One-way acoustic reciprocity and its applications in multiple elimination and time-lapse seismics

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Summary. In this paper we review our reciprocity theorems for one-way wave fields, modified for lossy media, and we discuss applications in multiple elimination and time-lapse seismics.

Introduction

One-way wave equations have played a prominent role in seismic processing since the pioneering work of Claerbout [4], Berkhout [2] and others. The reason for this is that a seismic experiment can be explained in terms of 'downgoing' waves traveling from the source at the Earth's surface to a target in the subsurface and 'upgoing' waves traveling from the target to the receivers at the surface. One-way wave equations naturally honor this distinction between downgoing and upgoing waves.

This paper starts with a review of reciprocity theorems for one-way wave fields. These reciprocity theorems formulate general relations between the one-way wave fields in two different 'states'. One of these states is an actual seismic experiment, while the other state can either be a computational state (e.g. a wave field propagator), a desired state (e.g. multiple-free data) or another seismic measurement (characterizing time-lapse differences in the target). Fokkema and van den Berg [8] derived seismic processing techniques from Rayleigh's reciprocity theorem for total acoustic wave fields. In the current paper the one-way reciprocity theorems form the starting point. These theorems provide a theoretical frame-work for current seismic processing techniques based on the one-way wave equations. Some applications will be indicated.

One-way reciprocity theorems in media with losses

The one-way wave equation and its symmetry properties. We review the acoustic one-way wave equation for downgoing and upgoing waves in an inhomogeneous medium with losses. We introduce a one-way wave vector $\mathbf{P}$ and a one-way source vector $\mathbf{S}$, according to

$$\mathbf{P} = \begin{pmatrix} P^+ \\ P^- \end{pmatrix} \quad \text{and} \quad \mathbf{S} = \begin{pmatrix} S^+ \\ S^- \end{pmatrix}. \tag{1}$$

The superscripts $+$ and $-$ stand for 'downgoing' and 'upgoing', respectively. In the space-frequency $(\mathbf{x}, \omega)$ domain, the one-way wave equation reads

$$\partial_3 \mathbf{P} = \hat{\underset{\sim}{\mathbf{B}}}\mathbf{P} + \mathbf{S}, \tag{2}$$

where the one-way operator matrix $\hat{\underset{\sim}{\mathbf{B}}}$ is defined as

$$\hat{\underset{\sim}{\mathbf{B}}} = \begin{pmatrix} -j\hat{\mathcal{H}}_1 & O \\ O & j\hat{\mathcal{H}}_1 \end{pmatrix} + \begin{pmatrix} \hat{T} & -\hat{R} \\ -\hat{R} & \hat{T} \end{pmatrix} \tag{3}$$

(the circumflex denotes a pseudo-differential containing ∂_1 and ∂_2). $\hat{\mathcal{H}}_1$ is the well-known square-root operator and $\hat{R}$ and $\hat{T}$ are the reflection and transmission operators.

We define the bilinear form $\langle\cdot,\cdot\rangle_b$ and the sesquilinear form $\langle\cdot,\cdot\rangle_s$ according to

$$\langle \mathbf{f}, \mathbf{g}\rangle_b = \int \mathbf{f}^t(\mathbf{x}_H)\mathbf{g}(\mathbf{x}_H)\mathrm{d}^2\mathbf{x}_H, \tag{4}$$

$$\langle \mathbf{f}, \mathbf{g}\rangle_s = \int \mathbf{f}^\dagger(\mathbf{x}_H)\mathbf{g}(\mathbf{x}_H)\mathrm{d}^2\mathbf{x}_H, \tag{5}$$

where ^{t} denotes transposition, † denotes adjoint (here: transposition and complex conjugation) and $\mathbf{x}_H = (x_1, x_2)$. We introduce the transposed operator $\hat{\underset{\sim}{\mathbf{B}}}^t$ and the adjoint operator $\hat{\underset{\sim}{\mathbf{B}}}^\dagger$ via

$$\langle \hat{\underset{\sim}{\mathbf{B}}}\mathbf{f}, \mathbf{g}\rangle_b = \langle \mathbf{f}, \hat{\underset{\sim}{\mathbf{B}}}^t\mathbf{g}\rangle_b \quad \text{and} \quad \langle \hat{\underset{\sim}{\mathbf{B}}}\mathbf{f}, \mathbf{g}\rangle_s = \langle \mathbf{f}, \hat{\underset{\sim}{\mathbf{B}}}^\dagger\mathbf{g}\rangle_s. \tag{6}$$

