Controlled-source seismic interferometry by multi-dimensional deconvolution: stability aspects with various numbers of sources and receivers

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Summary

In controlled-source Seismic Interferometry (SI) we redatum source locations at the surface to receiver locations in the subsurface without requiring information about the medium between sources and receivers. SI is generally based on cross-correlation (CC), but in particular cases it can also be obtained by multi-dimensional deconvolution (MDD). MDD requires the inversion of a general integral equation, which we implement in a least-squares sense. We apply SI by MDD to a laterally invariant medium and study the stability and accuracy of the inversion for various numbers of sources and receivers. Results are compared with CC-based SI. Both methods prove equally sensitive to reducing the number of sources, deteriorating the illumination of the target area. CC is independent on the number of receivers, but MDD is not. Limiting the amount of receivers reduces the number of sample points at which MDD is evaluated, which simplifies the inversion process and improves the stability. We show that accurate data can be retrieved by MDD with a limited number of receivers, given that decent wavefield decomposition is provided. Only for very short receiver arrays artifacts show up, which may be due to undersampling of the general integral that we aim to invert. For any number of sources and receivers we tested, MDD was able to improve CC-based SI.

Introduction

Various authors have shown that cross-correlation of two receiver registrations can yield a Green's function between these receivers, given that the receivers are surrounded by a closed boundary of sources, being sampled sufficiently densely and uniformly; see Wapenaar et al. (2006) for an overview. We refer to this principle as Seismic Interferometry (SI) by cross-correlation (CC). Applications can be found both in passive seismics, where the sources are typically either noise sources or other uncorrelated events (Draganov et al., 2006) as well as active seismics, where SI can be used to redatum sources from their actual locations at the surface to a receiver level in the subsurface, as is done in the Virtual Source (VS) method (Bakulin & Calvert, 2006) and related techniques; see Schuster & Zhou (2006) for an overview. Recently it has been shown that the procedure of CC can be replaced by multi-dimensional deconvolution (MDD) (Wapenaar et al., 2008, Schuster & Zhou, 2006). Advantages of MDD may include improved radiation characteristics of the retrieved (virtual) sources and a relaxation of some assumptions, including the absence of loss terms and knowledge of the source wavelet. Disadvantages include the higher costs, the need for accurate wavefield decomposition and instabilities that might occur in the matrix inversion that forms the core of MDD. Van der Neut et al. (2008) show applications for multi-component seismic data and compare MDD results with CC-based redatuming. Wapenaar (2008) derive an equivalent method for applying MDD to passive seismic data. The implementation of MDD requires a rigorous matrix inversion which can be difficult to stabilize. In this abstract we focus on the stabilization of the MDD by least-squares inversion. We introduce a misfit function to describe the performance of the inversion in comparison with CC-based redatuming. We study the behavior of this function for various numbers of sources and receivers in a laterally invariant synthetic elastic model.

Theory

Our aim is to redatum source locations from their original positions in an array at the earth surface to a receiver array that is situated in a horizontal borehole at depth, without requiring information about the medium between the sources and receivers. The source array consists of two-component sources, imposing vertical and horizontal forces, respectively. In the receiver array we register particle velocities and traction. We decompose the multi-component registrations of each multi-component shot at the receiver array into their flux-normalized downgoing constituents $\hat{\mathbf{p}}^{+}(\mathbf{x}_R, \mathbf{x}_S, \omega)$ and upgoing constitutents $\hat{\mathbf{p}}^{-}(\mathbf{x}_R, \mathbf{x}_S, \omega)$ (Wapenaar, 1998), where $\mathbf{x}_R$ is the receiver location, $\mathbf{x}_S$ the source location and ω the angular frequency (the circumflex denotes the frequency-domain). These wavefields are related to each other via the reflection response of the medium below the receiver array $\hat{\mathbf{R}}_0^{+}(\mathbf{x}_R, \mathbf{x}'_R, \omega)$ via the following integral equation (Wapenaar & Verschuur, 1996; Amundsen, 1999; Holvik & Amundsen, 2005; Schuster & Zhou, 2006; Wapenaar et al., 2008):

