Resolving well-log singularities from seismic data

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Summary

In well-logs we often encounter sharp outliers, or singularities, that are responsible for strong reflections. However, these reflectors are different from step-functions, in a sense that step-functions are scale invariant and these singularities are not. In this paper a scale analysis has been performed on synthetic and real well-logs, as well as on their reflection responses, in order to test a method that can distinguish between scale invariant and scale variant reflectors. We will show that it is possible to extract a stable local singularity exponent α from a well-log, and we can characterize the same exponent out of seismic reflection data.

Introduction

Multi-scale analysis is usually performed on well-logs, only recently this analysis is being applied to seismic data [1]. In this paper we will investigate media that contain singularities. If singularities are described in a self-similar way, it can be proven that, except for the step-function, they are scale variant. This should imply [4] that the angle-dependent seismic reflection response is scale variant as well, in a way that is directly linked to the scaling behavior of the well-log. In this paper we will try to extract a local singularity exponent both from the well-log as well as from the seismic reflection data and we will investigate its stability. In the first example we consider a synthetic well-log with well-defined self-similar singularities, in the second example the singularities will be taken from real well-logs.

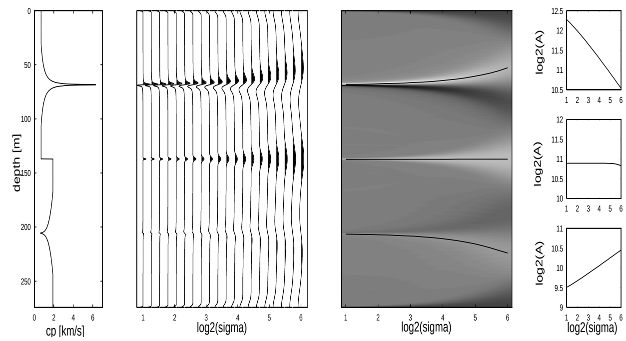
Example on a synthetic velocity well-log

We consider self-similar singularities in the velocity function $c(z)$ of the form

$$c(z) = \begin{cases} c_1|z/z_1|^\alpha & \text{for } z < 0 \\ c_2|z/z_1|^\alpha & \text{for } z > 0. \end{cases} \quad (1)$$

The synthetic well-log in Figure 1(a) consists of two shifted versions of this singularity, which are characterized by a singularity exponent α of -0.4 and 0.2 respectively, and $c_1 = c_2$ for both singularities. The stepfunction in between the two singularities can also be seen as a singularity, but its singularity exponent α must then be chosen as 0, but in this case $c_1 \neq c_2$. The density will be chosen constant throughout the complete depth-interval at a value of 1000 kg/m^3.

The multi-scale analysis will be done by means of the wavelet-transform. Mallat and Hwang [3] have shown that estimations of local regularity are possible with the *modulus maxima* representation. As proposed and investigated by Herrmann [2], we will use this transform to analyze the scale-properties of the well-log.



(a) (b) (c) (d)

Fig. 1: *(a) Synthetic well-log as used in this examle , (b) wavelet transform of the well-log, (c) position of modulus-maxima lines in wavelet transform, (d) log-log plot of amplitude along modulus-maxima lines vs. scale (top)* $\alpha = -0.4$, *(middle)* $\alpha = 0$, *(bottom)* $\alpha = 0.2$

The wavelet-transform maps $f(z)$ to $\check{f}(\sigma, z)$, according to

$$\mathcal{W}\{f,\psi\}(\sigma,z) = \check{f}(\sigma,z) = \int f(z')\frac{1}{\sigma}\psi(\frac{z'-z}{\sigma})dz' \quad (2)$$

in which σ is the scale dilatation parameter, $f(z)$ the function to be transformed and $\psi(z)$ the analyzing wavelet. In this study ψ has been chosen to be the first derivative of the Gaussian window.