Using these definitions, the transposed and adjoint one-way operator matrices obey

$$\hat{\underset{\sim}{\mathbf{B}}}^t\underset{\sim}{\mathbf{N}} = -\underset{\sim}{\mathbf{N}}\hat{\underset{\sim}{\mathbf{B}}} \quad \text{and} \quad \hat{\underset{\sim}{\mathbf{B}}}^\dagger\underset{\sim}{\mathbf{J}} = -\underset{\sim}{\mathbf{J}}\hat{\underset{\sim}{\mathbf{B}}}', \tag{7}$$

with $\underset{\sim}{\mathbf{N}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $\underset{\sim}{\mathbf{J}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. The first symmetry relation in equation (7) was derived in Wapenaar and Grimbergen [13] for lossless media. Using an approach modified after Dillen [5], it appears to hold in media with losses as well (Wapenaar et al. [12]). The prime $'$ in the second symmetry relation in equation (7) denotes that this operator is defined in the adjoint (=complex conjugate) medium. When a medium is passive, its adjoint medium is active and vice versa.

Reciprocity theorem of the convolution type for one-way wave fields. We introduce two different states that will be distinguished by the subscripts A and B. For these two states we consider the interaction quantity $\partial_3\{\mathbf{P}_A^t\underset{\sim}{\mathbf{N}}\mathbf{P}_B\}$, or, written alternatively, $\partial_3\{P_A^+P_B^- - P_A^-P_B^+\}$. [For comparison, Fokkema and van den Berg consider the interaction quantity $\partial_k\{P_A V_{k,B} - V_{k,A}P_B\}$]. Apparently, we consider the interaction between oppositely propagating waves. Applying the product rule for differentiation, substituting the one-way wave equation (2) for states A and B, integrating the result over a cylindrical volume $\mathcal{V}$ with boundary $\partial\mathcal{V}_0 \cup \partial\mathcal{V}_1$ (see Figure 1), applying the theorem of Gauss and using the first symmetry relation in equation (7) yields the following one-way reciprocity theorem

$$\int_{\mathbf{x}\in\partial\mathcal{V}_0} \mathbf{P}_A^t\underset{\sim}{\mathbf{N}}\mathbf{P}_B n_3 \mathrm{d}^2\mathbf{x}_H = \int_{\mathbf{x}\in\mathcal{V}} \mathbf{P}_A^t\underset{\sim}{\mathbf{N}}(\hat{\underset{\sim}{\mathbf{B}}}_B - \hat{\underset{\sim}{\mathbf{B}}}_A)\mathbf{P}_B\mathrm{d}^3\mathbf{x} + \int_{\mathbf{x}\in\mathcal{V}}\{\mathbf{P}_A^t\underset{\sim}{\mathbf{N}}\mathbf{S}_B + \mathbf{S}_A^t\underset{\sim}{\mathbf{N}}\mathbf{P}_B\}\mathrm{d}^3\mathbf{x}, \tag{8}$$

with $n_3 = -1$ at the upper surface of $\partial\mathcal{V}_0$ and $n_3 = +1$ at the lower surface of $\partial\mathcal{V}_0$.

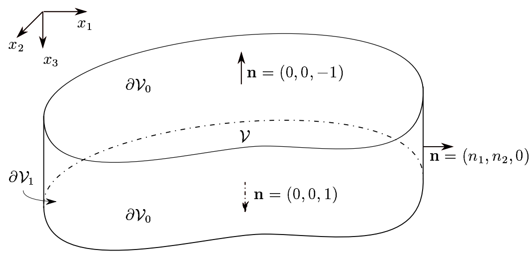


Fig. 1: *Configuration for the one-way reciprocity theorems. The combination of the two planar surfaces is denoted by $\partial\mathcal{V}_0$; the cylindrical surface is denoted by $\partial\mathcal{V}_1$.*

