

From order to disorder to order: a philosophical view on seismic interferometry

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Summary

We discuss the phenomenon of ‘turning noise into signal’ (one of the main properties of seismic interferometry) in the light of changing worldviews, starting with the ordered view of the nineteenth century, via the chaotic world of the twentieth century, to the present view, in which the chaos is tamed.

Introduction

It is by now well-known in ultrasonics (Weaver and Lobkis, 2001), geophysics (Campillo and Paul, 2003; Draganov *et al.*, 2007) and underwater acoustics (Roux *et al.* 2004) that the cross-correlation of diffuse wave fields recorded by two different receivers yields the response at one of the receiver positions as if there were a source at the other. This phenomenon is often named ‘Green’s function retrieval by cross-correlation’, whereas in the seismic literature the term ‘seismic interferometry’ is commonly used (after Schuster, 2004). Although seismic interferometry can be applied to noise as well as controlled source data (see also the 2006 July/August issue of Geophysics), in this paper we only consider its aspect of turning noise into signal. In the present age of chaos theory, one of the most striking properties of seismic interferometry is its robustness. We discuss the phenomenon of ‘turning noise into signal’ in the light of changing worldviews, starting with the ordered view of the nineteenth century, via the chaotic world of the twentieth century, to the present view, in which the chaos is tamed.

Worldview of the nineteenth century

In the 19th century, the world of physics was one of order. Laplace was a key proponent of the view that the physical world is a deterministic clockwork universe. In this model, the future is completely predictable if one knows the forces between all particles as well as their positions and velocities at any one moment in time. Take, for instance, a soccer player who kicks a ball into a forest (Figure 1). The soccer ball bounces repeatedly off the tree trunks, but if you know the original position of the ball, its velocity and the locations of the trees, the future motion of the ball is determined by the player's initial kick.

Twentieth century physics

With the advent of quantum mechanics in the 20th century, the dream of the universe as a deterministic machine was shattered. Heisenberg's uncertainty principle was the end of the worldview of Laplace: in the quantum world, only the

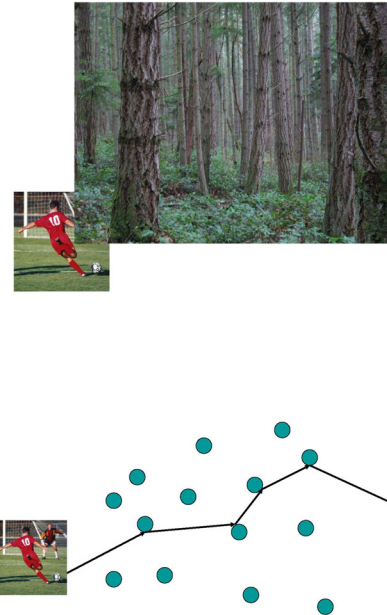


Figure 1. Laplace: the motion of the soccer ball is fully determined by the positions of the trees and the kick of the soccer player.

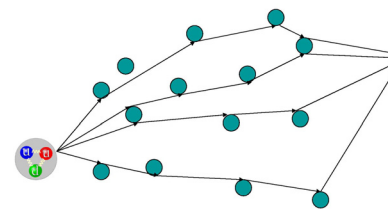


Figure 2. Heisenberg: for an atom-sized soccer ball only the probability of every imaginable trajectory is determined.

probabilities for events are determined by the laws of quantum mechanics. So if an atom-sized soccer ball is kicked into the forest, the trajectory is not determined, but the probability for every imaginable trajectory is (Figure 2). Even for macroscopic systems, determinism did not survive into the 20th century. At that point Poincaré, in a visionary anticipation of chaos theory, showed that even the tiniest uncertainty in the initial conditions can grow exponentially

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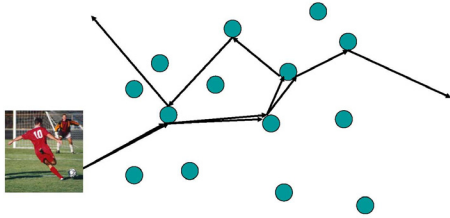


Figure 3. Poincaré: tiny differences in the initial conditions lead to large differences at later times.

with time to make the motion at a later time undetermined, for all practical purposes. So, when the soccer ball is kicked a number of times in slightly different directions, it hits the same trees during the first few bounces at slightly different positions; but the trajectories diverge over time, and after a few bounces the ball may move in completely different ways between the trees (Figure 3).

Particle scattering versus wave scattering

So much for particles: waves behave completely different. If a referee blows his whistle in the same forest repeatedly at slightly different positions, the sound waves scattering between the trees change far less than the motion of the ball (Figure 4). One reason is that waves have an intrinsic length scale, the wavelength, and any perturbations affecting the waves over this are effectively smoothed out. Snieder and Scales (1998) analyzed this mathematically. They showed for a specific configuration with random scatterers that particle scattering becomes chaotic after 8 scatterers whereas wave scattering is still stable after 30 and more scatterers.

