# C032 EXTRAPOLATION OPERATORS BY BEAM TRACING

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### Introduction

The key process in seismic imaging is given by migration. The accuracy and efficiency of the migration process is directly determined by the involved downward extrapolation operators. Taking into account that a downward extrapolation operator equals the response at the surface due to a point source in a subsurface grid point of a macro model, the computation of point source responses in macro models is of vital importance for the seismic method.

Sofar, two methods are used to simulate point source responses in macro models:

- 1. Finite difference method
- 2. Ray tracing method.

The finite difference method is accurate and can handle complicated macro models without any problem. However, the major disadvantage is that finite difference modeling is computationally expensive, particularly if one bears in mind that the modeling should be carried out for many grid points.

The ray tracing method is fast and, therefore, economically attractive. However, particularly in complex macro models, the ray tracing method has problems with delivering correct amplitudes and multi travel time events.

From the foregoing we may conclude that both methods are not ideal.

## Beamtracing

In this paper we propose a third alternative, called beam tracing, that may be considered as a compromise between finite difference modeling and modeling by ray tracing. Beam tracing is based on Huygens' principle, where the travelling wave field is recursively followed by looking at the response of secondary point sources. However,

instead of looking at the response of each individual Huygens source, a number of neighboring Huygens sources are combined into an areal source. Looking in the far field of this areal source, the directive response can be easily computed by analytically evaluating the source beam. Hence, by using the Rayleigh integral (quantification of Huygens' principle) we obtain accuracy and by combining Huygens point sources into areal sources (spatial discretization) we obtain speed. The above principle of generalized beam tracing was introduced by Berkhout (1987, p.279-284). Fig. 1 shows the three basic modules in beam tracing. First the wave field at depth level z1 is spatially subdivided in a number of (partially overlapping) sub wave fields, such that the sum of the sub wave fields equals the total wave field (decomposition). Next, each sub wave field is considered as an areal Huygens' source and the response is determined at z2 by making use of its beam. Finally, the contribution of all Huygens sources at z<sub>2</sub> are superimposed (composition). If a macro boundary exists between  $\boldsymbol{z}_1$  and  $\boldsymbol{z}_2$  , then the boundary conditions should be applied as well.

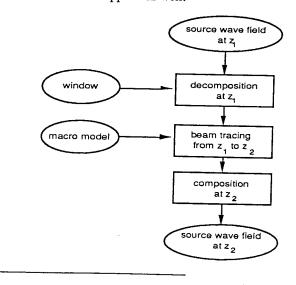


Fig. 1 The three basic modules in beam tracing.

#### Decomposition

To arrive at areal Huygens sources we need to spatially window the incident wave field at depth level  $z_1$ . The main restriction to the decomposition is that the sum of the overlapping window functions equals unity. We will show the response of Huygens sources, obtained by windowing with two different window functions: the cosine-square window (fig. 2a) and the Gaussian window (fig. 2b).

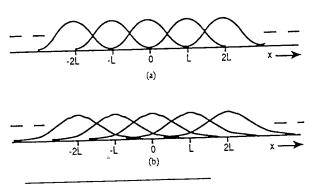
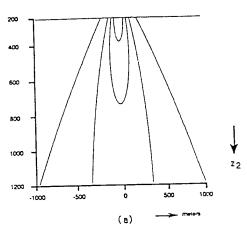


Fig. 2 Two types of window functions for the incident wave field.

## Extrapolation

The response at depth level z2, as a result of one Huygens source at depth level z<sub>1</sub>, is obtained by using the Rayleigh integral. Using the Fraunhofer approximation, it can be shown that in the far field region of the Huygens source this Rayleigh integral may be approximated by a (scaled) Fourier integral, which yields a considerable reduction in computational effort and clarifies the directivity properties of the areal Huygens source. If the incident wavefield at z1 for every Huygens source is approximated by a plane wave, the Fourier integral will be a function of the local amplitude and the local angle of incidence only and the value can be obtained from a table. In the extrapolation the energy per Huygens source remains spatially centralized (the beam concept). The center of this beam corresponds to the classical ray path. To illustrate this beam behaviour, figures 3a and 3b show constant amplitude contours of the response of one windowed Huygens source at every depth level  $z_2$ . In a layered medium, the extrapolation will be done from interface to interface. At every interface the boundary conditions have to be applied. If the interface is curved, the curvature function



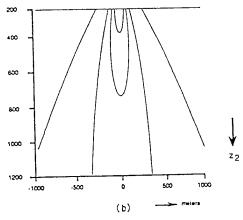


Fig. 3 (a)Constant amplitude contours of the response at every depth level z<sub>2</sub> of one cosine-square windowed Huygens source (b)Constant amplitude contours of the response at every depth level z<sub>2</sub> of one Gaussian windowed Huygens source.

can be linearized for each Huygens source, so that the Fourier integral approximation is still valid. Our general representation allows for redefinition of the beam decomposition at every interface. The spatial complexity of the interface determines the number of Huygens sources to be evaluated. The redefinition of the decomposition will all-ways be a trade-off between accuracy and efficiency.

#### Reference

Berkhout A.J., 1987, Applied Seismic Wave Theory: Elsevier, chapter IX, p.279-284.