$$\hat{\mathbf{p}}^{-}(\mathbf{x}_R, \mathbf{x}_S, \omega) = \int_{\partial D_R} \hat{\mathbf{R}}_0^{+}(\mathbf{x}_R, \mathbf{x}'_R, \omega) \hat{\mathbf{p}}^{+}(\mathbf{x}'_R, \mathbf{x}_S, \omega) d\mathbf{x}'_R, \quad (1)$$

where the integral takes place over the entire receiver array ∂D_R. The purpose of SI by MDD is to solve this integral equation by least-squares inversion. Therefore we rewrite equation 1 in vector-matrix notation (Berkhout, 1982) as

$$\hat{\mathsf{P}}^{-} = \hat{\mathsf{R}}_0^{+} \hat{\mathsf{P}}^{+}. \quad (2)$$

Here $\hat{\mathsf{P}}^{\pm}$ is a matrix of vectors $\hat{\mathbf{p}}^{\pm}(\mathbf{x}_R, \mathbf{x}_S, \omega)$, where the columns have fixed source type and location but variable receiver type and location and the rows have fixed receiver type and location but variable source type and location. $\hat{\mathsf{R}}_0^{+}$ is a matrix of multi-component reflection matrices $\hat{\mathbf{R}}_0^{+}(\mathbf{x}_R, \mathbf{x}'_R, \omega)$, holding the different wave-mode reflections as its components. Equation 2 can be solved by least-squares inversion as

$$\hat{\mathsf{R}}_0^{+}(\varepsilon) \approx \hat{\mathsf{P}}^{-}\left(\hat{\mathsf{P}}^{+}\right)^{\dagger}\left[\hat{\mathsf{P}}^{+}\left\{\hat{\mathsf{P}}^{+}\right\}^{\dagger} + \varepsilon^2 \mathsf{I}\right]^{-1}, \quad (3)$$

where ε is introduced as a stabilization factor, superscript $\dagger$ denotes the complex-conjugate transpose and I is the identity matrix. The stability and accuracy of the inversion strongly depends on our choice for the stabilization factor. For large ε, the term between the square brackets behaves like a scaled version of the identity matrix and the retrieved reflection response resembles nothing but a scaled version of $\hat{\mathsf{P}}^{-}\left(\hat{\mathsf{P}}^{+}\right)^{\dagger}$. This can be rewritten in integral notation as

$$\hat{\mathbf{R}}_0^{+}(\mathbf{x}_R, \mathbf{x}'_R, \omega) \approx \int_{\partial D_S} \hat{\mathbf{p}}^{-}(\mathbf{x}_R, \mathbf{x}_S, \omega)\left\{\hat{\mathbf{p}}^{+}(\mathbf{x}'_R, \mathbf{x}_S, \omega)\right\}^{\dagger} d\mathbf{x}_S . \quad (4)$$

We will refer to the retrieved reflection response by equation 4 as the CC-based result, which can be interpreted as a multi-component equivalent of the Virtual Source (VS) method applied to decomposed wavefields, as proposed by Mehta et al. (2007). Note however, that the best practice of the VS method requires time-gating of the downgoing wave field before applying equation 4, which we do not consider in this abstract. For smaller ε the response of MDD will be different from CC. To characterize the performance of MDD as a function of ε we introduce a relative misfit function. We compute the normalized retrieved amplitude spectrum of a virtual PP shot record by MDD in the frequency domain as a function of ε and refer to it as $\hat{A}_{MDD}(\varepsilon)$. The equivalent spectrum retrieved by CC is referred to as $\hat{A}_{CC}$. The misfit function $E(\varepsilon)$ is now defined as

$$E(\varepsilon) = \frac{\sum_f \sum_x \left|\hat{A}_{MDD}(\varepsilon) - \hat{A}_{GT}\right|}{\sum_f \sum_x \left|\hat{A}_{CC} - \hat{A}_{GT}\right|}, \quad (5)$$

where $\hat{A}_{GT}$ is the normalized amplitude spectrum of the PP Ground Truth (GT) response, that is computed by directly modeling the response that we want to retrieve. Summations take place over frequency and x, representing the receiver location. Note that for $\varepsilon \to \infty$ MDD converges to CC and the relative misfit converges to 1. Misfit E can thus be interpreted as a quantification of the relative improvements ($E < 1$) or deteriorations ($E > 1$) of MDD compared to CC.