Reciprocity theorem of the correlation type for one-way wave fields. We consider the interaction quantity $\partial_3\{\mathbf{P}_A^\dagger \underset{\sim}{\mathbf{J}} \mathbf{P}_B\}$, or, written alternatively, $\partial_3\{(P_A^+)^* P_B^+ - (P_A^-)^* P_B^-\}$, where * denotes complex conjugation. Following the same procedure as above, using the second symmetry relation in equation (7), yields the following one-way reciprocity theorem

$$\int_{\mathbf{x}\in\partial\mathcal{V}_0} \mathbf{P}_A^\dagger \underset{\sim}{\mathbf{J}} \mathbf{P}_B n_3 \mathrm{d}^2\mathbf{x}_H = \int_{\mathbf{x}\in\mathcal{V}} \mathbf{P}_A^\dagger \underset{\sim}{\mathbf{J}} (\hat{\underset{\sim}{\mathbf{B}}}_B - \hat{\underset{\sim}{\mathbf{B}}}{}'_A) \mathbf{P}_B \mathrm{d}^3\mathbf{x} + \int_{\mathbf{x}\in\mathcal{V}} \{\mathbf{P}_A^\dagger \underset{\sim}{\mathbf{J}} \mathbf{S}_B + \mathbf{S}_A^\dagger \underset{\sim}{\mathbf{J}} \mathbf{P}_B\} \mathrm{d}^3\mathbf{x}. \tag{9}$$

Representation theorem for one-way wave fields. We introduce a one-way Green's matrix $\underset{\sim}{\mathbf{G}}$ which satisfies the following one-way wave equation

$$\partial_3 \underset{\sim}{\mathbf{G}}(\mathbf{x}, \mathbf{x}') = \hat{\underset{\sim}{\mathbf{B}}}(\mathbf{x}) \underset{\sim}{\mathbf{G}}(\mathbf{x}, \mathbf{x}') + \underset{\sim}{\mathbf{I}} \delta(\mathbf{x} - \mathbf{x}'), \tag{10}$$

with $\delta(\mathbf{x}) = \delta(x_1)\delta(x_2)\delta(x_3)$ and $\underset{\sim}{\mathbf{I}}$ being the 2×2 identity matrix. $\hat{\underset{\sim}{\mathbf{B}}}$ is some reference one-way operator. The two columns of $\underset{\sim}{\mathbf{G}}(\mathbf{x}, \mathbf{x}')$ represent two independent Green's one-way wave vectors at observation point $\mathbf{x}$, related to two independent one-way sources at source point $\mathbf{x}'$. Using reciprocity theorem (8) we can derive $\underset{\sim}{\mathbf{G}}(\mathbf{x}', \mathbf{x}'') = -\underset{\sim}{\mathbf{N}}^{-1} \underset{\sim}{\mathbf{G}}^t(\mathbf{x}'', \mathbf{x}') \underset{\sim}{\mathbf{N}}$, or

$$\begin{aligned} \underset{\sim}{\mathbf{G}}(\mathbf{x}', \mathbf{x}'') &= \begin{pmatrix} G^{+,+}(\mathbf{x}', \mathbf{x}'') & G^{+,-}(\mathbf{x}', \mathbf{x}'') \\ G^{-,+}(\mathbf{x}', \mathbf{x}'') & G^{-,-}(\mathbf{x}', \mathbf{x}'') \end{pmatrix} \\ &= \begin{pmatrix} -G^{-,-}(\mathbf{x}'', \mathbf{x}') & G^{+,-}(\mathbf{x}'', \mathbf{x}') \\ G^{-,+}(\mathbf{x}'', \mathbf{x}') & -G^{+,+}(\mathbf{x}'', \mathbf{x}') \end{pmatrix}. \end{aligned} \tag{11}$$

