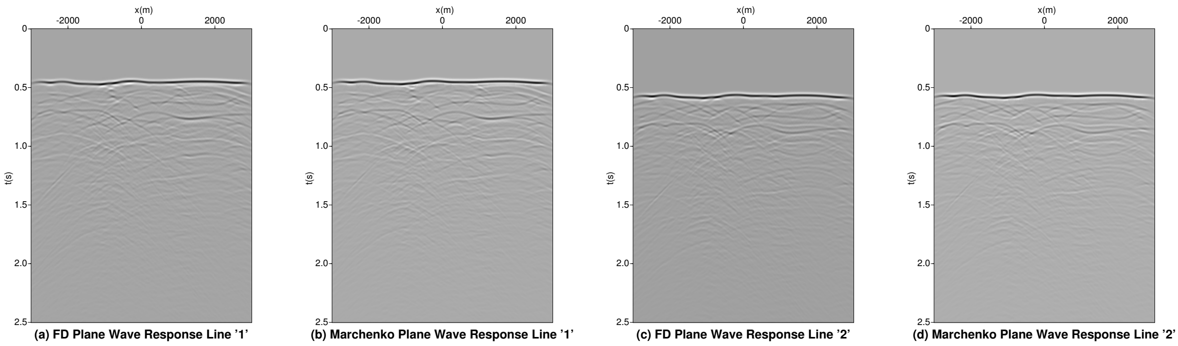

Figure 2 *(a) Forward modelled areal source response for Line '1', using the true model. (b) Retrieved areal source response for Line '1', using the Marchenko approach. (c) and (d), as for (a) and (b) but for Line '2'(FD stands for finite-difference forward modeling).*

datuming, can be additionally simplified invoking a separator operator Θ_f to annihilate the Green's functions terms:

$$\Theta_f \mathbf{f}_1^- = \Theta_f \mathbf{R}\mathbf{f}_1^+; \quad \Theta_f \mathbf{f}_1^+ = \Theta_f \mathbf{R}^\star \mathbf{f}_1^-. \tag{5}$$

This leads, after decomposing the focusing function into a direct and a coda component (i.e., setting $\mathbf{f}_1^+ = \mathbf{f}_{1d}^+ + \mathbf{f}_{1m}^+$, and using $\Theta_f \mathbf{f}_1^- = \mathbf{f}_1^-$ and $\Theta_\mathbf{f} \mathbf{f}_1^+ = \mathbf{f}_{1m}^+$), to the linear problem

$$[\mathbf{I} - \Theta_f \mathbf{R}^\star \Theta_f \mathbf{R}]\mathbf{f}_{1m}^+ = \Theta_f \mathbf{R}^\star \Theta_f \mathbf{R}\mathbf{f}_{1d}^+, \tag{6}$$

which, under standard convergence conditions, is solved by:

$$\mathbf{f}_1^+ = \sum_{k=0}^{\infty} (\Theta_f \mathbf{R}^\star \Theta_f \mathbf{R})^k \mathbf{f}_{1d}^+. \tag{7}$$

As outline above, the key ingredient to solve the underdetermined system in equation (4) is the existence of an appropriate annihilation operator. However, such an operator does not necessarily exist only for the space-time focusing system (4), as already preliminary observed in Broggini et al. (2012a) for slightly spatially-extended virtual sources.

Here we generalize the observation of Broggini et al. (2012a) within the context of the formalism of the system (4), now considering plane-wave spatially-extended sources. More precisely, we postulate

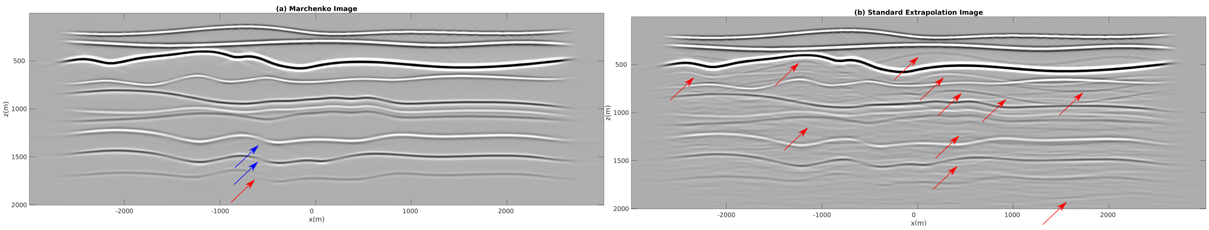

Figure 3 *(a) Migration result using the imaging condition of equation (11) and Marchenko redatumed virtual-plane wavefields. The red arrow points at a poorly imaged dipping layer, whereas the blue arrows point at similar structures that are properly imaged. (b) Migration result using standard one-way extrapolation of virtual-plane wavefields. Red arrows point at multiples-related artefacts.*

