Jan Thorbecke^{*} and Kees Wapenaar, Delft University of Technology, Department of Geotechnology, Section of Applied Geophysics and Petrophysics, The Netherlands

Summary:

Seismic interferometry can be used to construct the Green's function between two buried receivers in the subsurface. To construct the exact Green function sources should be placed at a surface enclosing the buried receivers. In practice sources could only be placed at the surface, and spurious events are introduced in the constructed Green's function. The origin of these spurious events are analysed and methods to reduce the amplitude of these events are investigated.

Introduction:

Seismic interferometry (SI) is the process of generating new seismic reflection responses by cross-correlating observations of sources at different receiver locations. This process can be used in passive seismics where seismic events of natural sources are correlated to construct reflection data (Draganov et al., 2006). In the case of one-sided illumination (Wapenaar, 2006)SI can be used to construct the Green function between two buried receivers (also called virtual source responses by Korneev and Bakulin (2006)). The basis of all these methods is the time-correlation between two measured traces followed, by a summation of the contribution from different sources (in the case of un-correlated noise sources this summation is implicitly included in the data). In practice the number of sources is always limited and artefacts (called virtual multiples in Schuster et al., 2003) are introduced in the reconstructed signal. Snieder et al. (2006b) used stationary phase analysis to give a physical interpretation of these spurious events. In this paper we will start by briefly mentioning the approximations introduced in SI. An artificial, but interesting, way to suppress the spurious events is shown and the influence of the in-homogeneity of the medium in the constructed Green's function is investigated. This research is part of a larger project in which the influence of source-radiation and -distribution patterns on the constructed signals from SI is investigated.

Approximations in Seismic interferometry

Wapenaar and Fokkema (2006) derived a mathematical expression which is directly suited for seismic interferometry in the field and is given by:

$$\Re\{\hat{G}(\mathbf{x}_A,\mathbf{x}_B,\boldsymbol{\omega})\} \approx \oint_{\partial \mathbb{D}} \frac{2}{c\rho} \hat{G}^*(\mathbf{x}_A,\mathbf{x},\boldsymbol{\omega}) \hat{G}(\mathbf{x}_B,\mathbf{x},\boldsymbol{\omega}) \mathrm{d}^2\mathbf{x}, \quad (1)$$

where \mathbf{x}_A and \mathbf{x} are Cartesian co-ordinate vectors for observation and source points, and $\boldsymbol{\omega}$ denotes angular frequency. In this equation the temporal Fourier transform is defined as $\hat{G}(\mathbf{x}_A, \mathbf{x}, \boldsymbol{\omega}) = \int_{-\infty}^{\infty} \exp(-j\omega t) G(\mathbf{x}_A, \mathbf{x}, t) dt$, where *j* is the imaginary unit and *t* denotes time. The Green's functions un-



Figure 1: For Green's function retrieval between two points within the closed surface (\mathbf{x}_A and \mathbf{x}_B), for example from buried receivers, sources are needed on both $\partial \mathbb{D}_0$ and $\partial \mathbb{D}_1$. In controlled source acquisition, $\partial \mathbb{D}_1$ does not contain sources and the effects of this missing part of the integral is investigated in this paper.

der the integral are responses of monopole sources at **x** on the boundary $\partial \mathbb{D}$, observed by receivers at \mathbf{x}_A and \mathbf{x}_B . Here $c = c(\mathbf{x})$ and $\rho = \rho(\mathbf{x})$ are the propagation velocity and mass density of the inhomogeneous medium. This equation simply states that time-correlation of measured signals, originating from sources surrounding the measurement stations, is used to construct the Green's function between the stations by summation of contributions from different sources.

To arrive at seismic interferometry equation 1 the following assumptions were made (explained in detail in Wapenaar and Fokkema, 2006):

- D is an inhomogeneous lossless (acoustic) medium,
- ∂D is an arbitrary closed surface with outward pointing normal vector n = (n₁, n₂, n₃) and encloses x_A and x_B,
- the medium may be inhomogeneous inside ∂D but homogeneous outside ∂D,
- the Fraunhofer far-field approximation is made,
- the sources are assumed to be impulsive point sources.