Next we derive a representation for the one-way wave field vector $\mathbf{P}$, obeying equation (2). This wave field vector will be used as state B in the reciprocity theorem (8). The Green's matrix $\underset{\sim}{\mathbf{G}}$, obeying equation (10), will play the role of state A. We thus obtain

$$\chi(\mathbf{x}')\mathbf{P}(\mathbf{x}') = \int_{\mathcal{V}} \underset{\sim}{\mathbf{G}}(\mathbf{x}', \mathbf{x}) \hat{\underset{\sim}{\mathbf{\Delta}}}(\mathbf{x}) \mathbf{P}(\mathbf{x}) \mathrm{d}^3\mathbf{x} + \int_{\mathcal{V}} \underset{\sim}{\mathbf{G}}(\mathbf{x}', \mathbf{x}) \mathbf{S}(\mathbf{x}) \mathrm{d}^3\mathbf{x} - \int_{\partial\mathcal{V}} \underset{\sim}{\mathbf{G}}(\mathbf{x}', \mathbf{x}) \mathbf{P}(\mathbf{x}) n_3 \mathrm{d}^2\mathbf{x}_H, \tag{12}$$

where the characteristic function χ is defined as

$$\chi(\mathbf{x}') = \begin{cases} 1 & \text{for } \mathbf{x}' \in \mathcal{V} \\ 1/2 & \text{for } \mathbf{x}' \in \partial\mathcal{V} \\ 0 & \text{for } \mathbf{x}' \notin \mathcal{V} \cup \partial\mathcal{V} \end{cases} \tag{13}$$

and the contrast operator $\hat{\underset{\sim}{\mathbf{\Delta}}}$ as

$$\hat{\underset{\sim}{\mathbf{\Delta}}}(\mathbf{x}) = \hat{\underset{\sim}{\mathbf{B}}}(\mathbf{x}) - \hat{\bar{\underset{\sim}{\mathbf{B}}}}(\mathbf{x}). \tag{14}$$

Note that the right-hand side of equation (12) contains, respectively, a volume integral over the 'contrast operator' $\hat{\underset{\sim}{\mathbf{\Delta}}}$, a 'direct wave' contribution and a boundary integral over the 'interaction quantity' $\underset{\sim}{\mathbf{G}}\mathbf{P}$.

Applications

The one-way reciprocity theorems (8) and (9) as well as the one-way representation theorem (12) have many applications. We mention forward and inverse extrapolation in finely layered media, multiple elimination, and time-lapse seismics. In the following, we discuss the latter two applications in more detail.

Reciprocity theorem for multiple elimination. Here we discuss multiple elimination, using a similar approach as Fokkema and van den Berg [8] and van Borselen et al. [11]. We start by deriving an integral equation for multiples, related to a reflecting boundary. This boundary may represent the Earth's free surface, the ocean bottom, or any reflector in the subsurface. We denote this reflecting boundary by Σ. We define the volume $\mathcal{V}$ entirely below Σ, in such a way that its upper boundary approaches Σ (as a limiting process) from below and its lower boundary lies below all inhomogeneities in the subsurface. We denote the upper boundary by Σ^+ and the lower by Σ_m, respectively, see Figure 2b. We employ representation (12) to this configuration, where $\mathbf{P}$ denotes the one-way wave vector related to the actual situation (hence, P^+ and P^- include the multiples related to the reflector Σ). Throughout $\mathcal{V}$ we choose the reference medium equal to the actual medium, so the first volume integral on the right-hand side of equation (12) vanishes. Moreover, we assume that the source is situated at or above Σ, so the second volume integral on the right-hand side of equation (12) vanishes as well. Finally,

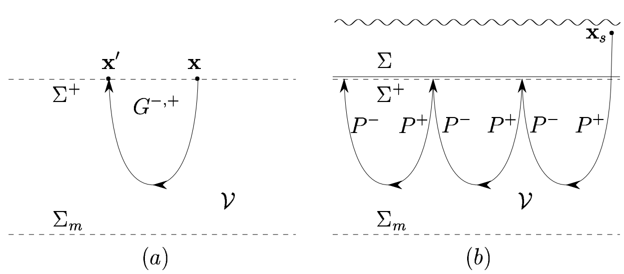


Fig. 2: *(a) State A: the multiple-free impulse response $G^{-,+}(\mathbf{x}', \mathbf{x})$. (b) State B: the actual one-way responses $P^+(\mathbf{x})$ and $P^-(\mathbf{x}')$, including multiples related to the reflecting boundary Σ.*

the boundary integral in equation (12) reduces to an integral over Σ^+ only, with $n_3 = -1$. So for $\mathbf{x}' \in \Sigma^+$ we obtain

$$\frac{1}{2}\mathbf{P}(\mathbf{x}') = \int_{\Sigma^+} \underset{\sim}{\mathbf{G}}(\mathbf{x}', \mathbf{x})\mathbf{P}(\mathbf{x})\mathrm{d}^2\mathbf{x}_H. \tag{15}$$

