

# Plane-Wave Marchenko Imaging Method: Field Data Application

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## Summary

Seismic imaging is often used to interpret subsurface formations. However, images obtained by conventional methods are contaminated with internal multiples. The Marchenko method provides the means to obtain multiple-free subsurface images. Due to the high computational cost of the conventional pointsource Marchenko imaging method, the less expensive plane wave Marchenko imaging method can be used to produce subsurface images along planes. This method can be repeated for different incident angles to produce images that account for the variable dip of the subsurface structures. In this abstract, we present the results of applying the plane wave Marchenko imaging method to a 2D marine dataset over the Vøring basin, the North Sea. The results show that, in comparison to the conventional plane-wave image, the plane-wave Marchenko imaging method successfully suppressed internal multiples, resulting in improvements in both the amplitude and continuity of the seismic events.



# Introduction

Seismic imaging is often the preferred method to identify subsurface structures, especially in unexplored regions. However, conventional seismic images are contaminated with multiples that might interfere with the subsurface formation interpretations. Conventionally, subsurface seismic images are obtained using methods that are based on the Born approximation, which assumes that the scattered energy consists of a single event. Consequently, subsurface images based on these methods are contaminated with internal multiples.

Multiple-free subsurface images can be obtained using the novel Marchenko method (Wapenaar et al. (2014)). The Marchenko method is a data-driven method that requires single-sided reflection data and a macro velocity model of the subsurface. It plays the role of redatuming both sources and receivers to a certain depth within the target zone. The redatuming is done by propagating the so-called focusing functions into the medium, resulting in redatumed data that does not include internal multiples from the overburden. Repeating the redatuming for different lateral positions and for different depth levels allows for constructing multiple-free subsurface images. Additionally, The Marchenko-based imaging method can be used to extract images between any two depth levels without the need for imaging the entire overburden. Hence, it provides an advantage over recursive multiple-free imaging methods, such as Full Waveform Inversion (Virieux and Operto (2009)).

Applying the standard point-source Marchenko imaging methods has proven to be computationally expensive as it requires imaging each lateral position at each imaging depth. Alternatively, the plane wave Marchenko method can be applied (Meles et al. (2018)). This method images the subsurface along planes, reducing the imaging outputs to the number of depth levels times the number of illumination angles. Consequently, the plane wave Marchenko method allows for obtaining images of the subsurface with less computational expense, while inheriting most of the standard Marchenko method advantages. In contrast to the standard point source Marchenko methods, the subsurface illumination of the plane wave Marchenko method is limited to the incident angle of the plane wave. However, the subsurface illumination can be enhanced by imaging with different plane wave incident angles and stacking the resulted images.

In this work we apply the plane wave Marchenko imaging scheme to field data using several incident angles. Then, we sum the individual images to produce the final image. In addition, we repeat the plane wave imaging procedure using the conventional RTM imaging method, and we compare the two results.

### Theory

Consider an acoustic inhomogeneous and lossless medium bounded by an upper transparent boundary  $\mathbb{S}_0$  defined at  $(\mathbf{x}_H, x_{3,0})$ . Here, and throughout this abstract,  $\mathbf{x}_H$  denotes the horizontal coordinates  $(x_1, x_2)$ , and  $x_3$  denotes the depth coordinate. Additionally, the upper half-space is assumed to be homogeneous. While the full derivation of the Marchenko method is beyond the scope of this abstract, at the core of the method are two focusing functions that focus the wavefield at a focal depth  $\mathbb{S}_A$ , which is defined at  $(\mathbf{x}_H, x_{3,A})$ . The plane wave focusing functions are defined as the integration of all point-source focusing functions along the focal depth  $\mathbb{S}_A$  (Meles et al. (2018)):

$$\tilde{f}_{1}^{\pm}(\mathbf{x},\mathbf{p}_{A},t) = \int_{\mathbb{S}_{A}} f_{1}^{\pm}(\mathbf{x},\mathbf{x}_{A},t-\mathbf{p}\cdot\mathbf{x}_{\mathrm{H},A}) \,\mathrm{d}\mathbf{x}_{A} \tag{1}$$

