Marchenko redatuming by MDD and by double focusing: a systematic comparison

Kees Wapenaar

Summary

In Marchenko redatuming, sources and receivers are moved from the acquisition surface to a new datum plane in the subsurface, thereby accounting for internal multiple reflections in the overburden. After estimating focusing functions from the reflection data at the surface, the sources are focused onto virtual sources at the new datum. Next, the receivers can be redatumed, either by multidimensional deconvolution (MDD) or by applying a second focusing step. From a theoretical point of view MDD is preferred over focusing, but the practical implementation is challenging and computationally expensive. When accurate redatumed data are required, applying MDD may be worth the effort, but when redatuming is combined with imaging at all depth levels of interest, the more straightforward focusing approach is preferred.

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Introduction

In Marchenko redatuming, sources and receivers are moved from the acquisition surface to a new datum plane in the subsurface, thereby accounting for internal multiple reflections in the overburden. The first step is an iterative procedure to estimate focusing functions between the acquisition surface and the new datum plane, from the reflection response at the surface and a smooth model of the overburden. Next, using these focusing functions, the sources at the surface are focused onto virtual sources in the subsurface. Hence, this process accomplishes source redatuming and results in Green's functions, with sources in the subsurface and receivers at the surface. The final step aims at redatuming the receivers from the acquisition surface to the new datum plane in the subsurface. Here in essence two approaches can be followed: multidimensional deconvolution (MDD) or applying a second focusing step. We review these two approaches and discuss their pros and cons.

Estimation of focusing functions

Figure 1(a) shows a cartoon of the focusing functions $f_1^+(\mathbf{x},\mathbf{x}_A,t)$ and $f_1^-(\mathbf{x},\mathbf{x}_A,t)$ (the plus- and minus-signs referring to the downgoing and upgoing parts) in a truncated version of an inhomogeneous medium. The downgoing focusing function $f_1^+(\mathbf{x}_S,\mathbf{x}_A,t)$ at the acquisition surface $\mathbb{S}_0$ (Figure 1(a), red rays, and Figure 1(b)) is tuned such that a single downgoing field $f_1^+(\mathbf{x}'_A,\mathbf{x}_A,t)$ focuses at $\mathbf{x}'_A=\mathbf{x}_A$ at the new datum plane $\mathbb{S}_A$ (Figure 1(c)). Figure 2(a) shows a cartoon of the Green's functions $G^{-,+}(\mathbf{x}_R,\mathbf{x}_A,t)$ and $G^{-,-}(\mathbf{x}_R,\mathbf{x}_A,t)$ for a downward and upward radiating source, respectively, at $\mathbf{x}_A$. The following relations hold between the focusing functions and Green's functions (Wapenaar et al., 2014; Slob et al., 2014)

$$G^{-,+}(\mathbf{x}_R,\mathbf{x}_A,t)+f_1^-(\mathbf{x}_R,\mathbf{x}_A,t) = \int_{\mathbb{S}_0}\mathrm{d}\mathbf{x}_S\int_0^\infty R(\mathbf{x}_R,\mathbf{x}_S,t')f_1^+(\mathbf{x}_S,\mathbf{x}_A,t-t')\mathrm{d}t', \tag{1}$$

$$G^{-,-}(\mathbf{x}_R,\mathbf{x}_A,-t)+f_1^+(\mathbf{x}_R,\mathbf{x}_A,t) = \int_{\mathbb{S}_0}\mathrm{d}\mathbf{x}_S\int_{-\infty}^0 R(\mathbf{x}_R,\mathbf{x}_S,-t')f_1^-(\mathbf{x}_S,\mathbf{x}_A,t-t')\mathrm{d}t', \tag{2}$$

where $R(\mathbf{x}_R,\mathbf{x}_S,t)$ is the reflection response at the surface $\mathbb{S}_0$. Note that the integrals on the right-hand sides represent multidimensional convolutions. The left-hand sides of equations (1) and (2) (fixed $\mathbf{x}_A$, variable $\mathbf{x}_R$) are shown in Figures 2(b) and 2(c), respectively. Note that the Green's functions and focusing functions are separated by the dashed lines in these figures, except for the direct part $f_{1,\mathrm{d}}^+(\mathbf{x}_R,\mathbf{x}_A,t)$ of the focusing function. In the following we simplify the notation and replace equations (1) and (2) by

