

# Obtaining angle-dependent reflectivity using the Marchenko redatuming method

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## Summary

When reflection images are studied, often only the zero-offset reflectivity is considered, however, taking into account the angle-dependent reflectivity can add additional information about the subsurface. This additional information can be used to extract the properties of the subsurface using amplitude variation with offset (AVO) analysis. However, the presence of a complex overburden can significantly deteriorate the AVO response, especially for deep targets. To overcome this problem, the overburden effects can be removed by redatuming the reflection response at a depth level below the overburden. In this paper, we show that the Marchenko redatuming method has the potential to correctly retrieve the angle-dependent reflectivity in an acoustic medium without distortions due to multiple scattering. The retrieved angle-dependent reflection coefficients can be used as input in a subsequent inversion process to obtain the velocity and density of the subsurface.

## Introduction

When reflection images are studied, often only the zero-offset reflectivity is considered, however, taking into account the angle-dependent reflectivity can add additional information about the subsurface. This additional information can be used to extract the properties of the subsurface using amplitude variation with offset (AVO) analysis. However, the presence of a complex overburden can significantly deteriorate the AVO response, especially for deep targets. To overcome this problem, the overburden effects can be removed by redatuming the reflection response at a depth level below the overburden. De Bruin et al. (1990) showed that for a laterally homogeneous lossless medium, the angle-dependent reflectivity can be correctly retrieved by means of prestack migration after using wavefield extrapolation. Following a similar approach to De Bruin et al. (1990), we evaluate the Marchenko redatuming method to obtain the AVO response at the reflectors in the subsurface, free of artefacts related to multiple scattering.

## Method and Theory

The Marchenko redatuming method estimates the downgoing Green's function  $G^+$  and upgoing Green's function  $G^-$  at a virtual receiver in the subsurface from a surface reflection response with limited information about the medium (Broggini and Snieder, 2012; Wapenaar et al., 2014). The Green's functions can be then used to estimate the redatumed reflection response  $R$  using the multidimensional deconvolution (MDD) (Wapenaar et al., 2014):

$$R(\mathbf{x}_F, \mathbf{x}'_F, t) = \int_{\partial D_0} G^-(\mathbf{x}_F, \mathbf{x}_S, t) * [G^+(\mathbf{x}'_F, \mathbf{x}_S, t)]^{inv} d\mathbf{x}_S \quad (1)$$

where  $\mathbf{x}'_F$  represents a focal point located in the medium at depth level  $z_f$ ,  $\mathbf{x}_F$  represents an observation points at depth  $z_f$ ,  $\mathbf{x}_S$  represents the location of the sources at the surface of the Earth,  $\partial D_0$  represents the surface of the Earth, and  $t$  represents time. The abbreviation *inv* stands for the inverse of the  $G^+$  and the asterisk denotes temporal convolution. According to De Bruin et al. (1990), the angle-dependent reflection coefficients can be recovered from  $R$  by summing the reflection coefficients along lines of constant angle  $\theta$  in the horizontal wavenumber-frequency ( $k_x, \omega$ ) domain.

## Numerical example

We consider an acoustic multi-layer model with a constant velocity and varying density as shown in plots A and B in Figure 1. The second layer of this model is characterized by a high impedance contrast to test the method in retrieving the angle-dependent reflectivity for a reflector below that layer. The surface reflection response is modeled without free-surface multiples, using a flat spectrum wavelet and a finite-difference modeling scheme (Thorbecke and Draganov, 2010), see plot C in Figure 1. The redatuming is applied slightly above the reflector. After obtaining the Green's functions with the Marchenko method, we apply muting to obtain the first arrival of both  $G^-$  and  $G^+$ . Next, we transform the  $G^-$  and  $G^+$  into the wavenumber-frequency domain and then the horizontal wavenumber is mapped into the angle domain using  $k_x/\omega = \sin(\theta)/c$ , where  $\omega$  is the angular frequency and  $c$  is the velocity of the layer. After that, the result is summed along lines of constant angle and divided by the number of frequency components. Figure 2 illustrates this process for the first reflector as an example. Finally, the MDD is applied by dividing  $G^-$  by  $G^+$  in the frequency-angle domain, producing the angle-dependent reflection coefficients for the reflector. The results for all reflectors are shown in plots D, E and F in Figure 1. It can be seen that the reflection coefficients, which are constant because there are only density contrasts, are correctly retrieved at each reflector. Although there is a high-contrast overburden layer, the method is still able to recover the angle-dependent reflectivity below that layer. The very far angles are set to zero by the user to indicate where the aperture is limited.

