Decomposition of multicomponent ocean-bottom data: inversion for the sub-bottom parameters

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Summary

The aim of wavefield decomposition of multicomponent seismic ocean-bottom data, is to separate the measurements into up- and downgoing P- and S-waves. To accomplish this separation, the medium parameters (compressional wave velocity c_P, shear wave velocity c_S and density ϱ) just below the receiver level need to be known. In this paper, estimates of these parameters have been obtained from the data itself. Using an adaptive decomposition scheme, least-squares decomposition filters are obtained. These filters are inverted into sub-bottom parameters. For the inversion two forward models for the ocean-bottom were investigated. In the first model, the direct arrival and the deeper reflection events are well separated in time. The second model consists of a thin layer beneath the ocean-bottom. Reverberations inside the thin layer interfere with the first arrival. For both models velocity and density estimates were obtained.

Introduction

When applying the wave equation based decomposition theory [1, 4] to field data difficulties are encountered; c_P, c_S and ϱ just below the ocean-bottom need to be known, and the geophone coupling is imperfect. As the very shallow sub-bottom is not of interest in oil exploration - where the target lies at greater depths - measurements of these shallow parameters are commonly not available. Therefore, estimates of c_P, c_S and ϱ just below the receiver level, have to be obtained in a different way. Here, it has been shown that estimates of these parameters can be obtained from the data itself. The medium parameters arise as a byproduct of decomposing the data into P- and S-waves. The parameter inversion in itself, as an alternative way to obtain estimates of c_P, c_S and ϱ, might be of interest in other applications besides decomposition, e.g. marine acoustics, statics.

A more practical formulation of the decomposition theory was obtained by splitting the decomposition in two steps - the first step consists only of a decomposition into up- and downgoing wavefields, the second step decomposes into P- and S-waves. The advantage of this two-step formulation is that less data components are required at a time The derivation of the two-step decomposition was shown in [2]. Furthermore, by applying these 'partial' decomposition equations in a certain order (the adaptive decomposition scheme) it is possible to estimate all unknowns from the data itself as you go along [2]. An additional advantage is the better control on the quality of the final decomposition result by obtaining good 'partial' decomposition results. In the following, a more detailed description of the inversion of the sub-bottom parameters from the optimization filters, arising from the adaptive decomposition procedure, will be given.

Adaptive decomposition procedure

The adaptive decomposition scheme consists of five stages given below (decomposition equations are written in the rayparameter-frequency domain for the 2-D situation):

Stage 1. 'Rotation' of the velocity components In this first stage the V_z measurement is corrected for an imperfection in the acquisition that is not addressed in the preprocessing nor further on in this scheme: 'mechanical cross-coupling', visible as converted waves recorded on the V_z component that cannot be found on the P component.

Stage 2. Acoustic decomposition just above the bottom The acoustic decomposition is used to resolve the calibration filter $a(\omega)$ between the pressure and vertical velocity component, caused by imperfect coupling of the geophone to the ocean-bottom:

$$\tilde{P}^{\pm} = \frac{1}{2}\tilde{P} \pm a(\omega)\frac{\varrho_1}{2q_1}\tilde{V}_z, \tag{1}$$

where ϱ_1 and $q_1 = \sqrt{c_1^{-2} - p^2}$ are the density and vertical slowness in the water-layer. To resolve $a(\omega)$, the criterion that there should be no primary reflections present in the decomposed downgoing wavefield *above* the bottom (P^+) is used.

Stage 3. Elastic decomposition into $\tau_{zz}^{\pm}$ just below the bottom The next stage is an elastic decomposition *below* the bottom, into up- and downgoing normal stressfields:

$$-\tilde{\tau}_{zz}^{\pm} = \frac{1}{2}\tilde{P} \pm a(\omega)\frac{\varrho_2\beta_2}{2q_{P,2}}\tilde{V}_z, \tag{2}$$

where ϱ_2 and $q_{P,2} = \sqrt{c_{P,2}^{-2} - p^2}$ are the density and vertical P-wave slowness of the medium just below the bottom, and $\beta_2 = c_{S,2}^4[4p^2 q_{P,2}q_{S,2} + (c_{S,2}^{-2} - 2p^2)^2]$. This time the unknown factor is the operator in front of the V_z component, as it depends on the medium parameters just below the bottom. To find the operator, the expression is replaced by a general rayparameter dependent filter $f(p)$:

$$-\tilde{\tau}_{zz}^{\pm} = \frac{1}{2}\tilde{P} \pm a(\omega)f(p)\tilde{V}_z. \tag{3}$$

The condition imposed on the decomposition result is that there should be no direct wave and water bottom multiples in the upgoing normal stressfield *below* the bottom.

