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#### **Summary**

Many important details of potential subsurface reservoirs that we wish to characterize are only indirectly present in the reflected wavefields measured at the Earth's surface. Therefore, the analysis of seismic data always presents an inversion problem.

Instead of analyzing the data trace by trace, we propose an automated procedure that adjusts the parameters of a *two-dimensional* geological model by minimizing the mismatch between the simulated and measured seismic. This approach differs from standard inversion problems in that the size of the required details of the 2D geological reservoir model is far below the limits of the seismic resolution.

#### Introduction

The possibility of getting detailed information on the size and location of hydrocarbon reservoirs in the subsurface has generated continued interest in seismic research. The details of potential reservoirs that we wish to characterize are only indirectly present in the reflected acoustic wavefields which we are able to measure at the surface. Therefore, the analysis of seismic data always presents an inversion problem. One of the possible inversion methods is the Bayesian approach, which incorporates data-independent *a priori* information in order to exclude unreasonable models.

Several recent studies have focused on a high resolution inversion technique, which deals with very thin reservoirs, with thicknesses at the sub-wavelength scale. Usually in a standard seismic experiment, a source wavelet with a central frequency in the range of 30 to 50 Hz is used, resulting in a vertical resolution in the order of tens of meters. This means that layers with smaller thicknesses are not easily recognizable on a seismic image as a result of wavelet interference. One-dimensional techniques estimate the parameters of a geological model on a trace-by-trace basis. Hence the typical shapes of geological bodies are not taken advantage of in these methods.

The primary focus of this paper is on the development of 2D geological models (in *x-t* cross-sections of the data set), and to incorporate these models in the high resolution Bayesian-based inversion process. We assume that geological objects are not random structures but that

they follow certain typical patterns caused by the depositional processes that formed them. To exploit these characteristics, we model the body as an entity instead of a set of independent traces as has previously been done. The Bayesian-based inversion method employs a nonlinear least-squares estimator for maximizing the *a posteriori* probability. The quasi-Newton method with the Broyden-Fletcher-Goldfarb-Shanno (BFGS) update formula for the Hessian matrix is used as a minimizing device. We chose the clinoform as a typical geological pattern that occurs on a wide range of scales and in a broad spectrum of depositional environments, all of which may be conducive to form potential reservoir rocks. Here we focus on clinoforms at the sub-seismic scale.

#### Theory and method

#### Geological model description

Clinoform is a term originally introduced by Rich (1951) to describe the shape of a depositional surface at the scale of the entire continental margin (Figure 1). In the current geologic literature, the term clinoform denotes stratal packages with oblique internal layering, best imaged on seismic reflection profiles, where three geometric elements are recognized: (1) the topset in the shallowest and flattest area; (2) the foreset in the central and steepest area; and (3) the bottomset in the flat but deepest area (Figure 1b, Mitchum et al., 1977). The break in sea floor slope between the topset and the foreset is often called the *rollover point*. In the study of modern continental margins, clinoforms are widely recognized as one of the fundamental building blocks of the stratigraphic record. Over millions of years, the entire continental margin can be viewed as a clinoform, including the continental shelf as a transfer area, the continental slope as the main area of sediment accumulation, and the base of the slope as a distal bottomset. Changes in clinoform thickness, internal geometry, and style of superposition of multiple clinoforms provide information regarding long-term margin subsidence, sea-level change, and short-term fluctuations of sediment supply. Clinoforms forming over a few thousands of years are observed on the inner shelf of diverse margins: tectonically passive settings, active-margin settings and broad epicontinental shelves. Typically, clinoforms are defined on vertical scales, ranging from several meters to several hundreds of meters and encompass intervals

ranging from hundreds to millions of years. Because the internal layers of the sediments of the clinoform do not differ much from an acoustic point of view, such features do not tend to show up very well on seismic images.

### Simulated field data

From a geometrical point of view clinoforms can be approximated by a number of sigmoidal curves. The sigmoid function  $f_i(x)$  may be described by four parameters.

$$f_i(x) = c_i + \frac{b}{1 + e^{(x - x_i)/a}}$$

Let us consider a single clinoform, bounded by two shifted sigmoid curves of equal slope a and height b. The horizontal and vertical (topset and bottomset thickness) shifts add two more parameters, and an entire clinoform body may thus be described by means of six parameters. In the present study, the thickness of the clinoform was set to be less than 1/7th of the wavelength. The parameters of the clinoform and their description are depicted in the Table 1.

Table 1 The parameters of the clinoform

- more F m-m		
Parameter	Value (m)	
L (length of the model)	100	
a (a lateral scaling)	4	
b (a height scaling	100	
$c_2 - c_1$ (thickness)	14	
$x_1 - x_2$ (lateral translation)	33	

It is obvious that sigmoids cannot be an exact description of geological reality. Therefore, some alterations were made to the sigmoid curves in order for them to be more realistic. We introduced a nonlinear geometric disturbance in the lateral direction by locally stretching or compressing the sigmoid. This geometrical disturbance was applied separately to both the upper and lower curves. After that the clinoform boundaries are mathematically no longer exact sigmoidal curves, but still resemble a sigmoid shape.

