B040 THE GENERALISED PRIMARY AND ITS WAVELET TRANSFORM

F.J. HERRMANN and C.P.A. WAPENAAR
Delft University of Technology, Laboratory of Seismics and Acoustics, PO Box 5046, 2600 GA Delft, The Netherlands

Introduction

All seismic processing techniques are based on the starting point that seismic waves inherit information of certain material properties of the earth's subsurface. From both well and seismic measurements it is evident that the earth's subsurface can be subdivided into a few main stratigraphic sections. The trend in and throughout these intervals gives information on the compaction properties of the subsurface. The remaining detail displays an irregular behaviour and gives information on the local properties of the subsurface (within the resolution of the measurements).

The seismic wavefield at the surface represents a mixture of propagation and reflection information. Contemporary considerations in seismics attribute the propagation effects to the macro layering in the subsurface and the reflection behaviour (upto the seismic wavelength) to the detail (Berkhout and Wapenaar 1990). An important issue in this matter is the deformation of the initial pulse shape of the incident wavefield, due to the erratic detail (Burridge 1989;Morlet 1982;Herrmann 1991). Techniques related to the wavelet transform offer a description suitable to describe these propagation effects in an elegant way.

It was shown by Herrmann et al. (1991) that the quantitative effects acting on a wavefield can be accounted for by including additional information on the stratification (fine layering) to the conventional macro model. This proposed formulation also implies that the distorted wavefield contains information on the stratification. In this paper it is argued that by finding an appropriate set of wavelet functions it should be possible to extract this information on the detail.

The generalised primary and its analytic solution

As a wavefield propagates through an irregular 1D medium its initial wave form changes due to the gradual transformation of primary arrivals into a train of delayed internal multiples. This process results in a dispersion, i.e. frequency dependent amplitude attenuation and time delay, of the wavefield compared to the WKB-solution. It is known that the signature of the "generalised primary" (Resnick 1986), i.e. the ensemble of the primary and its first multiples, gradually converges to a limiting solution (Burridge 1989), the form of which is determined by the second order statistics of the medium fluctuations. This convergence is the result of a spatial averaging along the wavepath.

Given the fact that wave propagation is inherently a spatial averaging process it is possible to find a formulation in terms of the stochastic expectation of the medium properties. Fractal Gaussian Noise (FGN) provides a random process of which the characteristics are very similar to the statistical properties of the reflection coefficients evidenced from well measurements (Walden and Hosken 1985). The combination of this stochastic model with the expression of the propagation operator leads to the definition of an extended macro model (Herrmann et al. 1991). This model relates the limiting pulse shape to global stochastic properties of the medium. Stochastic information can, in this way, be transferred from well measurements to the description of wave propagation and this relationship can be used to obtain information of the medium from seismic measurements.

Application to real well-logs

An example of the application of the proposed method to the description of wave propagation through a medium defined in terms of a real well-log[1] is given. Fig. 1 (a) contains the acoustic impedances as a function of one-way traveltime. The normal incidence reflection coefficients are computed from this log and subsequently submitted to a stochastic analysis, see Fig. 1 (b), where an estimate of the log-log power spectrum of the reflection coefficients is depicted. The linear fit of this power spectrum corresponds to the power spectrum of FGN and its slope determines the degree of anticorrelation. Finally, in Fig. 1 (c) a comparison is made of the exact convolved (Ricker) transmission response and the convolved analytic solution computed with the estimated fit of Fig. 1 (b). It appears that the signature of the transmission response is remarkebly well covered by the analytic solution.

The wavelet transform

Morlet et al. (1982) applied the Gabor transformation to the description of wave propagation in (quasi) periodic media. This transformation allows a formulation of the varying pulse shape in both time and frequency. Similar analysis can be conducted with the wavelet transform. Application of this type of transform to the generalised primary and limiting solution enhances the insight in the proposed formulation. The wavelet transform lends itself particularly well to the description of random fractals and it is possible to construct a set of wavelet functions in which the generalised primary can be decomposed. This decomposition could lead to a method to capture the second order statistics of the medium.

Conclusions

The effects of detail in the earth's subsurface on wave propagation can be accounted for by including a priori information in the macro model. This information can be captured from well measurements or other geologic sources and amounts to a better treatment of the amplitudes and traveltimes of the seismic wavefield. Applying the Gabor or wavelet transform to the generalised primary and its limiting solution leads to a description localized in both time and frequency. Moreover, the decomposition of the transmitted wavefield with a set of suitable wavelets may deliver information of the second order medium statistics.

References

Berkhout A.J. and Wapenaar C.P.A., (1990): Delphi: Delft Philosophy on acoustic and elastic inversion (part I); The leading edge

Burridge R. and Chang H.W., (1989): Multimode, one-dimensional wave propagation in a highly discontinuous medium; Wave Motion 11, pg. 231-249

Herrmann F.J., Wapenaar C.P.A., Berkhout A.J., (1991): Parameterisation of detail in macro models; paper at the E.A.E.G. in Florence, pg.314-315

Morlet J., Arens G., Fourgeau E., Giard D., (1982): Wave propagation and sampling theory-Part I&II; Geophysics, Vol 47, NO 2, pg. 222-236

Resnick J.R., Lerche I., Shuey R., (1986): Reflection, transmission, and the generalized primary wave; Geophys. J.R. astr. Soc. 87, pg. 349-377

Walden A.T. and Hosken J.W.J., (1985): An investigation of the spectral properties of primary reflection coefficients; Geophysical Prospecting 33, pg. 400-435

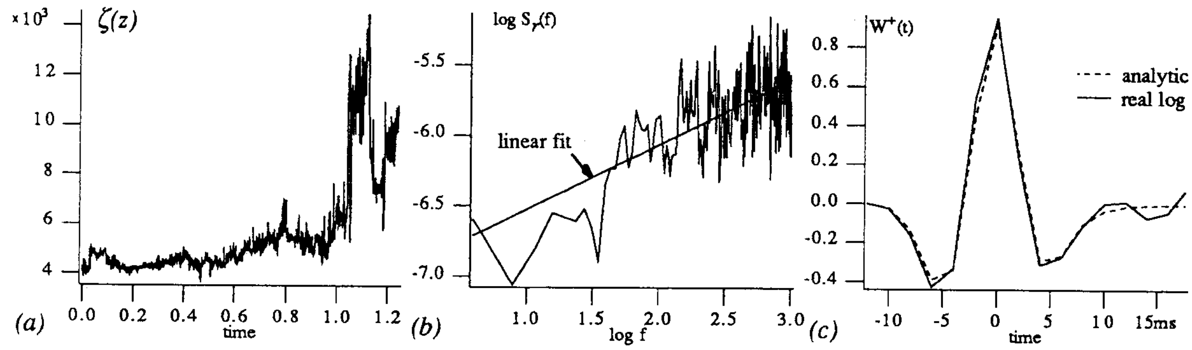

Fig. 1 Application of the proposed method to a real well-log; number of samples 2490; sample inter. 5.10^{-3}

a) acoustic impedances $\zeta(\tau)$

b) log-log power spectrum of the reflection coefficients with the linear fit; spectral window size 512

c) exact convolved transmission response and the convolved analytic solution

---

1. courtesy Faculty of Petroleum and Mining Engineering