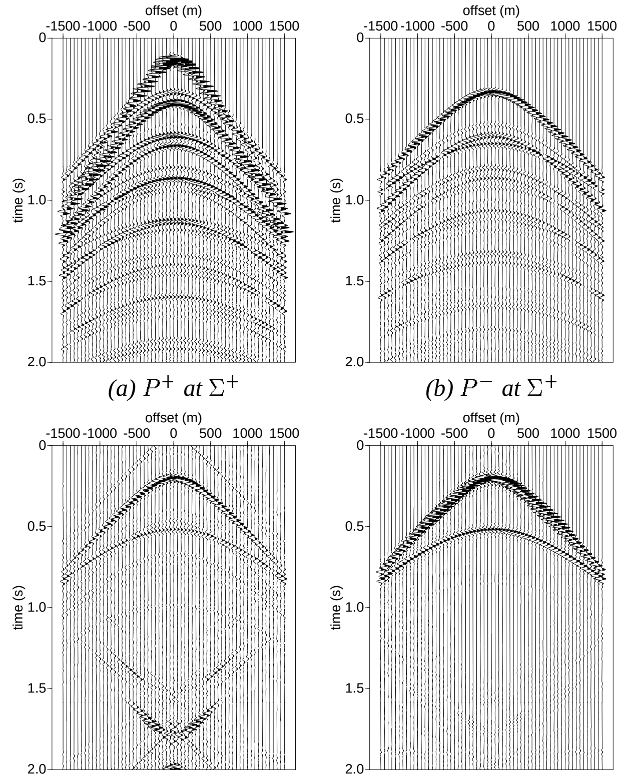
We have not yet specified the reference medium for the Green's matrix outside $\mathcal{V}$. In the following we choose a non-scattering reference medium in the half-space above Σ^+ (Figure 2a). The lower-left element of the Green's matrix, i.e., $G^{-,+}(\mathbf{x}', \mathbf{x})$ represents the reflection impulse response of the medium in $\mathcal{V}$ *without* multiples related to the reflector Σ; this is the response that we are after. The upper-right element is the reflection response of the medium above Σ^+, hence it equals zero. The diagonal elements of $\underset{\sim}{\mathbf{G}}$ are the direct downgoing and upgoing waves at Σ^+. From equation (10) it follows that they are given by $\pm\frac{1}{2}\delta(\mathbf{x}'_H - \mathbf{x}_H) = \pm\frac{1}{2}\delta(x'_1 - x_1)\delta(x'_2 - x_2)$. The factor $\frac{1}{2}$ is due to the fact that $\mathbf{x}'$ and $\mathbf{x}$ are both on Σ^+; the plus and minus signs follow from the boundary conditions (i.e., outgoing waves for $x_3 \to -\infty$ and $x_3 \to \infty$). Hence, we may rewrite equation (15) for $\mathbf{x}' \in \Sigma^+$ as

$$\frac{1}{2}\begin{pmatrix} P^+(\mathbf{x}') \\ P^-(\mathbf{x}') \end{pmatrix} = \tag{16}$$
$$\int_{\Sigma^+} \begin{pmatrix} \frac{1}{2}\delta(\mathbf{x}'_H - \mathbf{x}_H) & 0 \\ G^{-,+}(\mathbf{x}', \mathbf{x}) & -\frac{1}{2}\delta(\mathbf{x}'_H - \mathbf{x}_H) \end{pmatrix} \begin{pmatrix} P^+(\mathbf{x}) \\ P^-(\mathbf{x}) \end{pmatrix} \mathrm{d}^2\mathbf{x}_H,$$

from which we obtain

$$P^-(\mathbf{x}') = \int_{\Sigma^+} G^{-,+}(\mathbf{x}', \mathbf{x})P^+(\mathbf{x})\mathrm{d}^2\mathbf{x}_H, \tag{17}$$