Figure 1(b) shows the result of this transform for different values of σ, in the range of $[2, 64]$, hence $^2\log(\sigma)\epsilon[1,6]$. For the analysis of this transform we first take the modulus of the data and then we connect the local maxima of the neighboring traces. The lines that connect the local maxima are commonly known as *modulus maxima*-lines. We should be able to retrieve the scaling parameter α from a log-log plot of the amplitude along these lines versus the scale, because by (see [4]):

$$\log|\check{c}(\sigma, z_{\max})| = \alpha\log\sigma + \log|\check{c}(1, \frac{z_{\max}}{\sigma})| \quad (3)$$

it follows that α is given by the slope of this graph. We will further on refer to this method as the method of WTMML's (Wavelet Transform Modulus Maxima Lines). In Figure 1(d) the results of this analysis are given, for each of the singularities separately. These figures show clearly that the wavelet-transform is a powerful tool for the analysis of scaling effects in well-logs. It retrieves quite accurately the value of the singularity exponent.

However, as we hope that α is a reliable seismic indicator, we would like to retrieve α from seismic data.

For normal incidence this has been investigated by Dessing et. al. [1]. In this analysis we will incorporate also oblique incident waves, in order to get more stabel results. For this purpose seismic data has been modeled in the above mentioned synthetic well-log, making use of a "reflectivity method".

Figure 2a shows the plane wave response in the ray-parameter intercept-time (p, τ) domain as well as an image in the ray-parameter depth (p, z) domain. Note that for Figure 2 the reflection response $u^{\mathrm{refl}}(0, \mathrm{p}, \tau)$, which is modeled by a reflectivity method, has been convolved with a (symmetrical) wavelet, to represent the data in a more comfortable way.

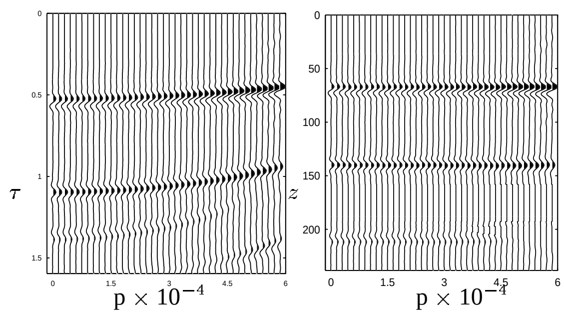


Fig. 2: *(a), (p, τ)- and (b), (p, z)- reflection response of synthetic well-log*

The imaged data set $R(\mathrm{p}, z)$(Figure 2b will now be given a third dimension, as visible in Figure 3, by a wavelet transform according to:

$$\check{R}(\mathrm{p}, \sigma, z) = \mathcal{W}\{R, \psi\}(\mathrm{p}, \sigma, z). \quad (4)$$

The wavelet-transform will be taken of the imaged impulse response, instead of the reflected wavefield. This is done, because otherwise we will be analyzing the combined effect of the reflectivity and the seismic wavelet, which will give results that are not easily interpretable. Note that if the wavelet is nearer to a spike, this effect becomes less deteriorative. For real seismic data this implies that if we want to perform this scaling analysis on the data, we have to deconvolve with the seismic wavelet as accurately as possible, in order to prevent a false characterization of its scaling behavior.

From the 3-dimensional cube $\check{R}(\mathrm{p}, \sigma, z)$ of Figure 3, we select cross-sections along *modulus maximum* planes which will be denoted $\check{R}(\mathrm{p}, \sigma, z_{max})$, and we will analyze contours of constant amplitudes in these planes. According to Wapenaar [4] we can expect to find curves of constant amplitude $|\check{R}(\mathrm{p}, \sigma, z_{max})|$ in the *modulus maximum* planes:

$$\mathrm{p}^{1-\alpha}\sigma^{\alpha} = \text{constant}. \quad (5)$$

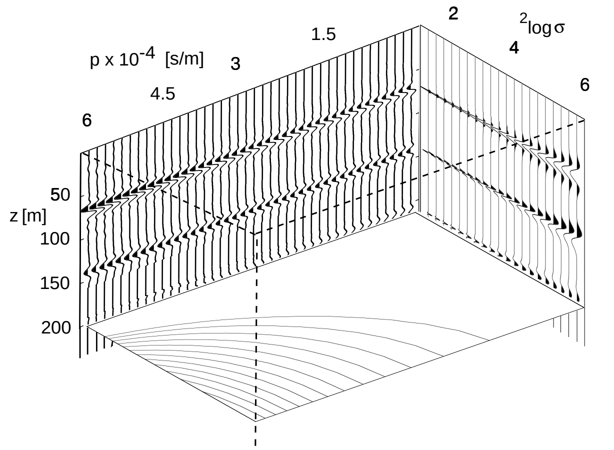


Fig. 3: *3-D representation of the scale analysis on imaged seismic data $\check{R}(\mathrm{p}, \sigma, z)$*

In figure 4 we can see the analytical curves for the three values of α in the (p, σ)-plane, as well as the results obtained from the numerical analysis we just discussed.

