Removal of Love waves without the use of a structural subsurface model

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Summary

A typical problem found when performing shear-wave seismic reflection experiments on land, is the occurrence of Love waves. Love waves are considered noise, because they are surface waves, bearing no subsurface reflection information. For several reasons it is hard to separate them from the reflections with conventional techniques.

This paper will present a technique, useful for removing Love waves from seismic shear-wave data, using a data driven approach. No model of the structure of the first layer is needed. The approach is similar to that of van Borselen et al. (1996), who used acoustic reciprocity to remove multiples from marine seismic data, where also no model of the structure of the water bottom is needed. In this case, elastic reciprocity will be used.

Introduction

In an elastic material, two types of waves can propagate, i.e. compressional waves, and two types of shear waves, being vertically and horizontally polarized, SV- and SH-waves respectively. In crossline (x_2) invariant media, the SH-waves are decoupled from the other two wave types.

The data of an SH-wave shot record are often polluted with Love waves. Love waves are surface waves and are considered noise because they bear no subsurface reflection information. Because they travel along the surface, they attenuate slowly, and make up for most of the energy in a seismogram. In shallow surveys, their wave speed is almost equal to the shear wave velocity of the upper layers, making it hard to separate the two kinds of waves with e.g. f-k analysis. Another problem is, that Love waves are dispersive, meaning that their phase velocity is frequency dependent. A full discussion on the behaviour of Love waves is given by Aki and Richards (1980).

The technique presented in this paper uses full wave theory and elastic reciprocity. Reciprocity is a way to relate two different states to each other. In this case, the difference between the two states is the free surface being present or not. The final result gives the desired data (without the surface) as a function of the recorded data (with the surface present). No subsurface model of the top layer is needed. This approach is different from that of Ernst et al. (1998), because they make a model of the shallow subsurface, calculate the resulting response, and the difference with the data is then minimized in a least-squares sense.

Theory

In this section, the Betti-Rayleigh reciprocity theorem is given. This will be applied to the SH-wave case, i.e. by using a crossline seismic source and crossline receivers. An integral equation of the second kind is derived. This equation is used to derive an algorithm for removing Love waves from SH-wave data. After some more manipulations simple equations are derived which can be used for horizontally layered media. These equations can be expanded in a Neumann series.

Integral transforms

The equations are required to be causal, linear and time invariant. The causality condition is enforced by using the Laplace transformation (Arfken, 1985), which is defined for causal functions as:

$$\hat{u}(\boldsymbol{x}, s) = \int_0^\infty e^{-st} u(\boldsymbol{x}, t) \mathrm{d}t. \tag{1}$$

Here, $\mathrm{Re}(s) > 0$. The Laplace transform has the following property with regard to differentiation to time: $\partial_t u(\boldsymbol{x}, t) \rightarrow s\hat{u}(\boldsymbol{x}, s)$.

The function $\hat{u}(\boldsymbol{x}, s)$ can be further transformed to the horizontal Fourier domain:

$$\tilde{u}(k_1, x_3, s) = \int_{-\infty}^\infty e^{jk_1x_1} \hat{u}(\boldsymbol{x}, s) \mathrm{d}x_1. \tag{2}$$

The transformation to the horizontal Fourier domain is useful for horizontally layered media.

The Betti-Rayleigh reciprocity theorem

Reciprocity in most general terms provides a means for comparing two different states. In this case, the states are wave fields in an elastic medium. The wave field in an elastic earth is described by the elasto-dynamic equations:

$$\partial_j \hat{\tau}_{i,j} - s\rho\hat{v}_i = -\hat{f}_i, \tag{3}$$

$$\frac{1}{2}\left(\partial_p \hat{v}_q + \partial_q \hat{v}_p\right) - sS_{p,q,i,j}\hat{\tau}_{i,j} = \hat{h}_{p,q}. \tag{4}$$

Note that these equations are in the Laplace domain, and that the Einstein summation convention is used. In these equations, $\hat{\tau}_{i,j}$ is the elastic stress tensor, $\hat{v}_i$ is the particle velocity vector, ρ is the volume density of mass, $S_{p,q,i,j}$ is the compliance tensor (the inverse of the stiffness tensor $C_{i,j,p,q}$), $\hat{f}_i$ is the volume-source density of external forces, and finally, $\hat{h}_{p,q}$ is the volume source density of deformation. A derivation of these equations can be found in de Hoop (1995).