for $\mathbf{x}' \in \Sigma^+$. Note that we have obtained an integral equation for the reflection impulse response $G^{-,+}(\mathbf{x}', \mathbf{x})$ which does not contain multiples related to the reflector Σ (Figure 2a). The one-way wave fields $P^+(\mathbf{x})$ and $P^-(\mathbf{x}')$ represent the actual situation including the multiples related to Σ. Since $G^{-,+}$ depends on $\mathbf{x}'$ as well as on $\mathbf{x}$, it can not be resolved from a single seismic experiment (except when there are no lateral variations in which case $G^{-,+}(\mathbf{x}', \mathbf{x})$ is invariant with respect to lateral shifts). Hence, in general equation (17) can only be resolved when many independent seismic experiments (related to different source positions) are available. In principle it involves a multi-dimensional deconvolution of the upgoing wave fields by the downgoing wave fields. The idea of resolving the multiple-free response by 'dividing' the upgoing waves by the downgoing waves has been discussed before by amongst others Kennett [9], Berkhout [3], Wapenaar and Verschuur [14], Ziolkowski et al. [15] and Amundsen [1]. The latter author also uses representation theory, analogous to Fokkema and van den Berg [8], to arrive at an integral equation for surface multiples in ocean bottom data, similar to equation (17). The present result is valid for surface multiples, ocean bottom multiples or internal multiples. Unfortunately, direct inversion of equation (17) is unstable, because the spectrum of the wave field $P^+(\mathbf{x})$ contains notches due to the multiples. In Wapenaar and Verschuur [14] we discussed a procedure to stabilize this inversion. An example is presented in Figure 3.



(c) $G^{-,+}$ at Σ^+ *(d) $G^{-,+}$ at Σ^+*

Fig. 3: *Downgoing (a) and upgoing (b) wave fields at Σ^+, obtained by decomposing the pressure and velocity at the ocean bottom Σ. (c) The multiple-free response $G^{-,+}$, obtained by inverting equation (17). (d) Idem, after stabilization.*

Reciprocity theorems for time-lapse seismics. Since in a reciprocity theorem two states interact, it is optimally fitted to formulate the relation between two measurements in a time-lapse seismic experiment (Fokkema et al. [7], Dillen [5], [6]). State A is associated with the reference wave field at, say, $t = t_1$, while state B is associated with the monitoring wave field at, say, $t = t_2 > t_1$. It is noted that $t_2 - t_1$ is much longer than the seismic experiment time. In our analysis $\mathbb{R}^3$ is divided in three domains (Figure 4): $\mathcal{V}_0$ is the domain where there are no differences between the material parameters in the two states, mostly associated with the domain above the reservoir (i.e., $x_3 \leq x_3^1$); the domain $\mathcal{V}_c$, for example associated with the reservoir ($x_3^1 < x_3 \leq x_3^2$), where there is a difference between the material parameters in the two states mostly due to the reservoir production history; and $\mathcal{V}'$ denotes the complement of

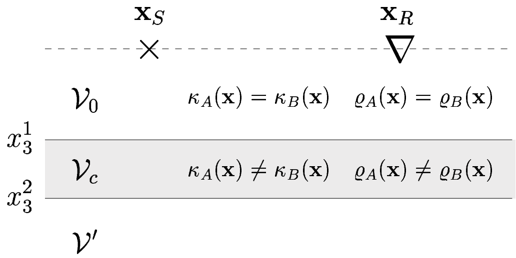


Fig. 4: *Configuration for time-lapse seismics.*

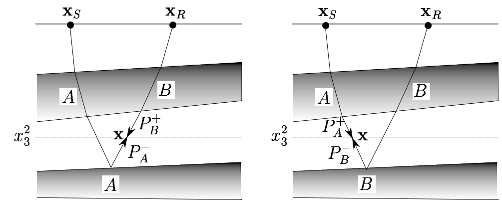
$\mathcal{V}_0 \cup \mathcal{V}_c$ (i.e., $x_3 > x_3^2$); the material parameters in this domain may or may not be different. In our one-way analysis we consider point-sources for downgoing waves in both states, according to

$$\mathbf{S}_A(\mathbf{x},\omega) = (s_A^+(\omega)\ 0)^t \delta(\mathbf{x} - \mathbf{x}_S), \quad (18)$$
$$\mathbf{S}_B(\mathbf{x},\omega) = (s_B^+(\omega)\ 0)^t \delta(\mathbf{x} - \mathbf{x}_R). \quad (19)$$