We now assume the presence of a nearby well with known rock types and intrinsic rock properties. They are chosen to be representative for a typical reservoir and homogeneously distributed within the layers. These properties are displayed in Table 2; their acoustic impedance contrasts are large enough to cause sufficient reflections.

The seismic forward model used consists of a 1D convolution with primary reflections only. The source wavelet was convolved with the normal-incidence reflectivity time-series in order to simulate a 2D seismic dataset.

Table 2: Properties of the simulated field data

Rock type	P-velocity,	Density,
	m/s	$kg/m^3$
		$\kappa_{\mathcal{S}}/m$
Layer 1. Sandstone (20%	3850	2320
porosity, water saturated)		
Layer 2. Shale	2900	2400
Layer 3. Sandstone (20%	3850	2320
porosity, water saturated)		

The convolution is carried out as a multiplication in the frequency domain. The parameters of acquisition were chosen as follows: time sampling step dt is 1ms, number of samples is 100, trace-to-trace distance dx is 1m and the number of traces is 160.

#### Noise addition

To be more realistic, noise was added to the synthetic clinoform seismic. The noise is uncorrelated and the amplitude is Gaussian-distributed with a zero mean. Tests for different noise levels were done and the one with the most realistic signal-to-noise ratio was chosen. For the given reflection coefficient, a noise standard deviation of 0.005 was selected.

#### Phase shift

The Ricker wavelet is a zero-phase wavelet, and corresponds to the second derivative of a Gaussian function. In our experiment the central frequency is 30Hz. In a real seismic experiment the wavelet is generally not precisely known, which complicates the inversion process. Either the amplitude spectrum or the phase spectrum (or both) can be disturbed. In order to simulate such an effect, we introduced phase distortion. The Ricker wavelet s(t) may be represented in the frequency domain as:

$$S(f) = |S(f)| \cdot \exp(j \cdot (\gamma(f) + \theta(f)))$$
, where  $|S(f)|$  and  $\gamma(f) + \theta(f)$  are the amplitude and the phase spectrum of  $s(t)$  respectively. In our experiment,  $\gamma(f)$  was a constant part and we used  $\theta(f) = a \cdot sign(f)$ , which is a phase distortion with  $a$  expressing the amount of phase distortion. We have applied phase-shifts in a range from 0 to  $2\pi$ . Testing as well as analysis of the seismic practice showed that a phase-shift with  $a = \pi/6$  is quite adequate in representing the real seismic situation. Figure 2 illustrates the simulated field seismic data with noise added

#### The forward model

and phase shift applied.

Although the sigmoid may qualitatively describe the global trend, such a simple analytical function cannot be expected to exactly match the geological body. Since our aim is to arrive at a sub-wavelength resolution in the

description of the clinoform needs to be modeled with a higher accuracy. We therefore propose to replace the sigmoid by a spline. The spline is spanned by a number of equi-lateral spaced control points (knots) along the sigmoid.

For the sake of simplicity we used a uniform spline function with the knots uniformly distributed over the entire interval. Only in areas where the discrepancy between the clinoform and the sigmoid curve exceeds a certain threshold, we displaced the knots upwards and downwards to improve the fit. The number of knots *nn* depends on the ratio between lateral size of the clinoform and amount of the seismic traces available on that area. It is clear that the more knots are used the more unknown variables would be needed in the minimization procedure. On the other hand, with more knots a better approximation of the clinoform will be achieved, and, therefore, an optimum model complexity needs to be strived for.

For the current clinoform model, a set of tests was done in order to find an optimum number of knots needed for the spline. The parameters of acquisition were taken to be the same as for the field data simulation. For the sake of simplicity, knots were uniformly distributed along the profile, and the x-coordinates of the knots for the two curves were assumed to be equal. The parameters (P-velocity, density, geometry) were set at several knot positions on the line only. The next step was to interpolate the parameters' values between the knots in order to obtain them for each seismic trace, not only at the knot positions. For this purpose, a cubic spline interpolation was used.

For the synthetic data the wavelet was chosen to be the same as for the field data, with a central frequency of 30 Hz. The only difference is that this wavelet is initially set to a zero-phase. This simulates the real situation when no knowledge of the phase-shift in the seismic data is available. Of course, no noise was added to the synthetic data.

### Construction of priors

Over the last 30 years, the Bayesian based seismic inversion has been extensively studied to enhance its accuracy and efficiency (Tarantola, 1984, Duijndam, 1988). Tikhonov and Arsenin (1977) developed a regularization method restricting the family of models that fit the data. Two main issues in the Bayesian approach that have been widely investigated are obtaining *a priori* information and parameter uncertainty examination. Gouveia and Scales (1998) showed an approach where *in situ* (borehole) measurements are used to derive an empirical priori for surface seismic data.

