# H032 WAVE FIELD EXTRAPOLATION IN THE SPACE-WAVENUMBER DOMAIN

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## Introduction

The forward model of seismic data that is used within our seismic research project DELPHI [1] is based on a number of operator multiplications. In its simplest form (i.e. after preprocessing) it is given by

$$\underset{\sim}{\mathbf{P}}^- = \underset{\sim}{\mathbf{W}}^- \underset{\sim}{\mathbf{R}} \underset{\sim}{\mathbf{W}}^+ \underset{\sim}{\mathbf{P}}^+ \tag{1}$$

where $\underset{\sim}{\mathbf{P}}^+$ contains the downgoing source wave fields at the surface, $\underset{\sim}{\mathbf{W}}^+$ describes downward propagation into the subsurface, $\underset{\sim}{\mathbf{R}}$ represents reflection in the subsurface, $\underset{\sim}{\mathbf{W}}^-$ describes upward propagation to the surface and, finally, $\underset{\sim}{\mathbf{P}}^-$ contains the upgoing wave fields registered at the surface. Equation (1) is a monochromatic description of the seismic response of a single reflector; for discretized wave fields it is a matrix multiplication; different elements in the matrices correspond to different lateral positions. The heart of our migration scheme consists of the inversion of equation (1) for the reflectivity, which yields

$$\underset{\sim}{\mathbf{R}} = [\underset{\sim}{\mathbf{W}}^-]^{-1} \underset{\sim}{\mathbf{P}}^- [\underset{\sim}{\mathbf{W}}^+]^{-1} \tag{2}$$

where we have assumed $\underset{\sim}{\mathbf{P}}^+$ to represent a series of normalized dipole sources, i.e. an identity matrix. Equation (2) says that the reflection at a certain depth level can be found by correcting the data $\underset{\sim}{\mathbf{P}}^-$ for the propagation through the overburden between the acquisition level and the imaging level. The correction matrices are the inverses of the wave field extrapolation matrices $\underset{\sim}{\mathbf{W}}^-$ and $\underset{\sim}{\mathbf{W}}^+$.

It has been shown [2] that it is possible and advantageous to transform equations (1) and (2) to the wavenumber-frequency domain even in the case that one is dealing with lateral variations of the subsurface parameters. In the wavenumber domain the extrapolation operators $\underset{\sim}{\tilde{\mathbf{W}}}^+$ and $\underset{\sim}{\tilde{\mathbf{W}}}^-$ become narrow band matrices, which make forward and inverse extrapolation very efficient.

The data model $\underset{\sim}{\mathbf{P}}^-$ is a pure spatial description of the data, angular information is not available. On the other hand the data model in the wavenumber domain is a pure angular description of the data [2] with infinite plane waves. In practical situations infinite plane waves do not exist. An intermediate description where both spatial and angular information are available and where one experiment gives the reflection of a region seems to be interesting. The transform that provides such description is the Gabor transform [3].

## The Gabor transform

The Gabor transform consists of a windowed Fourier transform, i.e. it consists of a multiplication of the data with a window function $g(x)$ and a subsequent Fourier transform of the windowed data:

$$\underset{\sim}{\hat{\mathbf{P}}} = \underset{\sim}{\mathbf{\Gamma}}_g \underset{\sim}{\mathbf{P}} \quad \text{or, for one column, in integral notation} \quad \hat{P}(p,q) = \int_{-\infty}^{\infty} P(x) \left[g(x - p\xi_0) e^{-jq\kappa_0 x}\right]^* dx \tag{3}$$

The window function $g(x)$ is spatially translated over a distance $p\xi_0$ and its Fourier transform over a distance $q\kappa_0$. This means that $\hat{P}(p,q)$ gives the information of $P(x)$ around $x = p\xi_0$ and $k_x = q\kappa_0$. A gaussian window function $g(x)$ is a convenient choice, because of the fact that it transforms the Green's function to Gaussian beams (this is shown analytically for a homogeneous medium [4]). A transform is only interesting if a stable reconstruction is possible. For a gaussian window function *stable* reconstruction is only possible for $\xi_0 \kappa_0 < 2\pi$ [5].

