Q-factor estimation and redatuming in a lossy medium using the Marchenko equation

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Summary

Marchenko Imaging is a new technology in geophysics, which enables us to retrieve Green's functions at any point in the subsurface having only reflection data. One of the assumptions of the Marchenko method is that the medium is lossless. One way to circumvent this assumption is to find a compensation parameter for the lossy reflection series so that the lossless Marchenko scheme can be applied. The main goals of this work are to: [1] use the Marchenko equation to estimate the attenuation in the subsurface, [2] find a compensation parameter for the lossy reflection series so that the lossless Marchenko scheme can be applied. We propose a novel approach which makes it possible to calculate an effective temporal Q-factor of the medium between a virtual source in the subsurface and receivers at the surface. This method is based on the minimization of the artefacts produced by the lossless Marchenko scheme. Artefacts have a very specific behavior: if the input data to the Marchenko equation is over- or under- compensated, the resulting artefacts will have an opposite polarity. Thus, they can be recognized. This approach is supported by a synthetic example for a 1D acoustic medium without a free surface.

Introduction

According to Green's Theorem, we can correctly retrieve the Green's functions of the medium if the data are available from all sides of the medium. Under several assumptions, this problem can be solved by using the 3D Marchenko equation (Wapenaar et al., 2014; van der Neut et al., 2015). The 3D Marchenko Equation's solution's main feature is the one-sided focusing function. It represents the injected wave field from one side of the medium which focuses inside the medium at a focal point. By applying this method to the reflection data, Marchenko imaging can be performed and Marchenko redatuming can be done at any depth level. One of the assumptions of the Marchenko method is that the medium is lossless. Slob (2016) extended the method for a dissipative acoustic medium but it requires double-sided data. Attenuation is one of the problems preventing the practical applications of the Marchenko imaging to real data; So circumventing this assumption is a worthwhile endeavor and paramount to this paper. If the lossy reflection response is used in Marchenko imaging, some artefacts in the Green's functions as well as in the seismic image are present. The main purposes of this study are to: [1] use the Marchenko equation to estimate the attenuation in the subsurface, [2] find a compensation parameter for the lossy reflection series so that the lossless Marchenko scheme can be applied. We propose the Artefact Removal Method which makes it possible to calculate an effective temporal Q-factor of the subsurface. This is achieved through minimization of the artefacts, which are produced by the lossless Marchenko scheme. The estimated attenuation of the medium can further be used to compensate for the attenuation in the lossy reflection series. This can then be applied back to the lossless Marchenko scheme.

Quantifying Attenuation using the Marchenko Equation

The method is based on the fact that the solution of the Marchenko equation is exact (NB: evanescent waves are excluded); However, there are artefacts present in the solution if some assumptions of the medium are not fulfilled. Artefacts in the solution are caused by: [A] numerical limitations (these artefacts are small) and [B] additional medium's assumptions. Properties which can cause some artefacts in the solution are: [i] the anisotropy of the medium, [ii] an incorrectly scaled source signature (Brackenhoff, 2016; Mildner et al., 2017), [iii] an incorrect background velocity model for the direct arrivals, [iv] an incorrect compensation for transmission losses and [v] an incorrect compensation for intrinsic losses (or, simply, lossy medium) etc. In this work, the medium assumptions [i]-[iv] are fulfilled. Therefore, the artefacts are present because the medium is lossy. These artefacts have a very specific behavior: if the input data to the Marchenko lossless scheme are over- or under- compensated, the resulting artefacts will have an opposite polarity (Alkhimenkov, 2017). Thus, they can be recognized. The artefacts are present in the upgoing Green's function and in the upgoing focusing function of the first kind proposed by Wapenaar et al. (2014) . Furthermore, even more artefacts are present in the redatumed reflection series. Thus, by applying different compensation parameters to the lossy reflection series, the artefacts can be removed and, hence, the correct compensation for the losses can be found. The method works very well for the medium where the losses can be compensated via an effective temporal Q-factor. If there is a layer/layers with a very strong attenuation, the effective temporal Q-factor can be found for an interval between the acquisition surface and the layer/layers with very strong attenuation. The layer with a very high attenuation can be found but this requires additional modeling.