Now consider two different states, call them state A and state B, and the following scalar interaction quantity:

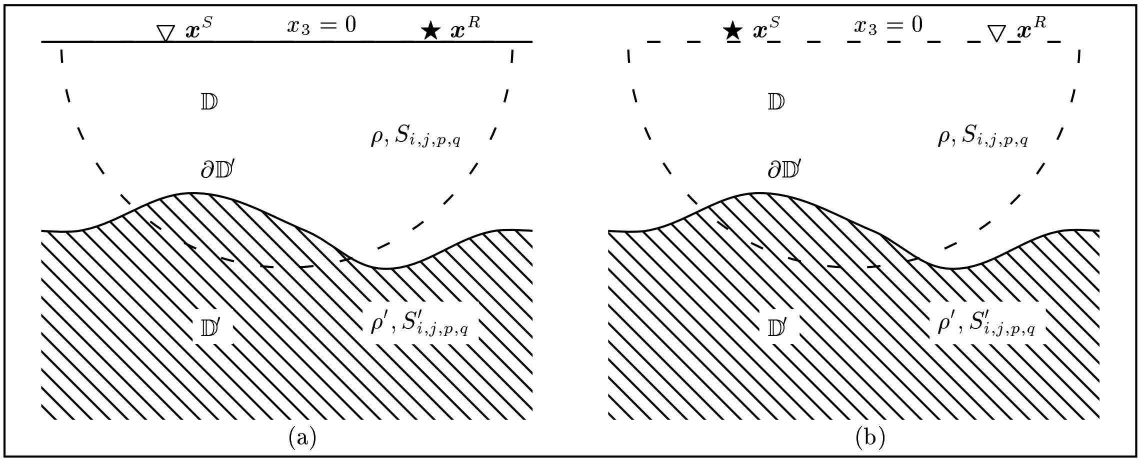

Fig. 1: The two states for the reciprocity theorem. a) with stress-free surface, b) without surface. The dashed line is the path of integration, which goes to infinity.

$\partial_j(\hat{\tau}^A_{k,j}\hat{v}^B_k - \hat{\tau}^B_{k,j}\hat{v}^A_k)$. The elasto-dynamic equations are substituted, the interaction quantity is integrated over a volume, called domain $\mathbb{V}$, and finally Gauss' theorem is applied. The following equation is thus obtained:

$$\int_{\boldsymbol{x}\in\partial\mathbb{V}} \left(\hat{\tau}^A_{k,j}\hat{v}^B_k - \hat{\tau}^B_{k,j}\hat{v}^A_k\right)\nu_j \,\mathrm{d}^2\boldsymbol{x} = \int_{\boldsymbol{x}\in\mathbb{V}} \left[s\left(S^B_{j,k,p,q} - S^A_{p,q,j,k}\right)\hat{\tau}^A_{j,k}\hat{\tau}^B_{p,q} - s\left(\rho^B - \rho^A\right)\hat{v}^A_k\hat{v}^B_k\right]\mathrm{d}^3\boldsymbol{x} + \int_{\boldsymbol{x}\in\mathbb{V}} \left[\hat{f}^B_k\hat{v}^A_k + \hat{h}^B_{j,k}\hat{\tau}^A_{j,k} - \hat{f}^A_k\hat{v}^B_k - \hat{h}^A_{j,k}\hat{\tau}^B_{j,k}\right]\mathrm{d}^3\boldsymbol{x}. \quad (5)$$

This is the global form of the Betti-Rayleigh reciprocity theorem. The media are assumed to be reciprocal, implying the symmetry relation: $S_{j,k,p,q} = S_{p,q,j,k}$. Eq. (5) can be simplified by taking the SH-wave case, where only the v_2 component is of concern. Also assuming no differences in material parameters, no sources of deformation and two-dimensional media, eq. (5) becomes:

$$\int_{\boldsymbol{x}\in\partial\mathbb{V}} \left(\hat{\tau}^A_{2,j}\hat{v}^B_2 - \hat{\tau}^B_{2,j}\hat{v}^A_2\right)\nu_j \,\mathrm{d}\boldsymbol{x} = \int_{\boldsymbol{x}\in\mathbb{V}} \left[\hat{f}^B_2\hat{v}^A_2 - \hat{f}^A_2\hat{v}^B_2\right]\mathrm{d}^2\boldsymbol{x}. \quad (6)$$