Stage 4. Elastic decomposition into $\tau_{xz}^{\pm}$ just below the bottom The fourth decomposition stage involves the P and

V_x components, making it possible to resolve the calibration filter $b(\omega)$ between them:

$$-\tilde{\tau}_{xz}^{\pm} = \pm\frac{\gamma_2 p}{2q_{S,2}}\tilde{P} \pm b(\omega)\frac{\varrho_2\beta_2}{2q_{S,2}}\tilde{V}_x. \qquad (4)$$

where $q_{S,2} = \sqrt{c_{S,2}^{-2} - p^2}$, $\gamma_2 = c_{S,2}^2[2q_{P,2}q_{S,2} - (c_{S,2}^{-2} - 2p^2)]$ and β_2 as above. From equation (4) we can see that first the decomposition operators need to be calculated with the medium parameters just below the ocean-bottom, before $b(\omega)$ can be obtained. An estimate of the medium parameters can be obtained by inverting the filter $f(p)$ found in the previous stage.

Stage 5. Elastic decomposition into $\Phi^{\pm}$ and $\Psi^{\pm}$ In the last stage the estimated parameters just below the ocean-bottom and the results of the elastic decomposition into up- and downgoing stressfields are simply combined to obtain the up- and downgoing P- and S-waves:

$$\tilde{\Phi}^{\pm} = \frac{c_S^2}{\beta}\{\mp 2pq_S\tilde{\tau}_{xz}^{\pm} - (c_S^{-2} - 2p^2)\tilde{\tau}_{zz}^{\pm}\}, \qquad (5)$$

$$\tilde{\Psi}^{\pm} = \frac{c_S^2}{\beta}\{(c_S^{-2} - 2p^2)\tilde{\tau}_{xz}^{\pm} \mp 2pq_P\tilde{\tau}_{zz}^{\pm}\}. \qquad (6)$$

The first three stages of the adaptive decomposition scheme are readily applicable to multicomponent ocean-bottom data. They consist of least-squares optimizations ensuring the best possible result for the particular dataset. If the ocean-bottom is not too shallow, the windows in which energy is minimized, are easily determined based on local water depth and velocity. To apply stages 4 and 5, however, an inversion for the medium paramers just below the ocean-bottom is necessary. The medium parameters are inverted from the least-squares filter $f(p)$ obtained in stage 3.

In the following, we will take a closer look at the inversion of $f(p)$. With field data usually a frequency-dependent filter $f(p,\omega)$ is found. Therefore, also an analytic expression for frequency-dependent inversion is derived.

Least-squares filter $f(p)$

To calculate the filter $f(p)$, a window is put over the direct arrival and/or water bottom multiples. The energy in this window is minimized in a least-squares sense by $f(p)$. From equations 2 and 3 it follows that theoretically $f(p) = \frac{\varrho\beta}{2q_P}$. In Figure 1 the filter $f(p)$ obtained from a synthetic dataset is shown together with the best-fitting decomposition operator (line with dots). An initial estimate of the medium parameters is obtained from the location of the singularities (giving c_P). From the amplitude at $p = 0$ (which is equal to half the impedance) an estimate of ϱ is obtained. An initial estimate for c_S is obtained by assuming a realistic velocity ratio (c_P/c_S). Once an initial estimate has been obtained the actual curve fitting is done in the pre-critical rayparameter domain $[-4\cdot 10^{-4}, 4\cdot 10^{-4}]$ s/m. Otherwise the large amplitudes of the singularities influence the curve fitting for other rayparameter values too much.

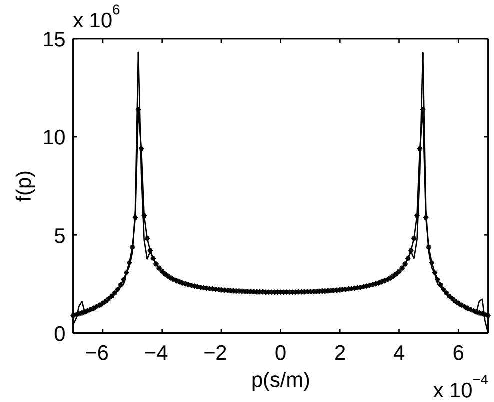

Fig. 1: *Least-squares operator $f(p)$ from synthetic data, together with the best-fitting theoretical decomposition operator (line with dots). The medium parameters just below the ocean-bottom are $c_P = 1700$ m/s, $c_S = 600$ m/s and $\varrho = 1470$ kg/m^3.*