where  $\mathbf{p}_A$  is the short notation of  $(\mathbf{p}, x_{3,A})$ ,  $\mathbf{p}$  are the horizontal ray parameters  $(p_1, p_2)$ , and  $\mathbf{x}_{H,A}$  is the horizontal coordinates along the focal depth  $\mathbb{S}_A$ . In Equation 1,  $\tilde{f}_1^+(\mathbf{x}, \mathbf{p}_A, t)$  is a focusing function that focuses as a plane wave at the focusing depth  $\mathbb{S}_A$ . The plane wave focusing condition is given by (Meles et al. (2018)):

$$\tilde{f}_{1}^{+}\left(\mathbf{x}_{A}^{\prime},\mathbf{p}_{A},t\right) = \delta\left(t-\mathbf{p}\cdot\mathbf{x}_{H,A}^{\prime}\right)$$
<sup>(2)</sup>

where  $\delta$  is the Dirac delta function. Similar to Equation 1, the plane wave Green's function is defined as the integration of all point-source Green's functions along the focal depth  $S_A$ :

$$\tilde{G}^{-,\pm}(\mathbf{x}_{R},\mathbf{p}_{A},t) = \int_{\mathbb{S}_{A}} G^{-,\pm}(\mathbf{x}_{R},\mathbf{x}_{A},t-\mathbf{p}\cdot\mathbf{x}_{\mathrm{H},A})\,\mathrm{d}\mathbf{x}_{A}$$
(3)



Inherently, the focusing functions are defined in the truncated medium while the Green's functions are defined in the physical medium. The truncated medium is identical to the physical medium above the focusing depth  $\mathbb{S}_A$  and reflection-free (i.e. homogeneous) below it. For Equation 1 and Equation 3, the inclusion of the term  $\mathbf{p} \cdot \mathbf{x}_{H,A}$  accounts for the plane waves dip in time. For horizontal plane waves, this term is equal to zero.

Based on the one-way reciprocity theorem, the plane wave equivalent of the Marchenko equations are defined as (Meles et al. (2018)):

$$\tilde{G}^{-,+}\left(\mathbf{x}_{R},\mathbf{p}_{A},t\right)+\tilde{f}_{1}^{-}\left(\mathbf{x}_{R},\mathbf{p}_{A},t\right)=\int_{\mathbb{S}_{0}}\,\mathrm{d}\mathbf{x}_{S}\int_{0}^{\infty}R\left(\mathbf{x}_{R},\mathbf{x}_{S},t'\right)\tilde{f}_{1}^{+}\left(\mathbf{x}_{S},\mathbf{p}_{A},t-t'\right)\mathrm{d}t',\tag{4}$$

$$\tilde{G}^{-,-}\left(\mathbf{x}_{R},\mathbf{p}_{A}^{\prime},-t\right)+\tilde{f}_{1}^{+}\left(\mathbf{x}_{R},\mathbf{p}_{A},t\right)=\int_{\mathbb{S}_{0}}\,\mathrm{d}\mathbf{x}_{S}\int_{-\infty}^{0}R\left(\mathbf{x}_{R},\mathbf{x}_{S},-t^{\prime}\right)\tilde{f}_{1}^{-}\left(\mathbf{x}_{S},\mathbf{p}_{A},t-t^{\prime}\right)\mathrm{d}t^{\prime}.$$
(5)

Here,  $\mathbf{p}'_A$  is the short notation of  $(-\mathbf{p}, x_{3,A})$ . This system of equation is solved via an energy-minimizing iterative scheme (Broggini et al. (2014)), which utilizes the difference in the causality properties between the focusing and Green's functions in the time domain. The subsurface images are obtained by applying plane wave Marchenko-based migration (Meles et al. (2018)):

$$\tilde{r}_{im}\left(\mathbf{x}_{B},\mathbf{p}_{A}\right) = \left\{ \int_{\mathbb{S}_{0}} \,\mathrm{d}\mathbf{x}_{R} \int_{-\infty}^{\infty} G_{d}^{+,+}\left(\mathbf{x}_{B},\mathbf{x}_{R},-t'\right) \tilde{G}^{-,+}\left(\mathbf{x}_{R},\mathbf{p}_{A},t-t'\right) \,\mathrm{d}t' \right\}_{t=\mathbf{p}\cdot\mathbf{x}_{\mathrm{H},B}} \tag{6}$$

where  $\tilde{r}_{im}$  is the angle-dependent local reflection coefficient at point  $\mathbf{x}_B$  along the focal depth  $\mathbb{S}_A$ ,  $G_d^{+,+}$  is the direct wave of a point source Green's function, excited as a down-going wave at point  $\mathbf{x}_R$  and received as a down-going wave at point  $\mathbf{x}_B$ ,  $t = \mathbf{p} \cdot \mathbf{x}_{\mathrm{H},B}$  is the imaging condition. Note that for a horizontal plane wave, the imaging condition is defined as t = 0. Alternative to computing the  $G_d^{+,+}$ , the image can be constructed by back-propagating  $\tilde{G}^{-,+}(\mathbf{x}_R, \mathbf{p}_A, t')$  by finite difference extrapolation to its corresponding depth prior to applying the imaging condition. The latter method is used to produce the results presented here.