When one or more of these assumptions are violated, the effects on the reconstructed Green's function are

- spurious events due to incomplete cancellation of contributions from different stationary points,
- an amplitude error.

The approximations used to derive equation 1 do not affect the phase and equation 1 is therefore still very useful to construct



Figure 2: At depth levels z = 0 and z = 3000 m, every 12 meter a source is placed between 0 and 10000 m. Two buried receivers are placed in the middle of the model. The top, bottom and sides of the model are absorbing the incoming energy.

Green's functions. However, the artificial spurious events are confusing if one wants to interpret constructed events as reflections from earth layers. In the following section two simple examples are used to illustrate the spurious events caused by an incomplete integration surface, and ways to reduce its amplitude.

Spurious events due to incomplete integration surface

Seismic interferometry equation 1 can be used to construct the Green's function between two buried receivers. As shown in Figure 1 the integral is carried out along a closed surface for a complete reconstruction of all events. Wapenaar (2006) showed that the contribution of $\partial \mathbb{D}_1$ can be neglected if the radius *r* becomes very large and the medium inside $\partial \mathbb{D}$ is inhomogeneous. The idea is that due to the inhomogeneities inside $\partial \mathbb{D}$ most of the energy transmitted from sources at $\partial \mathbb{D}_0$ is reflected back (multiple scattering) into $\partial \mathbb{D}$ and measured by the buried receivers.

Figure 2 shows a simple model with only a few layers. Two detectors are placed in the model; one just below the array of sources (\mathbf{x}_A) and another one below the deepest reflector (\mathbf{x}_B). To reconstruct the Green's function between those two detectors an array of sources at z = 0 and an array at z = 3000 m is used. Correlation panels are constructed by correlating the response of each source at \mathbf{A} with the response of each source at \mathbf{B} . The summation over the traces in a correlation panel gives a single trace representing the Green's function between \mathbf{x}_A and \mathbf{x}_B . This trace is shown in Figure 3 for the array of sources at z = 0. In the correlation panel one can clearly observe the stationary points around x = 5000, which contribute in the constructed (red) trace. The green trace show the correct Green's function. Note that before the first arrival spurious events (indicated with an arrow) are present in the constructed trace.



Figure 3: The correlation panel and the reconstructed trace from the source array at z = 0. Spurious events (indicated with an arrow) are present in the reconstructed (red) trace. The green trace represents the correct Green's function.

Figure 4a show again the constructed trace using the array of sources at z = 0. The spurious events in Figure 4a have their largest amplitude close to t = 0 and their amplitude decays strongly for increasing times. Following Snieder et al. (2006b) it is argued that spurious events have an amplitude proportional to the power of 2 of the reflection coefficient r. Figure 4b shows the constructed trace using the array of sources at z = 3000. The contribution from this array compensates the spurious events present in 4a. The summation of the result of 4a and 4b is shown in 4c. In practice one cannot place sources in depth and other ways have to be found to suppress spurious events. By correlation with the down-going wavefield at A, Mehta et al. (2007) showed that spurious events are also suppressed. The results of this approach is shown in Figure 4d. In this case the position of A is in the same layer as the sources and the reconstructed trace with the down-going field is identical to a downward extrapolating result. In the next example we will discuss a more complicated situation.

The spurious events are introduced in the constructed Green's function because the closed integral is only represented by sources at z = 0 m. Following the arguments of Wapenaar (2006) these spurious events should disappear when all downgoing energy is reflected back to the surface. In the simple model we can achieve this by turning on completely reflecting boundaries in the finite-difference modeling code. Although this is not realistic, this simple modeling exercise shows the principle well. In the next example we will consider a more realistic situation. Figure 5 shows the correlation panel and the constructed trace when the bottom boundary of the model is completely reflecting. Note that the exact Green's function



Figure 4: Comparison between reconstructed Green's functions using integration surfaces at z = 0 (a) and z = 3000 (b) m, both surfaces (c) and the first arrival in \mathbf{x}_{A} (d).