The input parameters for the spline function are coordinates of the knots. The x-coordinates for the two curves are assumed to be known since they are manually set by the user. Assuming all other parameters (the P-velocity and the

density) to be known from a nearby well, the parameters to be estimated are the z-coordinates of the knots of the two sigmoidal curves  $z_{jk}$ , j=1,2; k=1,2,...,nn, i.e. the geometry of the clinoform

The a priori knowledge of parameters often consists of an idea about values (mean) and the uncertainties (standard deviation) in these values. A frequently used probability density function to describe this type of information is the Gaussian distribution. A priori information makes the solution stronger only when it is close to the real value. In cases where this is not so, the shift may occur in the wrong direction. Therefore, the means of priors should be close to the true parameters values. A priori mean values corresponding to the sigmoid are  $z_{ik}^{i}$ , j = 1, 2; k = 1, 2, ..., nn, superscript i denotes the priors, the index j the number of the clinoforms, and nn the number of knots. The restrictions to a priori values are an increase of the  $z_{ik}^{\prime}$ value for the fixed j (to resemble the sigmoid shape) and  $z_{1k}^{i} < z_{2k}^{i}$  (to avoid intersections of the curves). The standard deviation  $\sigma(z_1)$  and  $\sigma(z_2)$  for the curve's zcoordinate priors were set to be  $\sigma(z_1) = \sigma(z_2) = 5m$ .

Clearly a priori information as close as possible to the true parameters, with small standard deviation, would help considerably in finding a global minimum that most closely corresponds to the true solution. We propose the following method to obtain  $z_{jk}^i$ : As a forward model a robust sigmoid model, that may be described by means of six parameters, is applied. Applying the exact sigmoid model to the generated field data cannot produce an exact match (even for low noise levels) due to the geometric stretching/compression of the clinoform. From the output of the minimization procedure, z-coordinates of the knots of the two sigmoid curves  $z_{jk}$ , j = 1, 2; k = 1, 2, ..., nn may then be extracted. They are represented on the right hand side of the graph in Figure 3 by a green line. These  $z_{ik}$ values were used as a priori input information  $z_{ik}^{l}$  and applied to the spline model (described above) to the generated field data. Furthermore, we used the output of the minimization procedure to update a priori information for the next iteration for the spline forward model. At all successive steps, the estimated values of the parameters obtained in the previous step were used as the means of the priors. The standard deviations of the distributions for all priors remained fixed throughout the complete analysis.

Test and results

The objective here is to obtain sub-seismic information and to estimate the parameters of the two-dimensional geological model, i.e. the clinoform.

Two tests were applied for a comparison. The first one used *a priori* information regarding the knot position  $z^i_{jk}$ 

obtained from the well or other sources. The second test used the method proposed above where a robust sigmoid model may be applied in order to get a priori  $z_{ik}^i$  values.

The results are depicted in Figure 3 in two columns. The graphs of the upper row plot the initial-ground truth (blue lines), a priori (green lines), and estimated (red lines) of the clinoform. The lower row shows the differences between the estimated and initial field data for both cases. As can be seen, the mismatch between the estimated and the real data decreases when the sigmoid model is applied. This is attributed to the fewer degrees of freedom.

#### **Conclusions**

The proposed automated procedure is capable of modeling a *two-dimensional* geological model by minimizing the mismatch between the simulated and measured seismic.

Incorporating a source wavelet phase disturbance in the inversion process makes this method applicable to real seismic data where the wavelet is never precisely known. By adding geometrical distortion the synthetic seismic becomes more realistic. Better parameter estimations may be obtained by increasing the number of knots. Updating the priors with the proposed method, which employs the sigmoid forward model, yields a better convergence and more accurate parameter estimation.

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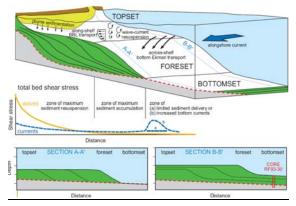


Figure 1. Three-dimensional sketch showing the main physical processes responsible for sediment transport across the shelf, resulting in clinoforms (Cataneo, 2004)

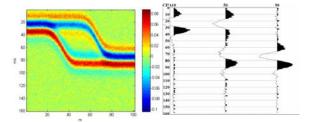


Figure 2. The simulated field seismic data with noise added and phase shift applied (left). The 10<sup>th</sup>, 50<sup>th</sup> and 90<sup>th</sup> seismic traces (right).

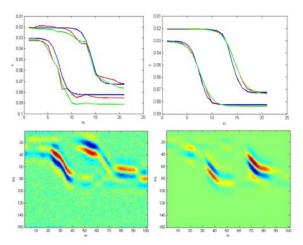


Figure 3. Comparison of two tests. The first one (left column) uses a priori information regarding the knots position  $z_{jk}^i$  obtained from a well or other external sources. The second test (right column) uses the method proposed above with a sigmoid model to define the a priori  $z_{jk}^i$  values. Lines in the upper graphs indicate the initial (blue), the estimated (red) and the a priori (green)  $z_j$  coordinates of the clinoform. The lower graphs show the differences between field and estimated data

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