In order to derive an expression which is able to recognize artefacts, we first calculated several upgoing Green's functions $G_i^{-,+}$ ("+" corresponds to the downgoing field of the source). Index i means that $G_i^{-,+}$ was calculated using the compensated lossy reflection response with different compensation parameter $\zeta_{compensation}^i$ applied to the lossy reflection series. The artefacts change polarity, therefore, the artefacts can be found by calculating the function $\mathscr{T}(\mathbf{x},t)$ (similar to (Mildner et al., 2017)):

$$\mathscr{T}(\mathbf{x},t) = \left[\left| \sum_i G_i^{-,+}(\mathbf{x},t) \right| - \sum_i \left| G_i^{-,+}(\mathbf{x},t) \right| \right] \cdot e^{bt} \qquad (1)$$

The exponent e^{bt} was added to this expression to increase the energy of the artefacts at longer times. Parameter b is a free parameter and can be set to 1, or another value. It depends on whether we want to have all artefacts be equal amplitudes or not. It is suggested to choose the value of b in such a way that all artefacts have similar amplitudes.

The approach is as follows: [1] calculate a set of compensated lossy reflection series R_i by multiplying the data by $exp\left(+\zeta_{compensation}^i\, t\right)$. Then, use this set of reflection series as input to the Marchenko equation and focus the wave field in the subsurface at different depth levels; [2] Calculate the function $\mathcal{T}(\mathbf{x},t)$ using equation 1. Identify the time intervals, t_l in $\mathcal{T}(\mathbf{x},t)$, where the artefacts occur; [3] Invert for the effective compensation parameter $\zeta_{compensation}$ which corresponds to the minimum in the function $\widehat{\mathcal{T}_G^i}(\mathbf{x},\omega) = FT\left[G_i^{-,+}(\mathbf{x},t_l)\right]$, where FT corresponds to the Fourier transform. The general approach is to focus at several focal points in the medium and apply the method to invert for the attenuation. This will yield the quantitative information about the attenuation in the subsurface. One will find the intervals where the attenuation is quite similar between the layers and where the attenuation is very strong. Moreover, this method is more stable if applied to the redatumed reflection series.

Inverting for Q-factor using Marchenko Redatuming

The main feature of Marchenko redatuming is that it takes into account all internal multiple reflections. It is a two-step process and a detailed explanation can be found in Wapenaar et al. (2014). The upgoing $G^{-,+}$ and downgoing $G^{+,+}$ Green's functions are related via :

$$G^{-,+}(\mathbf{x}_i,\mathbf{x}',t) = \int_{\partial\mathbb{D}_i} d\mathbf{x} \int_{-\infty}^{\infty} R^{\cup}(\mathbf{x}_i,\mathbf{x},\tau)G^{+,+}(\mathbf{x},\mathbf{x}',t-\tau)d\tau, \tag{2}$$

where $R^{\cup}(\mathbf{x}_i,\mathbf{x},\tau)$ is the reflection response of the medium below depth level $\mathbb{D}_i$. This reflection response is defined in a medium which is identical to the actual medium below $\mathbb{D}_i$ and is reflection-free above this depth level. In the 1D case, equation (1) can be solved via deconvolution in the frequency domain :

$$R^{\cup} = \frac{G^{-,+}}{G^{+,+}} = \frac{G^{-,+}\, G^{+,+*}}{|G^{+,+}|^2 + E} \tag{3}$$

where E is a parameter that accounts for stabilization and $R^{\cup}$ is the redatumed reflection response and the asterisk $(*)$ denotes complex conjugation. By focusing the wavefield at several depth levels we can use the proposed method to calculate the correct compensation parameter for the lossy reflection series.

Example

Two simple 1D models were created to demonstrate the method (Table 1). The models are identical except that the second model contains a thin layer with very high attenuation which will cause some artefacts. Each model consists of 5 plane-parallel layers. Each layer has its own P-wave velocity V_p, density ρ and attenuation. The lossless reflection response was modelled using the method proposed by Fokkema and Ziolkowski (1987). The reflection series was calculated by convolving the reflection coefficients with the Ricker wavelet. The central frequency of the source wavelet (Ricker wavelet) is $f_0 = 30$ Hz. The forward modelling of the lossy reflection series was done using the Maxwell model (Slob, 2016). The attenuation α_H is given in dB per wavelength at the center frequency of the source signal. The losses are compensated by multiplying the wavefields with the damping factor $exp\left(+\frac{\pi f_0}{Q_{tm}}t\right)$ (Draganov et al., 2010). Q_{tm} is the temporal Q-factor. Therefore, the compensation factor for amplitudes $u(t)$ can be written as:

$$u(t)_{compensated} = u_0(t)\, exp\left(+\frac{\pi f_0}{Q_{tm}}t\right) = u_0(t)\, exp(+\zeta_{compensation}\, t) \tag{4}$$