Turning noise into signal

Let us now apply the forest analogy to seismic interferometry and see how noise is turned into signal. Imagine it's raining in the forest: every raindrop excites acoustic waves that bounce among the trees in an apparently random fashion. Nevertheless, the trees leave an imprint on the wave field that is characteristic for the forest, just as a fingerprint identifies its owner. The unraveling of this imprint turns out to be surprisingly simple. Let's say there are two microphones in the forest, one of them replacing the whistle-blowing referee (Figure 5). With cross-correlation, we can reconstruct the whistle's sound from the recorded noise of falling raindrops. Take one raindrop that falls in line with the two microphones. The sound wave it generates travels forward, reaching the

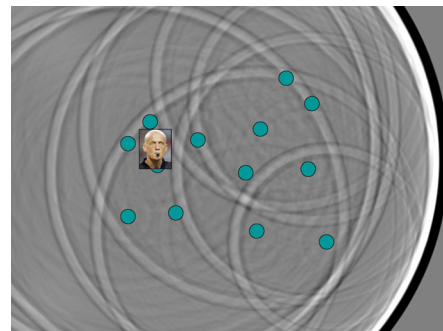
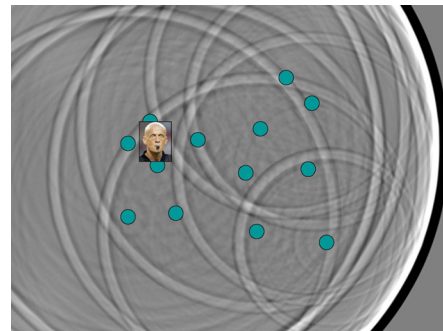


Figure 4. Waves behave much more stable than particles. Small changes in the initial conditions (here: different positions of the referee) are smoothed out at later times.

nearest microphone first and then continuing to the farther one. The difference in the time it takes for the wave to reach each microphone equals the time it takes for the wave to travel between the two microphones. The cross-correlation of the sound waves recorded by the microphones produces a signal at precisely this travel time. So it is as if the first microphone acts as a source, transmitting a weak sound wave to the second. This is enhanced by other raindrops falling in line with the microphones; the rest of the raindrops do not produce a coherent signal. Taking all the coherent waves together, the

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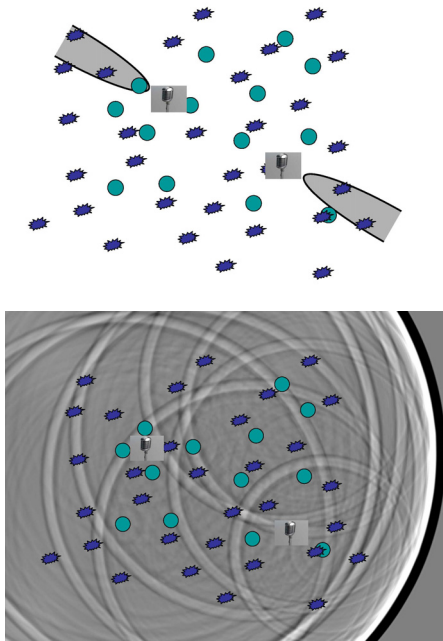


Figure 5. The sound field of the whistle blowing referee in the forest can be reconstructed from the noise of randomly falling raindrops. The raindrops falling in the shaded zones (top frame) contribute to the reconstruction of the direct wave. All scattered waves are reconstructed as well (bottom frame).

first microphone acts as if it is transmitting the sound of the whistle to the second. Mathematics shows that not only the direct wave, but the full wave field (i.e., the full Green's function, including all scattering between the trees), is reconstructed by cross-correlating the wave fields of the raindrops, observed by the two microphones. If this is repeated for multiple microphone positions, the reproduced sound field can be used for imaging – say, to determine the position of nearby trees. This principle creates the opportunity to do wave experiments without using active sources.

Extracting the deterministic response of a system from noise is amazing enough, but there is more. According to theory, noise sources must be distributed homogeneously throughout space, and be uncorrelated – that is, the raindrops must fall everywhere in the forest, and fall at statistically independent times and locations. Astonishingly, in many applications the extraction of the system response from noise has been shown to be fairly robust when noise sources are limited and irregularly distributed, probably because of the stability of wave propagation. Look again at Figure 5 (top frame). Due to the finite wavelength, the raindrops do not need to fall exactly

in line with the microphones to produce a coherent signal; all raindrops falling in the shaded areas contribute to the reconstruction of the direct wave of the whistle blowing referee. A similar type of reasoning holds for the reconstruction of the scattered waves.

