Generalized PP + PS = SS from seismic interferometry

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Summary

Using a novel form of seismic interferometry as a basis, we derive new, dynamically correct expressions for reflected and converted P- and S-wave responses to both P- and S-wave sources. We use these expressions to derive a generalized form of relationship between PP, PS, and SS waves, often referred to as PP + PS = SS. By relating this method to seismic interferometry, it is possible to see further applications of the new relationships in acquisition and processing of P- and S-waves, and also in the development of new imaging and inversion schemes related to interferometry.

Introduction

Generally, seismic interferometry refers to the process of generating responses to imagined or virtual impulsive sources by crosscorrelation (Wapenaar, 2003; van Manen et al., 2006; Wapenaar and Fokkema, 2006), crossconvolution (e.g., Slob et al., 2007), or deconvolution (e.g., Wapenaar et al., 2008) of wavefields from surrounding energy sources recorded at two different receiver locations. Recent work has shown that inter-source wavefields can be estimated by crosscorrelating recordings of a pair of sources at a range of azimuths (Curtis et al., 2009). Further, Curtis and Halliday (2010) showed that it is possible to use interferometry to estimate the wavefield between a source and a receiver, allowing interferometry to be used in any combination of source and receiver geometries. Halliday and Curtis (2010) also showed that the source-receiver relationships allow establishment of a direct link between seismic interferometry and seismic imaging. They show explicitly that these theorems are generalized forms of existing imaging methods, e.g., the methods of Oristaglio (1989) and Vasconcelos et al. (2009).

In this paper, we extend the applicability of these new source-receiver relationships by using the results of Curtis and Halliday (2010) to find interferometric relationships that describe the recovery of P- and S-wave responses between sources and receivers. The shear-wave component of the wavefield is important in determining the shear-wave velocities in any medium, for example, in the Earth's subsurface where shear-wave information allows fluid and rock properties to be discriminated, and also in studying anisotropic media. Typically, converted (PS) waves (P-waves down to a reflector at which the wave reflects and converts to S energy that propagates back to the surface) are used to infer shear-wave velocity structure. However, this is undesirable from several points of view. For example, Grechka and Tsvankin (2002) discuss the difficulty of velocity analysis for converted PS waves. Problems arise from the asymmetric moveout of the reflected and converted PS waves. Ideally, pure PP waves and pure SS waves would be analyzed independently. However, while sources of P-wave energy are available as standard industrial equipment, it is far more difficult to inject significant S-wave energy into the ground economically.

Grechka and Tsvankin (2002) and Grechka and Dewangan (2003) proposed a potential solution to this problem: by combining PP and PS responses, pseudoshear-wave data can be generated that has the same kinematics as pure SS waves. Presumably, as the interest in full waveform imaging and inversion grows, the recovery of shear-wave velocity profiles, and the study of anisotropic media will come under greater scrutiny. Therefore, it is important to consider approaches such as that presented by Grechka and Tsvankin (2002) and Grechka and Dewangan (2003).

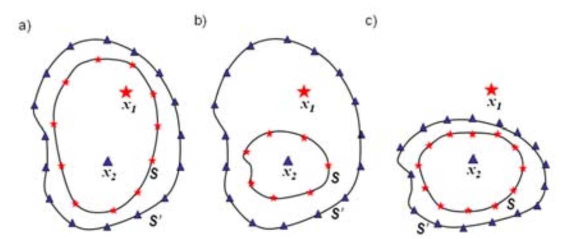

Figure 1: Canonical geometries for source-receiver interferometry for (a) the correlation-correlation, (b) the correlation-convolution, and (c) the convolution-convolution forms (Curtis and Halliday, 2010). Note the different positions of $\mathbf{x}_1$ and $\mathbf{x}_2$ relative to the boundaries.

The relationships between P- and S-wave energy sources and recordings developed in this paper may allow better understanding of such approaches, and also the development of new methods of P- and S-wave acquisition, processing, imaging, and inversion. We derive the PP + PS = SS equation of Grechka and Dewangan (2003) from source-receiver interferometry. While the previous derivation was in part heuristic and was purely kinematic, here, we show that this can also be derived from first principles in a fully dynamically consistent form. The source-receiver representations that we consider are derived directly from reciprocity and representation theorems (Curtis and Halliday, 2010). This approach reveals the key approximations and assumptions inherent in the approach of Grechka and Tsvankin (2002) and Grechka and

Dewangan (2003), and provides a theoretical framework to develop future processing, imaging, and inversion algorithms.