The removal equations

The two states that have to be compared in order to arrive at the algorithm to remove Love-waves, are a state with a stress free surface, as is the case in the field, and a state without a surface, where there are no surface effects. Figure 1 shows a graphical representation of these two states. In the surface case, a *volume*-source density of force cannot be defined when the sources are located on the surface. Instead, the source is introduced as a boundary condition on the stress-free surface. There are no such problems in the no-surface case, and a volume-source density of force can be defined normally. The states are summarized in Table 1. In this table, $\boldsymbol{x}^R$ and $\boldsymbol{x}^S$ are located on the surface ($x_3 = 0$).

	State A (surface case)	State B (no-surface case)
Field	$\{\hat{\tau}^{\text{surf}}_{2,j}, \hat{v}^{\text{surf}}_2\}(\boldsymbol{x}\vert\boldsymbol{x}^R, s)$	$\{\hat{\tau}^{\text{nosurf}}_{2,j}, \hat{v}^{\text{nosurf}}_2\}(\boldsymbol{x}\vert\boldsymbol{x}^S, s)$
Material	$\{\rho, S_{i,j,p,q}\}$ in$\mathbb{D}$ $\{\rho', S'_{i,j,p,q}\}$ in$\mathbb{D}'$	$\{\rho, S_{i,j,p,q}\}$ in$\mathbb{D}$ $\{\rho', S'_{i,j,p,q}\}$ in$\mathbb{D}'$
Source	0	$\hat{f}^{\text{nosurf}}_2(s)\times$ $\delta(x_1 - x^S_1)\delta(x_3)$
Boundary	Surface is stress free, except for a traction source: $\hat{\tau}^{\text{surf}}_{2,3} = \hat{t}^{\text{surf}}_2(s)\times$ $\delta(x_1 - x^R_1)$	Not applicable
Domain $\mathbb{V} = \mathbb{D}\cup\mathbb{D}'$ (see Figure 1)		

Table 1: States for the removal of Love-waves

These states can be substituted in eq. (6), and after applying physical reciprocity, the following equation is obtained, where all the vectors are located on the surface ($x_3 = 0$):

$$\int_{x_1\in\mathbb{R}} \hat{\tau}^{\text{nosurf}}_{2,3}(x_1\vert x^S_1, s)\hat{v}^{\text{surf}}_2(x^R_1\vert x_1, s)\mathrm{d}x_1 = \frac{1}{2}\hat{f}^{\text{nosurf}}_2(s)\hat{v}^{\text{surf}}_2(x^R_1\vert x^S_1, s) + \hat{t}^{\text{surf}}_2(s)\hat{v}^{\text{nosurf}}_2(x^R_1\vert x^S_1, s). \quad (7)$$

Note that there's no minus-sign in the equation above.

This is because the positive x_3 direction is down, yielding an extra minus sign. This is a correction of a previously published result (van Zanen et al., 1999). The factor $\frac{1}{2}$ on the right hand side of the equation is a result of integrating over a delta function located exactly on the path of integration: the surface. The integral at infinity in eq. (6) yields zero ($\mathcal{O}(\Delta^{-1})$ as $\Delta \to \infty$), due to causality (Fokkema and van den Berg, 1993).

When $\hat{v}_2^{\rm surf}$ is measured and $\hat{v}_2^{\rm nosurf}$ is to be determined, the only unknown term is the stress-component $\hat{\tau}_{2,3}^{\rm nosurf}$. The term can be rewritten with the help of eq. (4), and by assuming a reciprocal medium: $\hat{\tau}_{2,3}^{\rm nosurf} = (\mu/s)\partial_3\hat{v}_2^{\rm nosurf}$. Here, μ is defined as the shear modulus. When performing a Fourier transform to the horizontal Fourier domain, the differentiation with respect to the x_3 coordinate becomes a multiplication with either $+\gamma_s$ or $-\gamma_s$, depending on a differentiation of an up-going or down-going field respectively. γ_s is defined as $\sqrt{\frac{s^2}{c_s^2}+k_1^2}$, where c_s is defined as the shear-wave velocity of the top layer. Splitting $\hat{v}_2^{\rm nosurf}$ in a reflected and an incoming wavefield, and realizing that the derivative of the incoming field is zero, it is found that: $(\mu/s)\partial_3\hat{v}_2^{\rm nosurf} \to \frac{\mu\gamma_s}{s}(\tilde{v}_2^{\rm nosurf} - \tilde{v}_2^{\rm inc})$. Finally, for the incoming wavefield is written: $\tilde{v}_2^{\rm inc} = s\hat{f}_2^{\rm nosurf}(s)/(2\mu\gamma_s)$.