In Figure 2 an example of $f(p)$ obtained from a field dataset is shown. Note that the filter is frequency-dependent i.e. the different lines do not overlap. A possible model to explain the frequency-dependency in $f(p)$ can be given with the ocean-bottom model shown in Figure 3 a. This model consists of an ocean-bottom with a thin sediment layer on top. The thickness of the layer is chosen such that the arrival time of the reflection from the bottom of the layer cannot be distinguished from the direct source arrival. The layer merely causes a lengthening of the wavelet. Therefore, the optimization filter that is estimated not only removes the direct wave but also the reflection and reverberations inside the thin layer. Therefore, it is not a pure decomposition operator anymore. The filter $f(p,\tau)$ for this model is shown in Figure 3 b, an example of $f(p,\tau)$ for a field dataset is given in Figure 3 c. For this specific model an analytic expression for $f(p,\omega)$ can be derived, which will be done in the next section.

Analytic expression for $f(p,\omega)$

Acoustic case As $f(p,\omega)$ removes the energy from the direct source arrival as well as the reverberations from the thin layer, this is similar to obtaining the upgoing field just below depth level z_2 (see Figure 3 a). A decomposition at z_2 is given by

$$\begin{pmatrix}\tilde{P}^+(z_2)\\ \tilde{P}^-(z_2)\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & \frac{\varrho_2}{q_2}\\ 1 & -\frac{\varrho_2}{q_2}\end{pmatrix}\begin{pmatrix}\tilde{P}(z_2)\\ \tilde{V}_z(z_2)\end{pmatrix}, \qquad (7)$$

and $[P(z_2), V_z(z_2)]^T$ are related to the wavefields at the ocean-

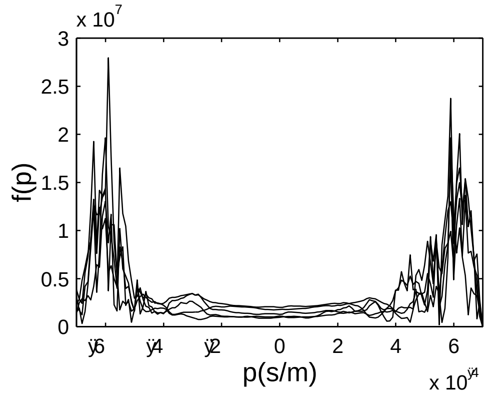

Fig. 2: *Least-squares operator $f(p)$ from a field dataset. The solid lines are different frequencies in the interval between 13–53 Hz.*

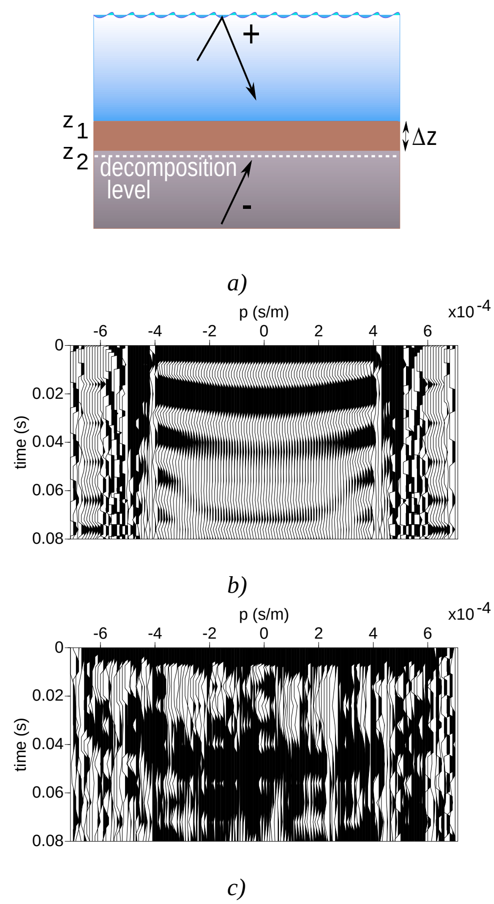

Fig. 3: *An ocean-bottom model with a thin layer on top of a half-space (a), b the filter $f(p,\tau)$ that optimally removes the first arrival, and c an example of $f(p,\tau)$ obtained from a field dataset.*

bottom (z_1) by the two-way wavefield extrapolation matrix [3]

$$\begin{pmatrix}\tilde{P}(z_2)\\ \tilde{V}_z(z_2)\end{pmatrix} = \begin{pmatrix}\cos(\omega q_1 \Delta z) & -\frac{j\varrho_1}{q_1}\sin(\omega q_1 \Delta z)\\ \frac{q_1}{j\varrho_1}\sin(\omega q_1 \Delta z) & \cos(\omega q_1 \Delta z)\end{pmatrix}\begin{pmatrix}\tilde{P}(z_1)\\ \tilde{V}_z(z_1)\end{pmatrix}. \tag{8}$$