Fluctuation-dissipation theorem

It has been known since Einstein's seminal 1905 paper on Brownian motion that the diffusion of a particle is related to the way it slows down after it is disturbed in some way, say by a kick. This principle was later generalized to the fluctuation-dissipation theorem, which states that for systems in thermal equilibrium, the deterministic response of the system is related to thermal fluctuations (Callen and Welton, 1951; Le Bellac *et al.*, 2004, see also Figure 6). The research on Green's function retrieval has shown that this principle can be extended to systems so large that thermal fluctuations are irrelevant, such as the sound waves generated by raindrops falling in the forest.

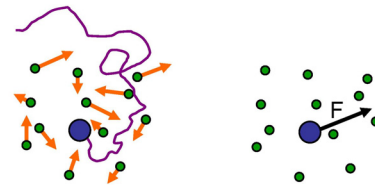


Figure 6. The fluctuations of a pollen particle in a fluid due to the Brownian motion of the fluid molecules are related to the friction forces acting on the same particle, when dragged through the fluid.

Unified Green's function retrieval by cross-correlation

Until recent it was commonly assumed that Green's function retrieval by cross-correlation only works for waves in lossless non-moving media, obeying a wave equation that is invariant for time-reversal. Recent research has shown that the principle holds for a much wider class of systems, where time-reversal invariance does not hold [as for general scalar diffusion phenomena (Snieder, 2006) electromagnetic waves in conducting media (Slob and Wapenaar, 2007), or acoustic waves in viscous media (Snieder, 2007, Extracting the Green's function of attenuating acoustic media from uncorrelated waves: Journal of the Acoustical Society of America, in press)]. With minor modifications it also works in moving fluids, where source-receiver reciprocity breaks down (Wapenaar, 2006; Godin, 2006).

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Snieder *et al.* (2007) investigated Green's function retrieval for systems described by scalar equations with arbitrary order time derivatives (note that – in case of real-valued coefficients – time-reversal invariance only holds for even order time derivatives and that this order is 2 in the acoustic wave equation). The general expression for scalar Green's function retrieval contains contributions from sources in a volume and sources on the boundary enclosing the volume. For any lossless system it appears to be sufficient to have sources on the boundary only; for systems with dissipation, sources are required to be present throughout the volume to overcome the dissipation. With these extensions, Green's function retrieval by cross-correlation becomes possible for

- acoustic waves in viscous media,
- electromagnetic waves in conducting media,
- pure diffusion phenomena (with applications in pore fluid pressure propagation in porous media, or diffusive transport of tracers and contaminants),
- Schroedinger's equation (where the 'zero-offset Green's function' is obtained from the intensity fluctuations of the quantum mechanical wave function),
- bending waves in beams (with applications in monitoring bridges, buildings and other mechanical structures, see Figure 7),

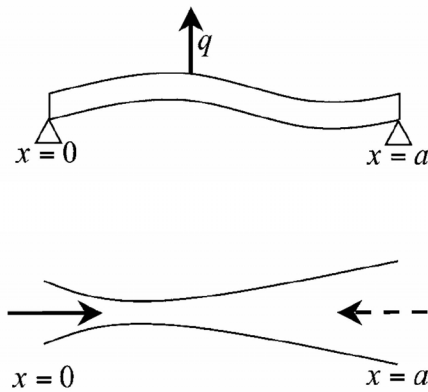


Figure 7: Two of the many new applications of scalar Green's function retrieval by cross-correlation (bending waves and advective transport).

- advective transport (with applications for monitoring the temperature in advective heat transport or the concentration of a nonreactive contaminant, see Figure 7), etc.

Wapenaar *et al.* (2006) investigated general vector equations and showed that Green's function retrieval by cross-correlation also holds for

- elastodynamic waves in viscous media,
- poroelastic waves in porous media,
- electroseismic waves in porous media (Figure 8),
- electrokinetic waves in piezoelectric media, etc.

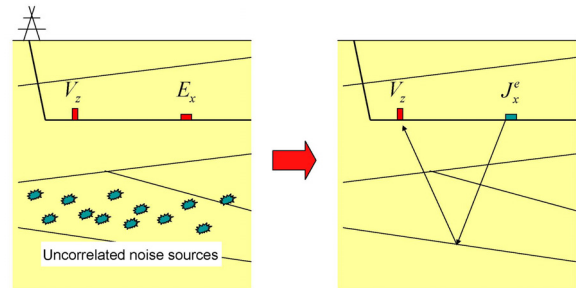


Figure 8: Vectorial Green's function retrieval by cross-correlation: in this example the vertical particle velocity is cross-correlated with the horizontal electric field, yielding the electroseismic response of a horizontal electric current source observed by a vertical geophone.

Conclusions

Our view of the universe may have shifted from the deterministic to the random, but since the turn of the last century physics itself has provided a less simplistic view. Fields generated by random sources can now be used for imaging and for monitoring of systems such as the Earth's subsurface or mechanical structures such as bridges. Randomness is no longer at odds with determinism, but has instead become a new tool providing insights into the deterministic response of the physical world.

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