Figure 1a shows the two reflection series. The first reflection series was calculated for a lossless medium (black curve). The second reflection series was calculated for a lossy medium (red curve). When the lossy reflection series was used as input for the Marchenko scheme, many artefacts and multiples were present and the amplitudes of the primary reflections were wrong. Then, we applied the proposed method for minimizing the artefacts in the upgoing Green's functions $G_i^{-,+}$ (Figure 2a) and in the redatumed reflection series (Figure 2b). The correct compensation parameter for the lossy reflection series was inverted and applied to the lossy reflection series (Figure 3). The compensation parameters for Models 1 and 2 were found as $\zeta_{compensation} = 0.4$ and $\zeta_{compensation} = 0.37$, respectively. Note, that the compensation for Model 2 is valid only for the first two layers.

$z_{top}(m)$	$z_{bottom}(m)$	$V_p(m/s)$	$\rho(g/cm^3)$	α_H
0	400	2500	2.5	$1.5 \cdot 10^{-3}$
400	1000	2800	2.7	$1.5 \cdot 10^{-3}$
1300	1500	4000	2.69	$1.2 \cdot 10^{-3}$
1500	2375	2650	2.7	$1.8 \cdot 10^{-3}$
2600	3000	2200	2.5	$1.65 \cdot 10^{-3}$

Table 1 *The model parameters (Model 1). Model 2 is identical except for the third layer, which has the properties $V_p = 2490$ (m/s), $\rho = 2.0$ (g/cm^3) and $\alpha_H = 15 \cdot 10^{-3}$.*
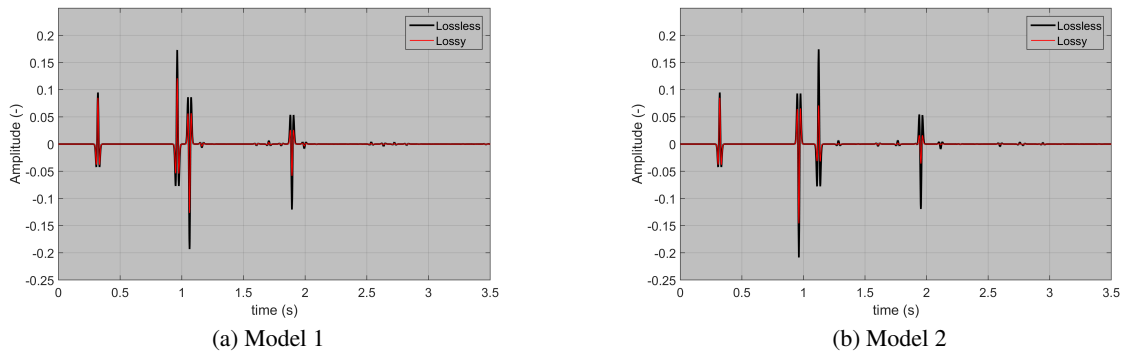

(a) Model 1

(b) Model 2

Figure 1 *a) Lossless (black curve) and lossy (red curve) reflection series for Model 1. b) Lossless (black curve) and lossy (red curve) reflection series for Model 2. The x-axis represents time in seconds. The y-axis represents the amplitude. The intrinsic loss was modelled using the Maxwell model.*
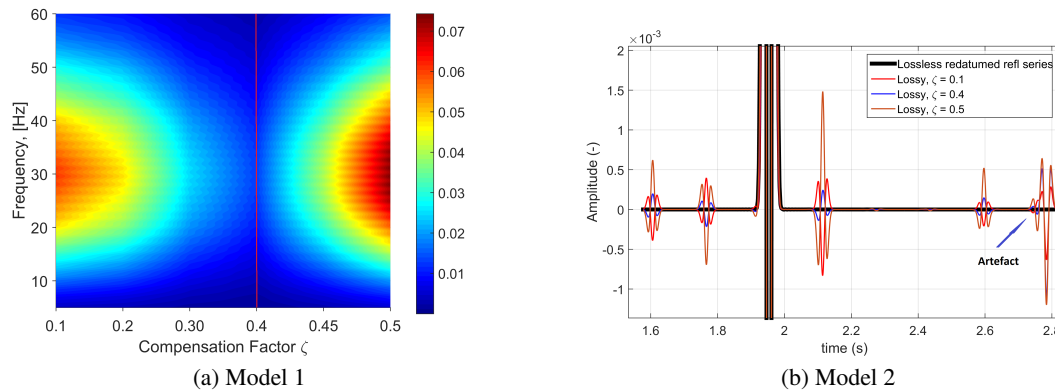