## Wave field extrapolation in the Gabor domain

Application of the forward and inverse Gabor transform to equation (1) yields

$$\underset{\sim}{\Gamma}_g \underset{\sim}{P}^- \underset{\sim}{\Gamma}_g^{-1} = \left[\underset{\sim}{\Gamma}_g \underset{\sim}{W}^- \underset{\sim}{\Gamma}_g^{-1}\right] \left[\underset{\sim}{\Gamma}_g \underset{\sim}{R} \underset{\sim}{\Gamma}_g^{-1}\right] \left[\underset{\sim}{\Gamma}_g \underset{\sim}{W}^+ \underset{\sim}{\Gamma}_g^{-1}\right] \left[\underset{\sim}{\Gamma}_g \underset{\sim}{P}^+ \underset{\sim}{\Gamma}_g^{-1}\right] \quad (4)$$

$$\hat{\underset{\sim}{P}}^- = \hat{\underset{\sim}{W}}^- \hat{\underset{\sim}{R}} \hat{\underset{\sim}{W}}^+ \hat{\underset{\sim}{P}}^+ \quad (5)$$

where $\underset{\sim}{\Gamma}_g$ and $\underset{\sim}{\Gamma}_g^{-1}$ are the forward and inverse Gabor transform operators. It is important to notice that no approximations are involved in going from the data model in the space domain to the data model in the Gabor domain. The efficiency, however, is better in the Gabor domain, because the extrapolation operators $\hat{\underset{\sim}{W}}^-$ and $\hat{\underset{\sim}{W}}^+$ have a sparse structure (Figure 1b). The interpretation of the data model in the Gabor domain is schematically explained in Figure 1a.

In an identical way as in equation (4), equation (2) can also be transformed to the Gabor domain. This means that migration can be carried out in the Gabor domain. In the space domain migration is carried out per point source experiment; in the wavenumber domain migration is carried out per plane wave experiment. In the Gabor domain migration is carried out per beam experiment. The full migration is the result of 'stacking' of single experiment migrations. For all three domains the full migration results are equal.

## Conclusions

The data model in the Gabor domain enables an elegant interpretation of the seismic data as beam experiments and enables an efficient wave field extrapolation. The three data models in the space domain, in the wavenumber domain and in the Gabor domain provide a generalized migration scheme. The interpretation and efficiency depend on the domain chosen. The full migration result is independent of the domain chosen. In the presentation numerical examples will be used to demonstrate the advantages of the Gabor domain.

## References


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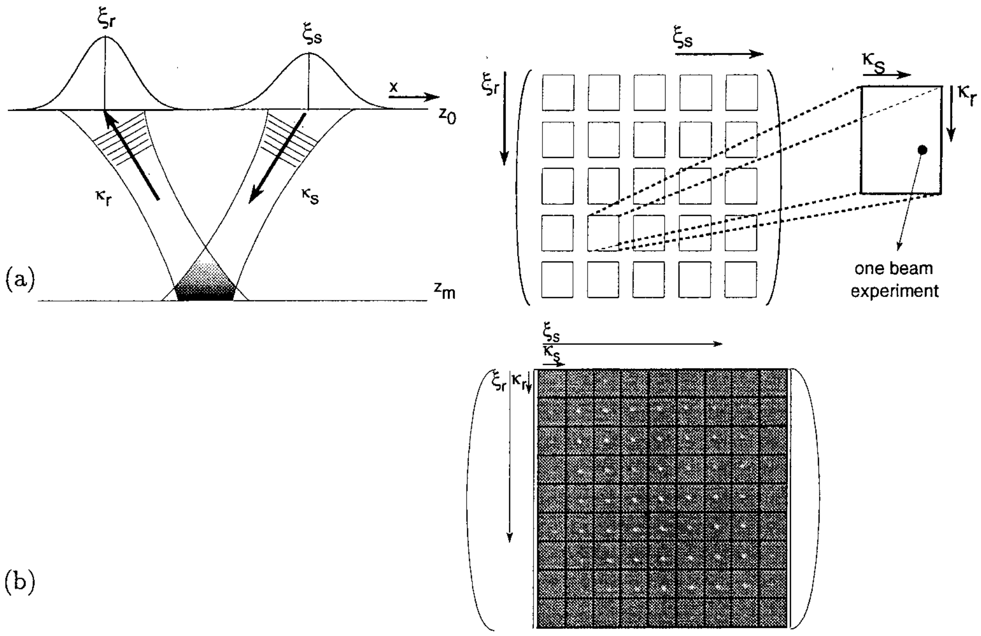


Figure 1: *(a) One point of the data matrix in the Gabor domain (right figure) represents the experiment of the left figure. A downgoing beam from $z_0$ to $z_m$ reflects at depth level $z_m$ into an upgoing beam from $z_m$ to $z_0$ and is registered as a beam at depth level $z_0$. The subscripts "s" and "r" denote the source side and the receiver side respectively. (b) An extrapolation operator in the Gabor domain for one frequency component. Note that the operator has a sparse structure.*