that when a focusing function $\mathbf{F}_1^+$ satisfies the time-focusing property discussed above, a separation operator Θ_F (based on the kinematics of the response of a plane-wave line-source corresponding to $\int_{\Lambda_f} d^2\mathbf{x} g(\mathbf{x}', z_a, t; \mathbf{x}, z_f, 0)$, where z_a and z_f indicate the depth levels of Λ_a and Λ_f, respectively) can be successfully applied to equation (3). The existence in this scenario of a separation operator reduces (3) into:

$$\Theta_F \mathbf{F}_1^- = \Theta_F \mathbf{R} \mathbf{F}_1^+; \quad \Theta_F \mathbf{F}_1^+ = \Theta_F \mathbf{R}^\star \mathbf{F}_1^-. \tag{8}$$

This leads, following again the decomposition into a direct and coda component of the down-going focusing function, to the standard solution for the focusing function:

$$\mathbf{F}_1^+ = \sum_{k=0}^{\infty} (\Theta_F \mathbf{R}^\star \Theta_F \mathbf{R})^k \mathbf{F}_{1d}^+. \tag{9}$$

However, since we crafted the focusing function to only focus in time, this scheme results in the retrieval of plane wave *up*- and *down*-going areal-receiver-responses (by invoking reciprocity, these responses can be related to the *down*- and *up*-propagating areal-sources-responses discussed in Rietveld et al. (1992)):

$$G^-(z_f; \mathbf{x}', z_a; t) = \int_{\Lambda_f} d^2\mathbf{x} g^-(\mathbf{x}, z_f; t; \mathbf{x}', z_a; 0); \quad G^+(z_f; \mathbf{x}', z_a; t) = \int_{\Lambda_f} d^2\mathbf{x} g^+(\mathbf{x}, z_f; t; \mathbf{x}', z_a; 0), \tag{10}$$

rather than standard *up*- and *down*-going Green's functions as in van der Neut et al. (2015). Once these plane-wave-responses are available, they could be used within the areal sources framework (Rietveld et al. (1992)).

Numerical Examples

We illustrate the potential of the iterative solutions algorithm for areal-sources-responses with a Finite Difference example (Thorbecke et al. (2017)) from a 2D inhomogeneous subsurface model (Figure 1). Figure 2 compares modelled and Marchenko areal-sources-responses at the surface for Lines '1' and '2', respectively. The match between the modelled and the retrieved areal-responses is very good, with mainly tapering-related minor differences in the left- and right-most portions of the gather.
With Marchenko areal-sources-responses we can use a single redatumed solution to image a whole line/plane at once, thus reducing considerably the computational burden of standard Marchenko imaging schemes.
To achieve this, we use the following redatumed reflectivity and standard migration imaging condition definitions:

$$R(\mathbf{x}, z_f, t) = \int_{\Lambda_a} d^2\mathbf{x}' g_d^+(\mathbf{x}, z_f, \mathbf{x}', z_a, t) \star G^-(z_f; \mathbf{x}', z_a, t); \quad Ic(\mathbf{x}, z_f) = R(\mathbf{x}, z_f, 0). \tag{11}$$

We apply our new imaging condition to the model discussed in the previous section. In this case we sample in depth every 5 meters, and consequently to image the entire domain we employ 400 hundred areal sources.

The migration associated with the imaging condition in (11) is shown in Figure 3(a). Each interface is properly imaged, while no multiples-related artefacts are present. Only a dipping layer in the bottom of the model is partially poorly imaged (red arrow in Figure 3(a)). However,others structures with similar geometry are properly imaged (blue arrows in Figure 3(a)). Multiples-related artefacts, on the other hand, contaminate the image if we migrate the up-going response associated with the same areal sources obtained through standard one-way wavefield extrapolation (Figure 3(b)). Note that in the migration step the same smooth models depicted in Figure 1(c) and (d) employed for Marchenko redatuming were used.

Conclusions

We have demonstrated that Marchenko methods can be successfully applied beyond conventional space-time focusing. We have discussed how a modified focusing condition relates areal-sources-responses to standard reflection data. A separation operator based on specifically designed direct focusing functions can then be applied to representation theorems to retrieve areal-sources-responses at the surface through standard Marchenko algorithms. The retrieved wavefields can be used for imaging at a fraction of the cost of standard Marchenko approaches, thus potentially being applicable also for 3D data-sets. An analysis of different focusing conditions and assessment of the resolution power of the proposed method with respect to standard Marchenko imaging will be the topic of future research.

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