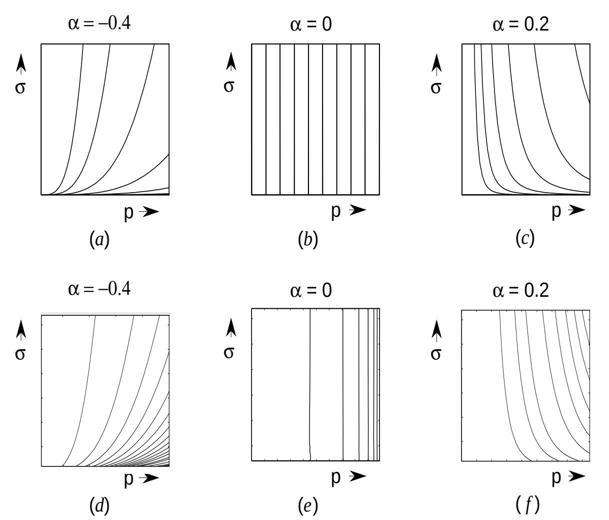


Fig. 4: *Analytical contours along which $\mathrm{p}^{1-\alpha}\sigma^{\alpha}$ = constant (a) $\alpha = -0.4$, (b) $\alpha = 0$, (c) $\alpha = 0.2$ and the corresponding results obtained from the image in Figure 2(b): (d), (e) and (f)*

We can clearly see that the curves in figure 4d,e,f follow quite accurately the predicted curves of 4a,b,c.

Examples on a real velocity well-log

The forementioned method has been performed on certain strong reflectors in an actual sonic P-velocity log. In the first example a piece of a log (Figure 5a) has been chosen of

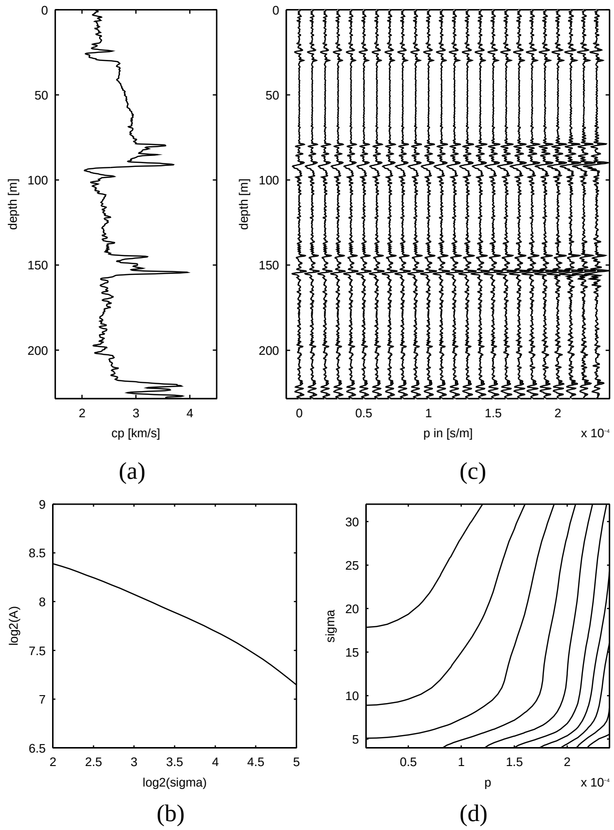


Fig. 5: *(a) Velocity function as used in this example (b) log-log plot of amplitude along modulus-maxima lines vs. scale at a depth of around 160 m (slope =* $\alpha \approx -0.32$*) (c) Imaged reflection response of this velocity function (d)* $|\check{R}(\mathrm{p},\sigma,z_{max})|$ *in a modulus maxima plane at the same depth as the analysis in (b)*