Results

In the following example we evaluate the results of MDD applied to a laterally invariant elastic model for different numbers of sources and receivers. A source array of 1600 meter consists of N_S vertical as well as horizontal force sources, situated at the surface. The upper 200 m of the subsurface consist of finely layered material. An array of 400 m consisting of N_R multi-component receivers is situated in a homogeneous layer at 250 m depth. Below the array we find four strong reflectors that we want to reveal. The data are decomposed into flux-normalized up- and downgoing P- and S-wave fields (Wapenaar, 1998). We apply MDD with several stabilization factors and compute the misfit function through equation 5 for a default scenario where $N_S = 321$ and $N_R = 81$. The misfit function for this case is shown as a solid line with a logarithmic ε-axis in Figure 1. For large ε MDD converges to CC, resulting in $E \to 1$. Around $\varepsilon = 1$ we notice a drop in the misfit, representing the improvements brought by the inversion in comparison with CC. Lowering ε even further is counterproductive as we quickly face instabilities, resulting in relatively high values of E. The optimal $\varepsilon = 1.23$ can be found as the minimum in the misfit-curve. In Figure 2 we demonstrate the effect of ε on MDD in the FK-domain. Figure 2A represents the GT response that we want to retrieve. Figure 2B shows an attempt with $\varepsilon = 1000$, being analogous to CC. In Figure 2C we show the result with the optimal $\varepsilon = 1.23$, showing a better convergence to the GT than CC. If we choose ε too small, we face instabilities causing local amplifications in the FK-domain – see Figure 2D for an illustration. In Figure 3 we illustrate that the optimal ε-value leads to a good match with the GT response in the time domain, slightly improving CC (Figure 4).

Both MDD and CC responses are sensitive to the number of sources in the array. In CC-based methods the source spacing determines the sampling of the integral over the source locations, as it appears in equation 4. In MDD the number of sources determines the number of evaluations of integral equation 1 that are used in the inversion. Each source can thus be interpreted as an extra equation in the MDD, providing additional information that stimulates the convergence of the inversion process. The default case of 321 sources and 81 receivers can be considered as an over-determined least-squares problem. Next we consider the even determined problem by reducing the number of sources to 81 and a strongly underdetermined problem with only 21 sources – see the dashed lines in Figure 1. The curve for 81 sources shows little difference to the default curve with 321 sources, but for the case with 21 sources it can be seen that the misfit gets slightly worse but still does not exceed the level $E=1$ representing the CC-based response. Since the misfit is normalized with the misfit of the CC method (see equation 5), which suffers from increasing the source spacing, the general quality of MDD deteriorates when reducing the number of sources in a similar way as CC, as we demonstrate in Figure 5. In conclusion MDD and CC seem about equally sensitive to reducing the number of sources.

Next we limit the number of receivers in the array, while keeping 321 sources. Since CC can be interpreted as a trace-to-trace correlation process, it is independent on receiver spacing. In MDD, the number of receivers depicts the number of sample points of the major integral as given in equation 1, but also the number of unknowns that we aim to solve for. In Figure 6 we show that limiting the number of receivers simplifies the choice of the stabilization factor as the system becomes more overdetermined. Instead of a sharp minimum we find a range of ε-values where the inversion yields desirable results, making it easier to stabilize MDD. We can thus show that with 21 receivers we can retrieve a MDD response being as good as, or even better, than what we obtained with 81 receivers – see Figure 7. Removing even more receivers deteriorates the response slightly as we see both in Figures 6 and 8. This may be due to undersampling of the general integral equation 1 that we aim to solve.