(green trace) is also changed. The red trace shows the constructed signal which still contain spurious events, clearly observed before the first arrival. In Figure 6 the left and right boundaries of the model are also made completely reflecting. In this case the reconstructed Green's functions (red trace) matches very well with the exact Green's functions and the spurious events have disappeared. Note that the stationary points which cancel the spurious events are not easy to recognize in Figure 6.

In a strongly inhomogeneous medium, e.g. a medium with many layers, most of the energy will be reflected back to the surface and measured by buried receivers. In a similar way, with the totally reflecting boundaries in the previous example, we expect that the constructed Green's function will be more accurate when more reflecting layers are present. To get an idea how inhomogeneous the medium must be for accurate construction of the Green's function, the model in Figure 7 with a varying number of layers, is used. The model consists of flat thin layers where each layer has a smoothly laterally varying propagation velocity. As in the previous example two buried receivers are placed within the model and sources at depth level z = 0 m.

The constructed Green's functions, with different numbers of layers in the propagation model, are shown in Figure 8 and 9. It is surprising to see that by even by using only 30 (9b) layers the reconstruction is still acceptable. Before the first arrival strong spurious events are observed in all constructed traces. These events originate from 'leakage' to the sides of the model. They will disappear if additional sources at the sides can be used (and close the integration surface) or if the medium is stronger inhomogeneous in the lateral sense. Figure 8b shows in red the



Figure 5: At the bottom of the model a completely reflecting boundary was placed and the correlation panel and trace were constructed. The spurious events (indicated with an arrow) are still present in the constructed (red) trace). The green trace represents the correct Green's function.

reconstructed trace when only the first arrival of the wavefield at **A** is used. The first arrival was obtained by time windowing the total recording at **A**. This reconstructed trace contains some of the main events, but also contains spurious events. Another way to reduce the amplitude of the spurious events is to use deconvolution instead of correlation (Snieder et al., 2006a; Wapenaar et al., 2008).

Conclusions

Seismic interferometry can be used to construct the Green's function between two buried receivers using only sources at the surface. From the theory a closed surface integral, of source positions surrounding the buried receivers, is required. Replacing the closed surface by an open surface (at which the sources are located) introduces artefacts. There are two complementary ways of suppressing these artefacts. In mildly inhomogeneous media it is sufficient to select the first arrival of the wave field at A and correlate it with the field at B. The accuracy of this approach decreases with increasing complexity of the medium. On the other hand, we can correlate the total fields at A and B, relying on the assumption that the downgoing energy is reflected back by the inhomogeneities of the medium. The accuracy of this second approach increases with increasing complexity of the medium.

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Figure 6: At the bottom, left and right sides of the model completely reflecting boundaries are used and a correlation-trace and panel were constructed. The spurious events are not present anymore in the constructed (red) trace. The green trace represents the correct Green's function.



Figure 8: SI constructed traces from buried detectors in the laterally variant model shown in Figure 7 with 200 layers (only the first 30 layers are shown in Figure 7). In the correlations the complete wavefield is used at position **B**. In the left trace (a) the total recording at position **A** is used. The reconstructed trace (red line) shows almost no visible difference with the Green's function (green line). In the right trace (b) a first arrival (obtained by time windowing) at position **A** is used. In this case the reconstructed trace is different from the actual Green's function.



Figure 7: Each layer in the model shown is 20 m thick and has a propagation velocity in the range 1300-3200 m/s. Within each layer the velocity changes laterally with 400 m/s around a central velocity between 1500 and 3000 m/s. To suppress the effects of the edges of the model the fields (p, v_x, v_z) , calculated by the finite difference code, are surrounded by a large homogeneous layers and tapered with 198 points (594 m) on all sides. The tapered area is indicated with slanted lines.



Figure 9: SI constructed traces from buried detectors in the laterally variant model shown in Figure 7. The number of thin layers is different for the two results shown. In the correlations the complete wavefields at position **A** and **B** are used. For the reflecting part there is no significant difference between the Green's function (green line) and the constructed trace (red line) in a model with 200 and 50 layers. For 30 layers differences are becoming visible. Note that the Green's function has also changed.

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