### **Data Pre-Processing**

The plane wave Marchenko imaging method was applied to a 2D marine dataset acquired by SAGA A.S. (currently Equinor) over the Vøring basin, the North Sea. This dataset suffers from a number of limitations that were corrected for to accommodate the Marchenko method requirements. The corrections include regularizing the receiver offsets, interpolating missing shots, constructing the missing positive offsets using source-receiver reciprocity, constructing the missing near-offsets using parabolic Radon interpolation, suppressing surface multiples using Surface Related Multiple Elimination (Verschuur et al. (1992)), reducing air-gun bubble effects by predictive deconvolution, correcting for spherical divergence and absorption effects by applying time-dependent amplitude corrections factors. The latter factor was validated by the convergence of the Marchenko iterative scheme and the continuous decrease of energy for each iteration.

### Results

Utilizing Equation 6, the imaging condition was applied using incident angles ranging from  $-10^{\circ}$  to  $10^{\circ}$ , with an increment of 5°. The individual images were, then, summed to form an image that accounts for the variable dip of the deep reflectors. Similarly, the counterpart RTM-based plane wave images were created using the same incident angles range, by limiting the Marchenko iterative scheme to one iteration. Figure 1 shows a comparison between the RTM-based and the Marchenko-based plane wave images. The red and green boxes highlight areas of the most apparent difference, and are shown separately in Figure 2 and Figure 3. As expected, the RTM-based images are contaminated with internal multiples which interfere with primary reflections and degrade the amplitude and continuity of the subsurface events. After applying the plane wave Marchenko imaging method, a noticeable improvement in both the amplitude and continuity of the events can be observed.





**Figure 1:** The imaging results of the SAGA A.S. dataset over the Vøring basin, the North Sea. (a) the RTM-based plane wave image, (b) the Marchenko-based plane wave image. Both images are the results of summing plane wave images with incident angles ranging from  $-10^{\circ}$  to  $10^{\circ}$  with an increment of  $5^{\circ}$ . Both images are scaled using the same scaling factor. The red and green boxes highlight areas that are magnified in Figure 2 and Figure 3 for a more detailed comparison.

### Conclusions

The Marchenko method provides the means to redatum single-sided reflection data to a specific point in the subsurface without internal multiples from the overburden. The redatumed data can be used to extract a multiple-free subsurface image. To reduce the high computational cost of the conventional Marchenko-based imaging method, the plane wave Marchenko imaging method can be used instead. This method redatums the single-sided reflection data to a focal plane at the target zone, which can be used to extract an angle-dependent multiple-free subsurface image. Lateral resolution can be enhanced by summing images obtained from plane wave virtual sources of different incident angles. The plane wave Marchenko imaging method was applied to a 2D marine field data, acquired by SAGA A.S. Images of five angles ranging from  $-10^{\circ}$  to  $10^{\circ}$  with an increment of  $5^{\circ}$  were constructed and summed to improve lateral resolution. Comparing the Marchenko-based images to their RTM-based counterparts shows that the plane wave Marchenko imaging method successfully produced multiple-free images with improved amplitude and continuity of events affected by multiples.

### Acknowledgement

The authors thank Equinor ASA for granting permission to use the vintage seismic reflection data of the Vøring Basin. This research has received funding from the European Research Council (grant no. 742703).





**Figure 2:** Magnification of the red box in Figure 1. (a) the RTM-based plane wave image, (b) the Marchenko-based plane wave image. The red ellipsoid and arrow indicate internal multiples that have been suppressed by the plane wave Marchenko imaging method. All images are scaled using the same scaling factor.



**Figure 3:** Magnification of the green box in Figure 1. (a) the RTM-based plane wave image, (b) the Marchenkobased plane wave image. The red arrows indicate internal multiples that have been suppressed by the plane wave Marchenko imaging method. All images are scaled using the same scaling factor.

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