For the further analysis of the removal equations it is remarked that traction is defined as the opposite of force. This means that for the source functions the following is taken: $\hat{t}_2^{\rm surf}(s) = -\hat{f}_2^{\rm nosurf}(s) = -\hat{f}_2(s)$.

When transforming eq. (7) to the horizontal Fourier domain, taking only horizontally layered media (so-called 1-D media), and following a similar approach as van Borselen (1995), this equation becomes, for the forward problem (i.e. generating a Love wave):

$$\tilde{v}_2^{\rm surf}(k_1, x_3=0, s) = \frac{s\hat{f}_2(s)\tilde{v}_2^{\rm nosurf}(k_1, x_3^S=0, s)}{s\hat{f}_2(s) - \mu\gamma_s\tilde{v}_2^{\rm nosurf}(k_1, x_3^S=0, s)}, \tag{8}$$

and for the inverse problem (i.e. removing the Love wave):

$$\tilde{v}_2^{\rm nosurf}(k_1, x_3=0, s) = \frac{s\hat{f}_2(s)\tilde{v}_2^{\rm surf}(k_1, x_3^S=0, s)}{s\hat{f}_2(s) + \mu\gamma_s\tilde{v}_2^{\rm surf}(k_1, x_3^S=0, s)}. \tag{9}$$

Notice that in this last equation the quantities to be known are: the measured data with surface effects ($\tilde{v}_2^{\rm surf}$), the wavelet ($\hat{f}_2(s)$), and the material parameters of the top layer (via $\mu\gamma_s$). No model is needed for the structure (in the case of horizontally layered media: depth) of the first layer.

The last two equations can be expanded in a Neumann series. For the forward problem, this becomes:

$$\tilde{v}_2^{\rm surf} = \tilde{v}_2^{\rm nosurf}\left[1 + \left(\frac{\mu\gamma_s}{s\hat{f}_2(s)}\tilde{v}_2^{\rm nosurf}\right) + (\cdots)^2 + \cdots\right], \tag{10}$$

while for the inverse problem, this is:

$$\tilde{v}_2^{\rm nosurf} = \tilde{v}_2^{\rm surf}\left[1 - \left(\frac{\mu\gamma_s}{s\hat{f}_2(s)}\tilde{v}_2^{\rm surf}\right) + (\cdots)^2 - \cdots\right]. \tag{11}$$

The terms in the expansion can be seen as multiples, the same as in the marine case. But a difference exists. For the deeper reflections they are the same, namely propagating waves, but for shallow layers, the main contribution of these "multiples" are evanescent waves.

Results

In this section the possibilities of the theory are shown. First, a dataset was made that included a Love wave, using finite difference modeling developed by Falk (1998). This dataset can be seen in Figure 2b). The model for this dataset is as follows: first there is a small layer of 1.2 m with a shear-wave velocity of 200 m/s, then a layer of 22.0 m thick with a shear-wave velocity of 300 m/s, and finally the lower half-space which has a shear-wave velocity of 350 m/s. Figure 2a) shows a graphical representation of this model. The source and receivers are placed on the surface. The amplitudes in the pictures are clipped, in order to provide a better view of the data.

The Love wave is the most obvious event present in Figure 2b). As explained, it has the most energy, and due to its dispersiveness, it obscures the reflection of the deeper layer.

Figure 2c) shows the data after application of eq. (9). For the implementation of this formula a complex Laplace parameter was used: $s = \varepsilon + j\omega$, where ω is the radial frequency, and a value of $\varepsilon = 6.0$ was taken. The Love wave has been removed. The reflection of the deeper layer has become more clearly visible.

The difference between the data with the Love wave removed and theoretical data is shown in Figure 2d). The theoretical data is also obtained with finite difference modeling. The error is minimal, only some artifacts due to the spatial windowing of the input data are introduced.