$$G_{R,A}^{-,+}+f_1^- = Rf_1^+, \tag{3}$$

$$G_{R,A}^{-,-\star}+f_1^+ = R^\star f_1^-, \tag{4}$$

(Van der Neut et al., 2015), where * denotes time-reversal. These are two equations for four unknowns (assuming the reflection response R is known from measurements at the surface). We suppress the Green's functions from these equations by applying time windows Θ_b and Θ_a (defined by the dashed lines in Figures 2(b) and 2(c)) to equations (3) and (4), which yields

$$f_1^- = \Theta_b R f_1^+, \tag{5}$$

$$f_1^+ - f_{1,\mathrm{d}}^+ = \Theta_a R^\star f_1^-. \tag{6}$$

Substitution of equation 5 into equation 6 gives

$$\{\delta-\Theta_a R^\star\Theta_b R\}f_1^+ = f_{1,\mathrm{d}}^+, \tag{7}$$

where δ is the identity operator. This can be solved iteratively, according to

$$f_{1,k}^+ = \Theta_a R^\star \Theta_b R f_{1,k-1}^+ + f_{1,\mathrm{d}}^+, \quad \text{starting with} \quad f_{1,0}^+ = f_{1,\mathrm{d}}^+. \tag{8}$$

Note that this scheme requires the reflection response R and the direct part $f_{1,\mathrm{d}}^+(\mathbf{x}_R,\mathbf{x}_A,t)$ of the focusing function, which in turn requires a smooth model of the overburden. Once $f_{1,k}^+$ has sufficiently converged, $f_{1,k}^-$ follows from equation (5).

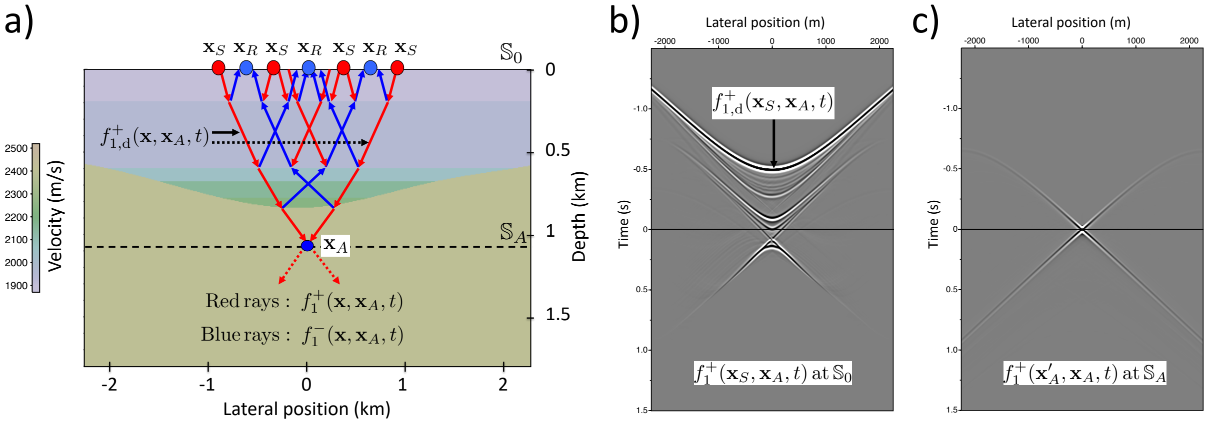

Figure 1 *Focusing functions $f_1^{+}(\mathbf{x},\mathbf{x}_A,t)$ and $f_1^{-}(\mathbf{x},\mathbf{x}_A,t)$ in a truncated version of the actual medium.*