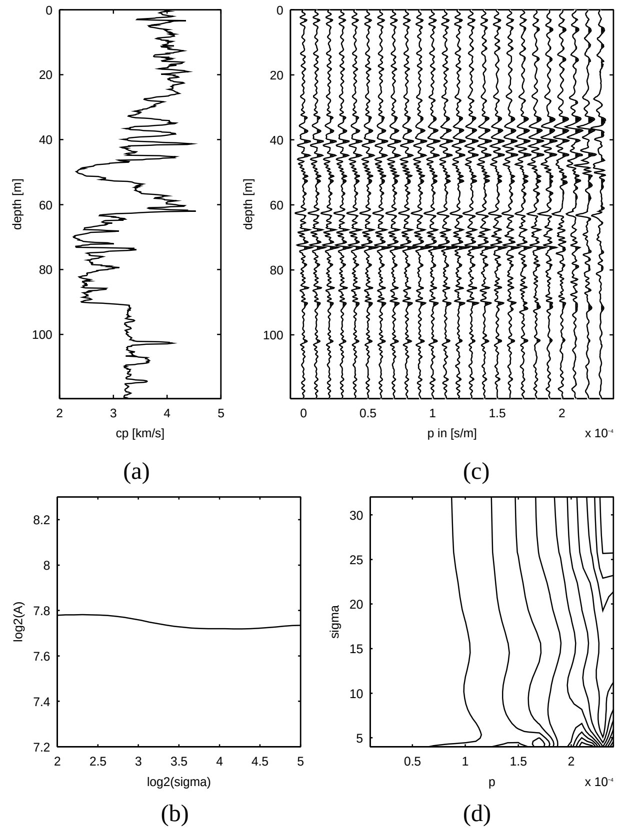


Fig. 6: *(a) Velocity function as used in this example (b) log-log plot of amplitude along modulus-maxima lines vs. scale at a depth of around 90 m (slope =* $\alpha \approx 0$*) (c) Imaged reflection response of this velocity function (d)* $|\check{R}(\mathrm{p},\sigma,z_{max})|$ *in a modulus maxima plane at the same depth as the analysis in (b)*

1500 samples (i.e. 250 m.) in which the acoustic reflection response was modeled. Making use of the method of WT-MML's a reflector was found at a depth of around 160m with a constant singularity exponent α=-0.32 (see Figure 5b), in the $^2\log\sigma$-range [2,5]. This scale range is approximately equal to a wavelength of 8 to 60 m, which corresponds to the higher frequencies in the seismic spectrum. The imaged reflection response is visible in Figure 5c. As we have used the same sampling for the velocity function as for the imaging, we can state that the scales in all pictures are directly comparable. We can see that there is a strong reflector which conforms to the one already pointed out. At this depth level a wavelet transform has been performed and we have measured the amplitude of the envelope of the reflectivity along *modulus maxima* planes. The result of this analysis is given in Figure 5d. We can see that the overall trend of the curves is consistent with the fact that the local singularity exponent in the log is negative (compare Figure 4a)

We have also performed the method on a log in which a step-function was present ($\alpha \approx 0$). This log which is visible in Figure 6a, contains this step-function at a depth of about 90 m. At this depth the method of WTMML's was performed. The results of this analysis are in Figure 6b. We can clearly see that the singularity exponent is approximately equal to 0 for the logarithmic scale range [2,5]. Just as in the foregoing case, the (imaged) reflection response is given in Figure 6c, and the value of the reflectivity along the modulus maxima planes has been given in Figure 6d. In this picture we can clearly see that the pattern is different from Figure 5d. If we compare these curves with the analytical curves in Figure 4a,b,c, we can see that the trend of the curves is best represented by a value of $\alpha = 0$.

Conclusion and discussion

We have shown that it is possible to retrieve a scale parameter from the reflection response of a medium, which is also consistent with the one derived from a sonic velocity log, in the same scale range. Up till now, the method has not been tested on real reflection data.

Note that we only used local amplitude information (retrieved from modulus maxima planes), to estimate the singularity exponent. We will investigate the possibility to include local phase information as well, in order to obtain more stable estimates of the local singularity exponent.

When looking at the results obtained from seismic data modeled in the real well-logs, we expect that the singularity exponent α may prove to be a useful seismic indicator, in addition to other parameters in AVA inversion.

Acknowledgement

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