Conclusions

Seismic Interferometry (SI) by multi-dimensional deconvolution (MDD) can be an alternative for various cross-correlation (CC) based redatuming methods that recently appeared in applied geophysics. We showed that MDD can indeed, in some cases, improve CC by testing both techniques on an elastic laterally invariant model. We quantified these improvements by a relative misfit function, providing insight in both the accuracy and stability of the least-squares inversion compared to CC-based methodology. The number of sources determines the illumination at the receiver array which is crucial for convergence to the desired reflection response. MDD and CC seem equally sensitive for this effect. The CC method, requiring only a trace-to-trace cross-correlation process, is independent on the amount of receivers, but MDD is not. The presence of more receivers results in more sample points in the general integral equation. However, it also results in more unknowns that need to be solved for, while the number of evaluations (sources) remains the same, making the least-squares problem less determined. Therefore, including extra receivers can make MDD more difficult to stabilize and does not automatically imply better results. Only very short receiver arrays produce additional artifacts that are possibly due to undersampling of the general integral equation that MDD aims to solve. For all the tested number of sources and receivers, MDD was able to produce better results than CC, given that an optimal stabilization factor could be chosen. Finally it should be noted that reducing the number of receivers generally deteriorates the quality of elastic decomposition, which is crucial for MDD. This effect has not been accounted for in this study.

Acknowledgements

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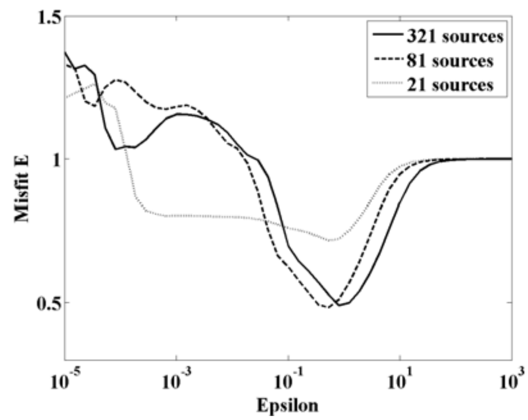

Figure 1: Relative misfit curves for source arrays of 321, 81 and 21 sources, respectively.

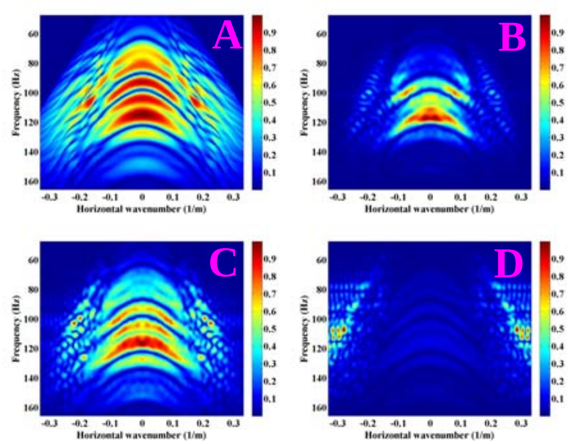

Figure 2: FK-representations of the Ground Truth (panel A) and retrieved data by MDD for $\varepsilon=1000$ (panel B; equivalent to CC), the optimum fit $\varepsilon=1.23$ (panel C) and an instable case $\varepsilon=0.0285$ (panel D).

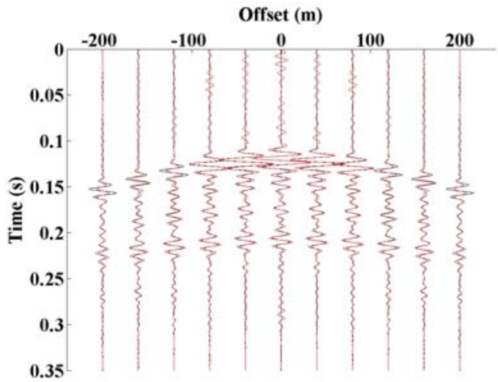

Figure 3: Results of MDD-based interferometry (red) versus the Ground Truth response (black) for a case with 321 sources and 81 receivers (only 11 receivers are shown).