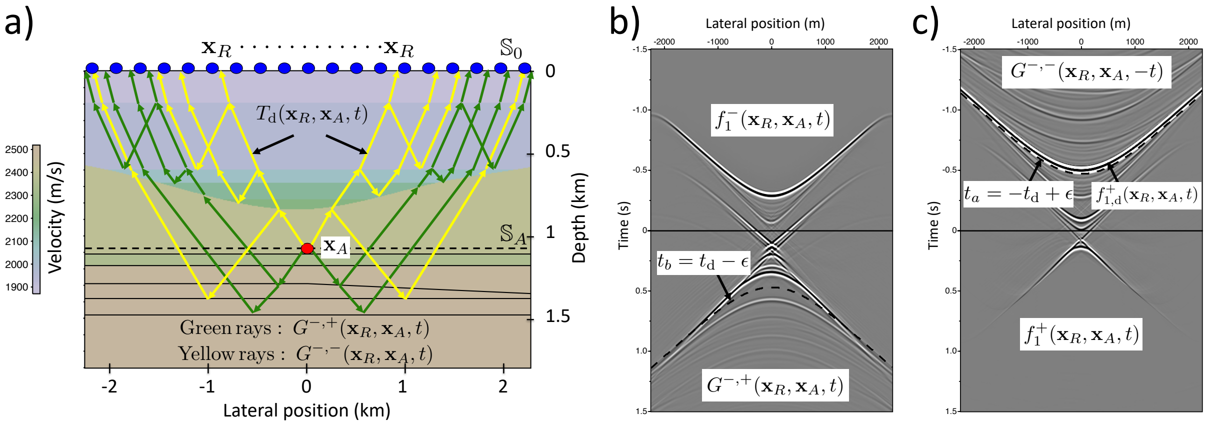

Figure 2 *Green's functions $G^{-,+}(\mathbf{x}_R,\mathbf{x}_A,t)$ and $G^{-,-}(\mathbf{x}_R,\mathbf{x}_A,t)$ in the actual medium.*

Source redatuming by focusing

Let us define time windows Ψ_a and Ψ_b as the complements of Θ_a and Θ_b, according to $\Psi_{a,b} = 1 - \Theta_{a,b}$. Applying these windows to equations (3) and (4) they pass the Green's functions and suppress the focusing functions on the left-hand sides (except $f_{1,\mathrm{d}}^{+}$), hence

$$G_{R,A}^{-,+} = \Psi_b R f_1^{+}, \tag{9}$$

$$G_{R,A}^{-,-\star} + f_{1,\mathrm{d}}^{+} = \Psi_a R^{\star} f_1^{-}. \tag{10}$$

The products Rf_1^{+} and $R^{\star} f_1^{-}$ on the right-hand sides are multidimensional convolutions, as defined in equations (1) and (2). Figure 3(a) shows a cartoon of the reflection response $R(\mathbf{x}_R,\mathbf{x}_S,t)$ and Figure 3(b) is a visualization of equation (9). The latter figure shows that the multidimensional convolution Rf_1^{+} accomplishes focusing of the sources from $\mathbf{x}_S$ at the surface onto a virtual source at $\mathbf{x}_A$ in the subsurface.

Receiver redatuming by MDD

The retrieved Green's functions on the left-hand side of equations (9) and (10) are mutually related via

$$G_{R,A}^{-,+} = -G_{R,B}^{-,-}\bar{R}_{\mathrm{tar}} \tag{11}$$

(Wapenaar, 1996; Amundsen, 2001), which is a short notation for the multidimensional convolution

$$G^{-,+}(\mathbf{x}_R,\mathbf{x}_A,t) = -\int_{\mathbb{S}_A} \mathrm{d}\mathbf{x}_B \int_0^t G^{-,-}(\mathbf{x}_R,\mathbf{x}_B,t')\bar{R}_{\mathrm{tar}}(\mathbf{x}_B,\mathbf{x}_A,t-t')\mathrm{d}t', \tag{12}$$