Application of reciprocity theorem (8) to domain $\mathcal{V} = \mathcal{V}_0 \cup \mathcal{V}_c$ yields

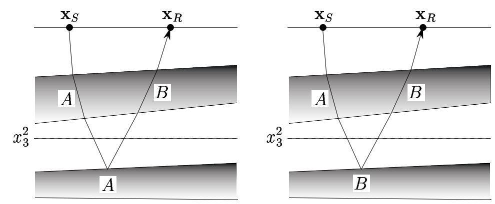
$$s_B^+(\omega) P_A^-(\mathbf{x}_R|\mathbf{x}_S) - s_A^+(\omega) P_B^-(\mathbf{x}_S|\mathbf{x}_R) \quad (20)$$
$$= \int_{\mathbf{x}\in\mathcal{V}_c} \mathbf{P}^t_A(\mathbf{x}|\mathbf{x}_S) \underset{\sim}{\mathbf{N}} (\hat{\underset{\sim}{\mathbf{B}}}_B(\mathbf{x}) - \hat{\underset{\sim}{\mathbf{B}}}_A(\mathbf{x})) \mathbf{P}_B(\mathbf{x}|\mathbf{x}_R) \mathrm{d}^3\mathbf{x} +$$
$$\int_{x_3 = x_3^2} \{P_A^-(\mathbf{x}|\mathbf{x}_S) P_B^+(\mathbf{x}|\mathbf{x}_R) - P_A^+(\mathbf{x}|\mathbf{x}_S) P_B^-(\mathbf{x}|\mathbf{x}_R)\} \mathrm{d}^2\mathbf{x}_H.$$

Using physical reciprocity, the term $P_B^-(\mathbf{x}_S|\mathbf{x}_R)$ may be replaced by $P_B^-(\mathbf{x}_R|\mathbf{x}_S)$. Hence, when $s_A^+(\omega) = s_B^+(\omega)$, the left-hand side represents the difference wave field. The first term on the right-hand side is related to the changes in the domain $\mathcal{V}_c$. The second term on the right-hand side is related to any changes below $x_3 = x_3^2$. Let us analyze this boundary integral further. Figure 5 shows a configuration with two regions in which changes occur (the grey areas). Figure 5a shows some wavepaths in the integral $\int_{x_3=x_3^2} P_A^-(\mathbf{x}|\mathbf{x}_S) P_B^+(\mathbf{x}|\mathbf{x}_R) \mathrm{d}^2\mathbf{x}_H$. If P_B^+ is interpreted as a Green's function for state B (multiplied by the source function $s_B^+(\omega)$), then it is understood that this integral performs an upward extrapolation of P_A^- in state A from the depth level x_3^2 to $\mathbf{x}_R$ at the acquisition surface. This results in a virtual experiment (see Figure 6a), in which the downgoing and upgoing waves propagate through the medium before (A) and after (B) the changes took place, respectively. A similar interpretation is shown in Figures 5b and 6b for the integral $\int_{x_3=x_3^2} P_A^+(\mathbf{x}|\mathbf{x}_S) P_B^-(\mathbf{x}|\mathbf{x}_R) \mathrm{d}^2\mathbf{x}_H$. The only difference between the two virtual experiments in Figures 6a and b is the change of material parameters below x_3^2 (region 2). Note that the two terms of the boundary integral cancel when no changes occur below x_3^2 and when $P_A^\pm$ and $P_B^\pm$ at x_3^2 have been obtained correctly. This yields a verification criterion for the estimated changes above x_3^2 (Dillen [5], [6], Scherpenhuijsen [10]).



(a) (b)

Fig. 5: *Analysis of the two terms in the boundary integral in equation (20). Both terms accomplish a forward extrapolation of upgoing waves from x_3^2 to the surface. The results are shown in Figure 6.*



(a) (b)

Fig. 6: *Virtual experiments, corresponding to Figures 5a and b. A and B: situations before and after the changes took place.*

Conclusions

We have derived reciprocity and representation theorems for one-way wave fields in media with anelastic losses. Applications are found in forward and inverse extrapolation in finely layered media, multiple elimination (free-surface, ocean bottom or internal) and in time-lapse seismics.

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