(a) Model 1

(b) Model 2

Figure 2 *a) Function $\widehat{\mathcal{T}_G^i}(\boldsymbol{x}, \omega) = FT\left[G_i^{-,+}(\boldsymbol{x}, t_l)\right]$, where FT corresponds to the Fourier transform and t_l corresponds to the time intervals when the artefacts occur (Model 1). Each vertical line corresponds to the function $\widehat{\mathcal{T}_G^i}(\boldsymbol{x}, \omega)$ for a single $\zeta_{compensation}$. The function $G_i^{-,+}(\boldsymbol{x}, t_l)$ is non zero only at time intervals where the artefacts are present. The x-axis represents the compensation parameter $\zeta_{compensation}$. The y-axis represents the frequency. The color denotes the amplitude. The correct compensation parameter is $\zeta_{compensation} = 0.4$, which corresponds to the minimum (vertical red line). b) Redatumed reflection response for lossless medium (black) and the redatumed reflection response for compensated lossy medium (Model 2). The focal point is 2100 meters depth. The figure is zoomed in to 1.5-2.9 seconds to show the artefacts (primary reflection and five artefacts, which change polarity). The artefact caused by a thin layer with a very high attenuation is shown by an arrow.*
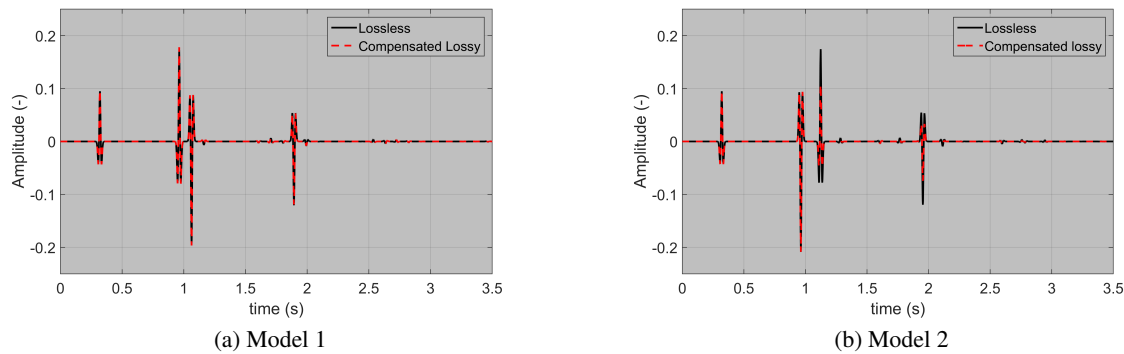
(a) Model 1   (b) Model 2

Figure 3 *a) Lossless (black curve) and compensated lossy (dashed red curve) reflection series for Model 1. The x-axis represents time in seconds. The y-axis represents the amplitude. The compensation parameter applied to the lossy reflection series is $\zeta_{compensation} = 0.4$. b) Lossless (black curve) and compensated lossy (dashed red curve) reflection series for Model 2. The compensation parameter applied to the lossy reflection series is $\zeta_{compensation} = 0.37$. The compensation is valid only for the first two layers.*

Conclusions

We propose the method to quantify attenuation in the subsurface. This method is based on the minimazation of the artefacts in the upgoing Green's functions $G_i^{-,+}$ and in the redatumed reflection series, which were both produced by the Marchenko scheme. The method works very well for the medium where the losses can be compensated via an effective temporal Q-factor. If there is a layer/layers with a very strong attenuation, the effective temporal Q-factor can be found for an interval between the acquisition surface and the layer/layers with very strong attenuation. Marchenko redatuming for lossy medium works very well when the losses can be compensated via an effective temporal Q-factor. If there is a layer/layers with a very strong attenuation, the redatumed reflection series will contain some undesired artefacts. Therefore, further research is required.

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