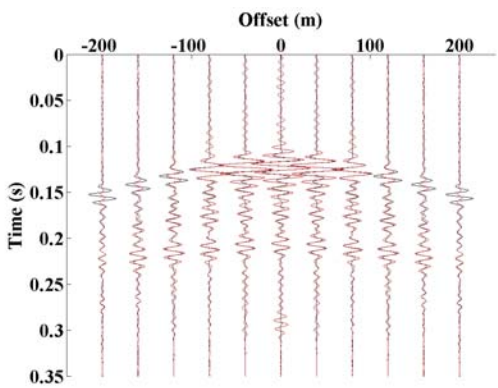

Figure 4: Results of CC-based interferometry (red) versus the Ground Truth response (black) for a case with 321 sources (only 11 receivers are shown).

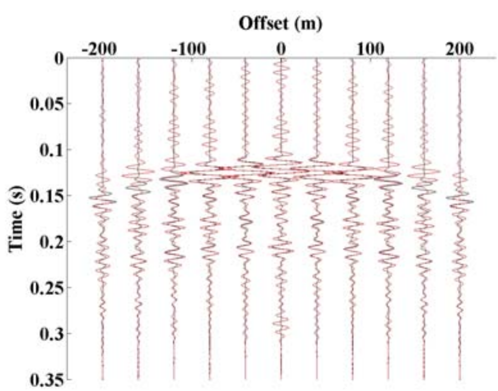

Figure 5: Results of MDD-based interferometry (red) versus the Ground Truth response (black) for a case with 21 sources and 81 receivers (only 11 receivers are shown).

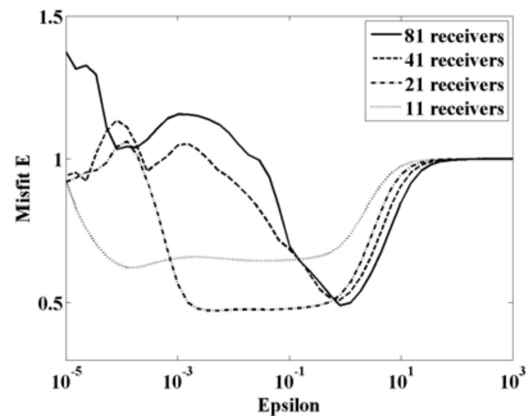

Figure 6: Misfit curves for arrays of 81, 41, 21 and 11 receivers, respectively.

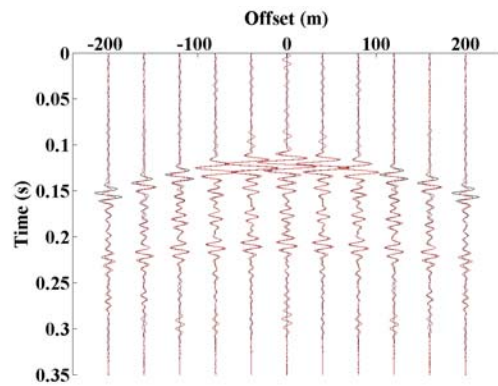

Figure 7: Results of MDD-based interferometry (red) versus the Ground Truth response (black) for a case with 321 sources and 21 receivers (only 11 receivers are shown).

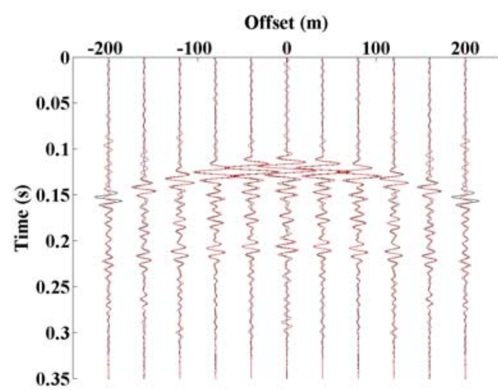

Figure 8: Results of MDD-based interferometry (red) versus the Ground Truth response (black) for a case with 321 sources and 11 receivers.