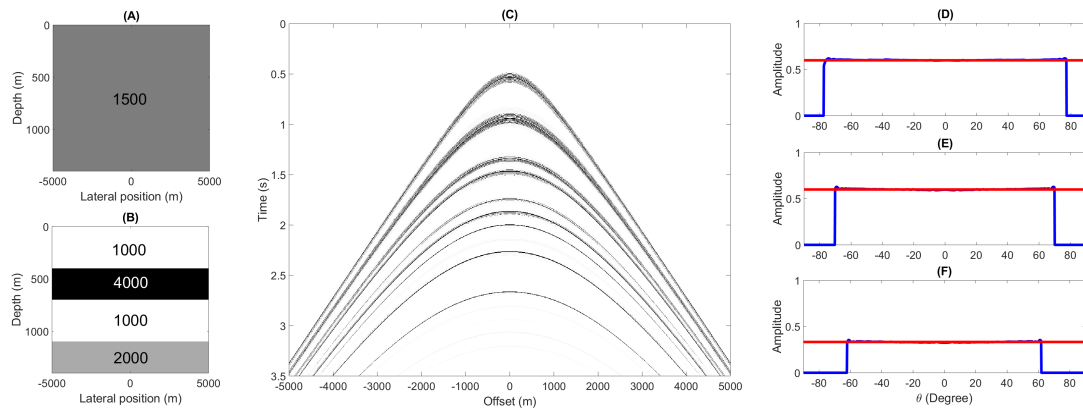
## Conclusion

We have demonstrated that the Marchenko method can potentially retrieve the angle-dependent reflectivity in an acoustic medium without distortions due to multiple scattering. The numerical example has shown that even with a high-contrast overburden, the method is still able to preserve the AVO response.

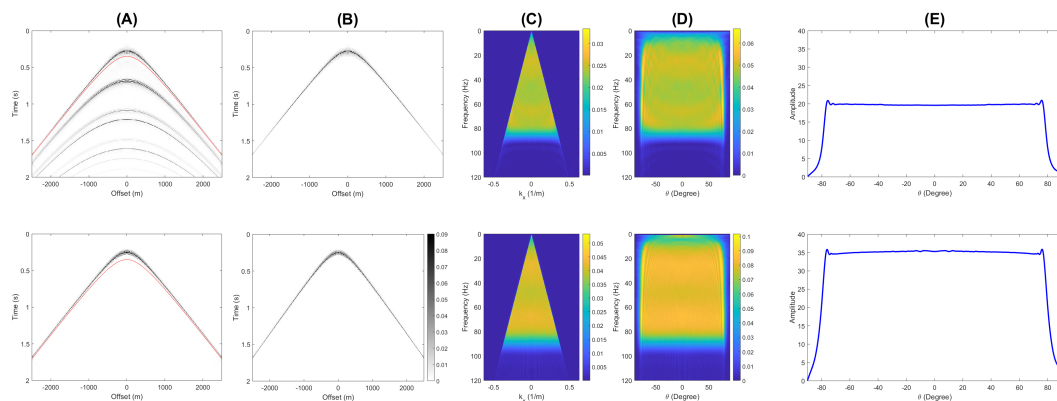
With angle-dependent reflection coefficients, the velocity and density can be obtained by a subsequent inversion process. Future work should consider elastic media, dipping layers, and velocity variations.

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**Figure 1** (A) Velocity model [m/s]. (B) Density model [kg/m<sup>3</sup>]. (C) Modeled surface reflection response. Retrieved angle-dependent reflectivity at (D) depth 400m, (E) depth 700, and (F) depth 1100m. The red curves represent exact reflectivity whereas the blue curves represent estimated reflectivity by MDD.



**Figure 2** The transforms of the Green's functions: the top row represents the  $G^-$  and bottom row represents the  $G^+$ . (A) Before muting. (B) After muting. The red hyperbola represents the muting window. (C) wavenumber-frequency domain. (D) angle-frequency domain. (E) Summing along lines of constant angle and dividing by the number of frequency components.

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