Conclusions

In this paper a procedure is presented for removing Love waves from SH-wave data. As in the acoustic case, the source wavelet is needed to eliminate the surface effects. But no subsurface model of the first layer is needed, just its physical properties. For a synthetic data set, with a simple model of the subsurface, Love waves can be removed successfully.

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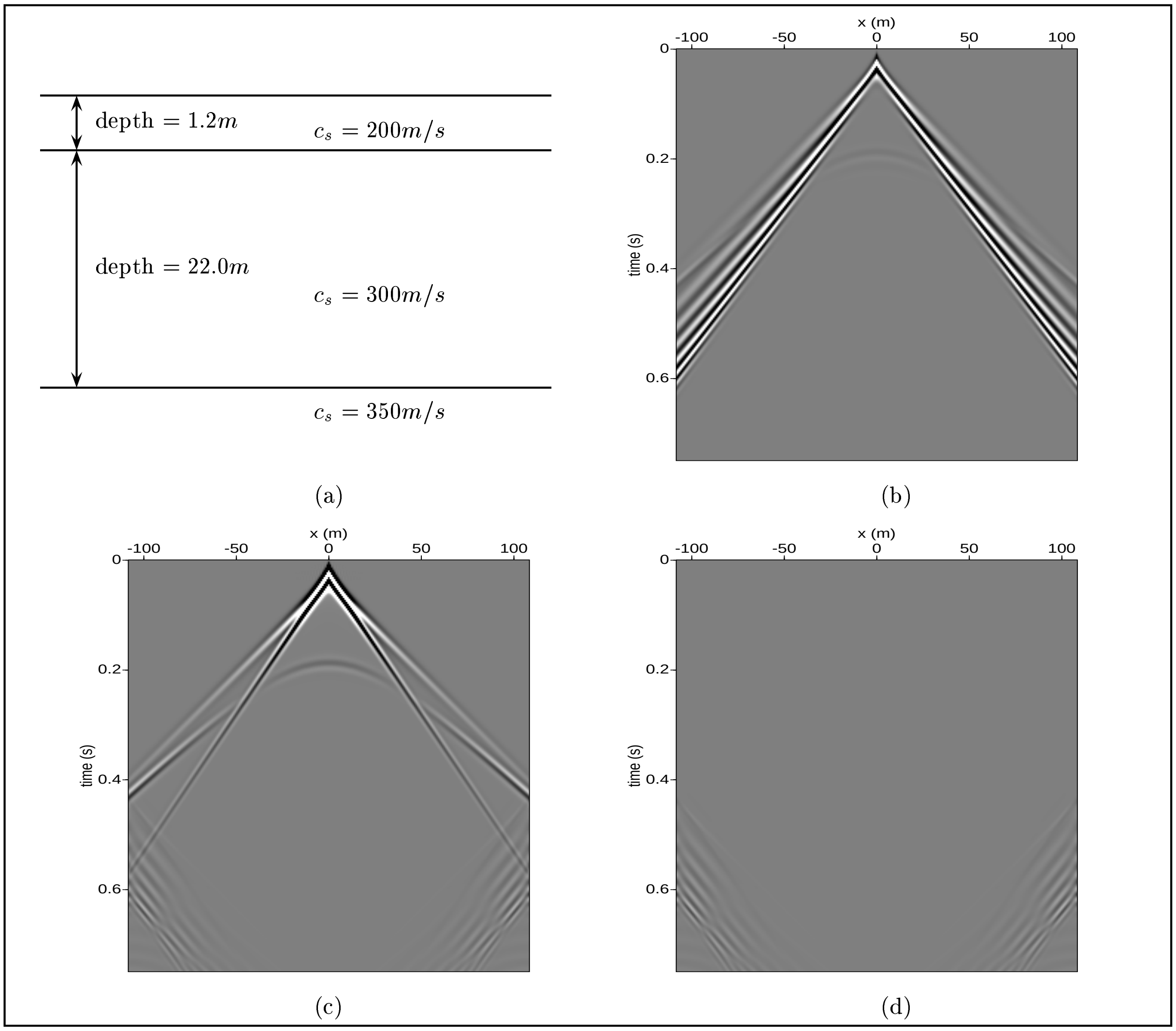

Fig. 2: Inverse problem, the removal of the Love-wave, a) model of the subsurface, b) input data, obtained with finite difference modeling, c) result of the removal procedure, d) difference between c) and theory

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