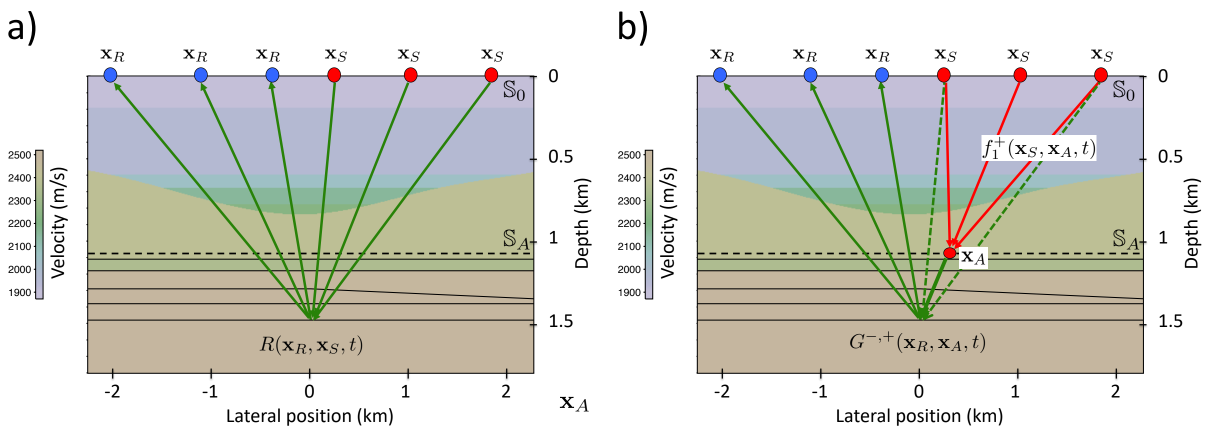

__Figure 3__ (a) Reflection response. (b) Source redatuming, as described by equation (9). For simplicity these and subsequent figures only show some primary rays. The focusing function f_1^+ and the Green's function $G^{-,+}$ are shown in more detail in Figures 1(a) and 2(a).

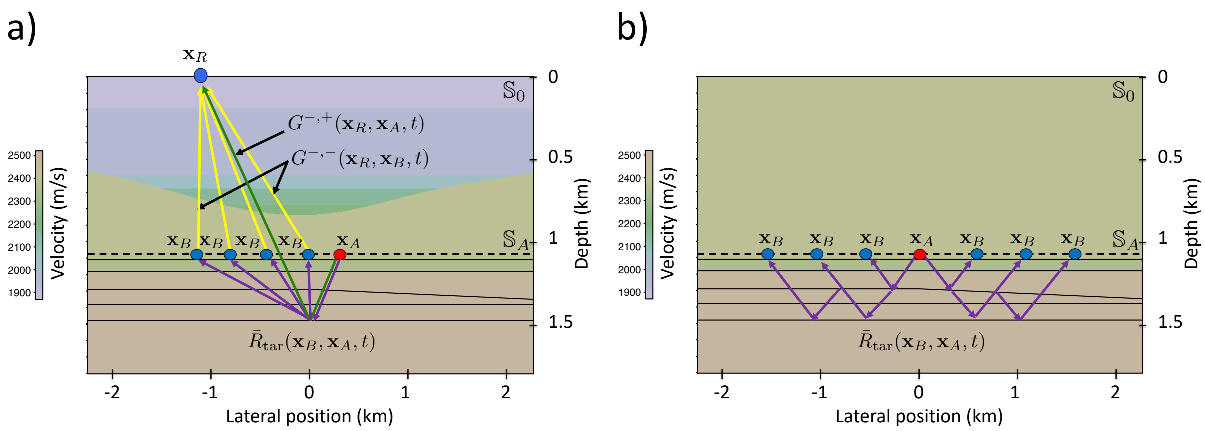

__Figure 4__ (a) Visualization of equation (12). The Green's functions are shown in more detail in Figure 2(a) and the target reflection response $\bar{R}_{\rm tar}(\mathbf{x}_B, \mathbf{x}_A, t)$ in Figure 4(b).