Substituting equation (8) into equation (7) gives

$$\begin{pmatrix}\tilde{P}^+(z_2)\\ \tilde{P}^-(z_2)\end{pmatrix} = \frac{1}{2}\begin{pmatrix}A & -\frac{\varrho_1}{q_1}B\\ C & -\frac{\varrho_1}{q_1}D\end{pmatrix}\begin{pmatrix}\tilde{P}(z_1)\\ \tilde{V}_z(z_1)\end{pmatrix}, \tag{9}$$

where

$$A = \cos(\omega q_1 \Delta z) - \frac{j\varrho_2 q_1}{\varrho_1 q_2}\sin(\omega q_1 \Delta z), \tag{10}$$

$$B = j\sin(\omega q_1 \Delta z) - \frac{j\varrho_2 q_1}{\varrho_1 q_2}\cos(\omega q_1 \Delta z), \tag{11}$$

$$C = \cos(\omega q_1 \Delta z) + \frac{j\varrho_2 q_1}{\varrho_1 q_2}\sin(\omega q_1 \Delta z), \tag{12}$$

$$D = j\sin(\omega q_1 \Delta z) + \frac{j\varrho_2 q_1}{\varrho_1 q_2}\cos(\omega q_1 \Delta z). \tag{13}$$

The acoustic decomposition equation for the situation without the thin layer reads

$$\tilde{P}^- = \frac{1}{2}\tilde{P} - f(p)\tilde{V}_z, \tag{14}$$

where

$$f(p) = \frac{\varrho}{2q_1}. \tag{15}$$

For the situation with the thin layer, the upgoing wavefield below the layer is obtained with

$$\tilde{P}^-(z_2) = \frac{1}{2}C\tilde{P} - \frac{\varrho_1}{2q_1}D\tilde{V}_z, \tag{16}$$

see equation (9). In practice the parameters of the thin layer and the layer underneath are unknown and therefore the operator C is unknown. When both sides of equation (16) are divided by C to obtain

$$\frac{\tilde{P}^-}{C} = \frac{1}{2}\tilde{P} - f(p,\omega)\tilde{V}_z, \tag{17}$$

$f(p,\omega)$ can be obtained in the same way as $f(p)$ in equation (14), only this time the expression to invert is:

$$f(p,\omega) = \frac{\varrho_1 D}{2q_1 C}. \tag{18}$$

Note that in the case of a thin layer the upgoing wavefield obtained by matching P and V_z is a scaled version of the upgoing pressure wavefield when no thin layer is present.

Elastic case In the elastic case a decomposition just below z_2 into up- and downgoing wavefields is given by

$$\begin{pmatrix} -\tilde{\tau}^+_{zz}(z_2) \\ -\tilde{\tau}^-_{zz}(z_2) \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -\frac{\gamma_2 p}{q_{P,2}} & 1 & 0 & \frac{\varrho_2 \beta_2}{q_{P,2}} \\ \frac{\gamma_2 p}{q_{P,2}} & 1 & 0 & -\frac{\varrho_2 \beta_2}{q_{P,2}} \end{pmatrix} \begin{pmatrix} -\tilde{\tau}_{xz}(z_2) \\ -\tilde{\tau}_{zz}(z_2) \\ \tilde{V}_x(z_2) \\ \tilde{V}_z(z_2) \end{pmatrix}, \quad (19)$$

see [2]. Furthermore, $[-\tau_{xz}(z_2), -\tau_{zz}(z_2), V_x(z_2), V_z(z_2)]^T$ are related to the wavefields at the ocean-bottom (z_1) by the two-way elastic wavefield extrapolation matrix [5]

$$\begin{pmatrix} -\tilde{\tau}_{xz}(z_2) \\ -\tilde{\tau}_{zz}(z_2) \\ \tilde{V}_x(z_2) \\ \tilde{V}_z(z_2) \end{pmatrix} = \begin{pmatrix} W_{11} & W_{12} & W_{13} & W_{14} \\ W_{21} & W_{22} & W_{23} & W_{24} \\ W_{31} & W_{32} & W_{33} & W_{34} \\ W_{41} & W_{42} & W_{43} & W_{44} \end{pmatrix} \begin{pmatrix} 0 \\ \tilde{P}(z_1) \\ \tilde{V}_x(z_1) \\ \tilde{V}_z(z_1) \end{pmatrix}. \quad (20)$$