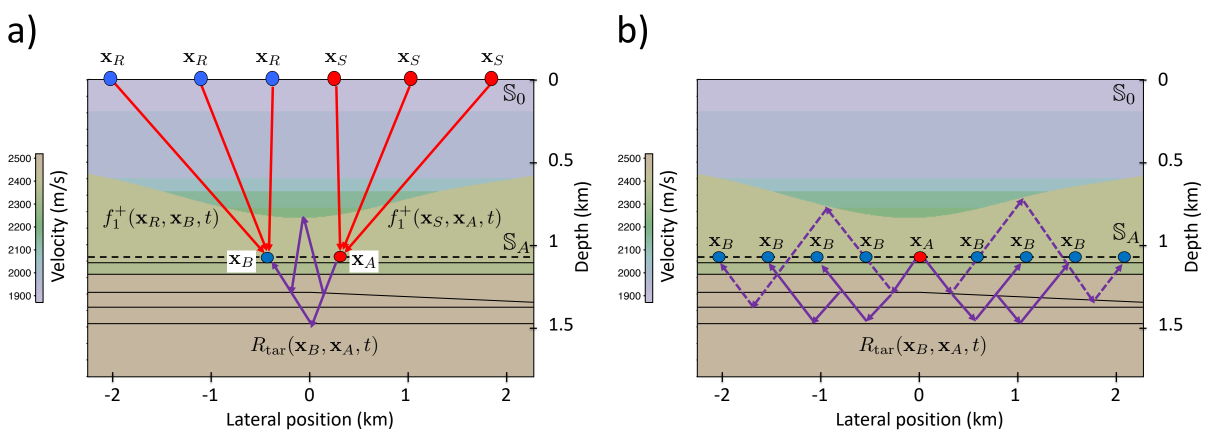

__Figure 5__ (a) Visualization of equation (15). The focusing functions are shown in more detail in Figure 1(a) and the target reflection response $R_{\rm tar}(\mathbf{x}_B, \mathbf{x}_A, t)$ in Figure 5(b).

see Figure 4(a). Here $\bar{R}_{\rm tar}(\mathbf{x}_B,\mathbf{x}_A,t)$ is the response of the target zone below the new datum $\mathbb{S}_A$, see Figure 4(b). Retrieving $\bar{R}_{\rm tar}(\mathbf{x}_B,\mathbf{x}_A,t)$ involves multidimensional deconvolution (MDD), which in the notation of equation (11) is described by

$$\boxed{\bar{R}_{\rm tar} = -(G^{-,-}_{R,B})^{-1}G^{-,+}_{R,A}.} \tag{13}$$

The retrieved target response $\bar{R}_{\rm tar}(\mathbf{x}_B,\mathbf{x}_A,t)$ is defined in a truncated version of the actual medium, which is homogeneous above the new datum $\mathbb{S}_A$ (Figure 4(b)). Ravasi et al. (2016) show a field data application.

Source and receiver redatuming by double focusing

Although in theory receiver redatuming by MDD yields the target response, free from all overburden effects, its practical implementation is challenging and computationally expensive. Here we discuss an alternative approach to receiver redatuming. Analogous to equation (9), which describes source redatuming by focusing the sources onto a virtual source, receiver redatuming can be accomplished by focusing the receivers onto a virtual receiver, according to

$$\boxed{R_{\rm tar} = f_1^{+t}G^{-,+}_{R,A},} \tag{14}$$

where the superscript ^{t} denotes transposition (which in essence means that f_1^{+t} acts on the receiver coordinate instead of on the source coordinate). Obviously equation (14) avoids a lot of the complexity of equation (13). Upon substitution of equation (9) into equation (14) we obtain

$$R_{\rm tar} \;=\; f_1^{+t}\Psi_b R f_1^{+}. \tag{15}$$

This expression describes source and receiver redatuming by double focusing, accounting for internal multiples in the overburden, see Figure 5. The retrieved target response $R_{\rm tar}(\mathbf{x}_B,\mathbf{x}_A,t)$ is defined in the actual medium, hence, it contains multiples between reflectors below and above $\mathbb{S}_A$ (Figure 5(b)). Staring et al. (2018) discuss a field data application, in which the multiples between the overburden and the target zone do not play a significant role. The effect of these multiples disappears altogether when redatuming is combined with imaging, i.e., by selecting $R_{\rm tar}(\mathbf{x}_B,\mathbf{x}_B,t=0)$ at all depth levels of interest.

Conclusions

In Marchenko redatuming, the receivers can be redatumed either by MDD (equation 13), or by focusing (equation 14). Whereas MDD yields the target response below a homogeneous overburden (Figure 4(b)), the target response after focusing still contains multipes between the overburden and the target zone (Figure 5(b)). From a theoretical point of view MDD is preferred over focusing, but the practical implementation is challenging and computationally expensive. When accurate redatumed data are required, applying MDD may be worth the effort, but when redatuming is combined with imaging at all depth levels of interest, the more straightforward focusing approach is preferred.

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