The elements of the two-way elastic extrapolation matrix are dependent on the frequency, medium parameters and thickness of layer 1 and are given in [5]. Substituting equation (20) into equation (19) gives

$$\begin{pmatrix} -\tilde{\tau}^+_{zz}(z_2) \\ -\tilde{\tau}^-_{zz}(z_2) \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -\frac{\gamma_2 p}{q_{P,2}} & 1 & 0 & \frac{\varrho_2 \beta_2}{q_{P_2}} \\ \frac{\gamma_2 p}{q_{P,2}} & 1 & 0 & -\frac{\varrho_2 \beta_2}{q_{P_2}} \end{pmatrix} \begin{pmatrix} W_{11} & W_{12} & W_{13} & W_{14} \\ W_{21} & W_{22} & W_{23} & W_{24} \\ W_{31} & W_{32} & W_{33} & W_{34} \\ W_{41} & W_{42} & W_{43} & W_{44} \end{pmatrix} \begin{pmatrix} 0 \\ \tilde{P}(z_1) \\ \tilde{V}_x(z_1) \\ \tilde{V}_z(z_1) \end{pmatrix}, \quad (21)$$

or

$$-\tilde{\tau}^-_{zz}(z_2) = \frac{1}{2} A\tilde{P}(z_1) - B\tilde{V}_z(z_1) - C\tilde{V}_x(z_1), \quad (22)$$

where

$$A = \frac{\gamma_2 p}{q_{P,2}} W_{12} + W_{22} - \frac{\varrho_2 \beta_2}{q_{P,2}} W_{42}, \quad (23)$$

$$B = -\frac{\gamma_2 p}{2q_{P,2}} W_{14} - \frac{1}{2} W_{24} + \frac{\varrho_2 \beta_2}{2q_{P,2}} W_{44}, \quad (24)$$

$$C = -\frac{\gamma_2 p}{2q_{P,2}} W_{13} - \frac{1}{2} W_{23} + \frac{\varrho_2 \beta_2}{2q_{P,2}} W_{43}. \quad (25)$$

The elastic decomposition equation for the situation without the thin layer reads

$$-\tilde{\tau}^-_{zz}(z_1) = \frac{1}{2} \tilde{P}(z_1) - f(p)\tilde{V}_z(z_1) \quad (26)$$

where

$$f(p) = \frac{\varrho \beta}{2q_P}. \quad (27)$$

For the situation with the thin layer the upgoing wavefield is given by equation (22). In practice the parameters of the thin layer and the layer underneath are unknown and therefore the operator A is unknown. When both sides of equation (22) are divided by A to obtain

$$-\frac{\tilde{\tau}^-_{zz}(z_2)}{A} = \frac{1}{2}\tilde{P}(z_1) - \frac{B}{A}\tilde{V}_z(z_1) - \frac{C}{A}\tilde{V}_x(z_1), \quad (28)$$

$f(p)$ can be obtained in the same way as in equation (26), provided the contribution of the V_x component can be neglected. Note that in the case of a thin layer the upgoing wavefield obtained by matching P and V_z is a scaled version of the upgoing normal stressfield when no thin layer is present. The expression for $f(p)$ becomes: $f(p, \omega) = \frac{B}{A}$. Inversion using this expression results in seven medium parameters - the velocities and densities of the thin layer and the medium beneath, and the thickness of the thin layer.

Conclusions

Experiences with the adaptive decomposition procedure on field data have resulted in frequency-dependent optimization filters obtained in the stage of the decomposition into up- and downgoing normal stressfields (stage 3). The optimization filter is then inverted for the medium parameters just below the ocean-bottom as if it were a pure decomposition operator, i.e. in a frequency-independent way. To better represent the ocean-bottom, a forward model is necessary that can explain the frequency-dependent behaviour of the optimization filter and is simple enough to provide an analytic expression. The model proposed here consists of a thin layer over the ocean-bottom. For this model analytic expressions are derived that describe frequency-dependent behaviour of the optimization filter. Furthermore, it will be investigated how well the parameters are determined in the inversion problem. A better representation of the ocean-bottom will lead to a more reliable decomposition result.

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