Integrals for P- and S-waves

Following Curtis and Halliday (2010, Appendix A), by using two correlation-type representation theorems, the response between a real source and a receiver can be expressed as,

$$G^*_{im}(\mathbf{x}_2,\mathbf{x}_1) - G_{im}(\mathbf{x}_2,\mathbf{x}_1) = \int_S \{ G_{in}(\mathbf{x}_2,\mathbf{x}) n_j c_{njkl} \partial_k \Phi_{ml}(\mathbf{x}_1,\mathbf{x}) - n_j c_{njkl} \partial_k G_{il}(\mathbf{x}_2,\mathbf{x}) \Phi_{mn}(\mathbf{x}_1,\mathbf{x}) \}\, dS\,, \quad (1)$$

where

$$\Phi_{ml}(\mathbf{x}_1,\mathbf{x}) = -\int_{S'} \{ G^*_{n'l}(\mathbf{x}',\mathbf{x}) n_{j'} c_{n'j'k'l'} \partial_{k'} G_{l'm}(\mathbf{x}',\mathbf{x}_1) - n_{j'} c_{n'j'k'l'} \partial_{k'} G^*_{l'l}(\mathbf{x}',\mathbf{x}) G_{n'm}(\mathbf{x}',\mathbf{x}_1) \}\, dS'\,. \quad (2)$$

Here, $G_{im}(\mathbf{x}_2,\mathbf{x}_1)$ is the Green's function representing the ith component of particle displacement at $\mathbf{x}_2$ due to a unidirectional point force in the m-direction at $\mathbf{x}_1$, n_j is the jth component of the normal vector on the boundary S, ∂_k denotes a spatial derivative in the k-direction, and c_{njkl} is the stiffness tensor. Primed and unprimed quantities indicate that these relate to the primed and unprimed boundaries, respectively (Figure 1a), and Einstein's summation principle for repeated indices applies throughout.

Equation 1 describes the recovery of a Green's function (and its time reverse due to the complex conjugate on the left side) between a source at $\mathbf{x}_1$ and a receiver at $\mathbf{x}_2$ in elastic media, using only Green's functions from $\mathbf{x}_1$ to a surrounding boundary S' of receivers, and Green's functions from a surrounding boundary S (Figure 1a). The integral in equation 2 describes a first step where the boundary S' is used to determine the Green's functions (and their time reverse) between the source at $\mathbf{x}_1$ and each source on the boundary S; hence, this first step turns the source $\mathbf{x}_1$ into a virtual receiver. In a second step, the boundary S is used to determine the Green's function between the receiver at $\mathbf{x}_2$ and the newly generated virtual receiver $\mathbf{x}_1$. Thus, this interferometric integral uses both surrounding sources and receivers to reconstruct source-receiver wavefields. This specific form of the integral is derived by combining two representation theorems of the correlation type. Figures 1b and 1c show other configurations that can be derived using both correlation- and convolution-type representation theorems, and two convolution-type representation theorems, respectively (Curtis and Halliday, 2010).

To extend equation 1 to describe the recovery of P and S responses, we recall from Wapenaar and Fokkema (2006), that the P- and S-wave components of the wavefield can be expressed as a sum of partial derivatives of the displacement,

$$G_{\psi_0 m}(\mathbf{x}_2,\mathbf{x}_1) = \frac{\rho c_P^2}{\omega^2} \partial_i G_{im}(\mathbf{x}_2,\mathbf{x}_1)\,, \quad (3)$$

$$G_{\psi_k m}(\mathbf{x}_2,\mathbf{x}_1) = -\frac{\rho c_S^2}{\omega^2} \varepsilon_{kij} \partial_j G_{im}(\mathbf{x}_2,\mathbf{x}_1)\,, \quad (4)$$

where c_S is the local S-wave velocity at $\mathbf{x}_2$, c_P is the local P-wave velocity at $\mathbf{x}_2$, ρ is the density, ω is the angular frequency, $G_{\psi_k m}(\mathbf{x}_2,\mathbf{x}_1)$ is the Green's function representing the S-wave polarized in the plane with normal n_k, due to a point force in the m-direction at $\mathbf{x}_1$, and $G_{\psi_0 m}(\mathbf{x}_2,\mathbf{x}_1)$ is the equivalent Green's function for a P-wave at $\mathbf{x}_2$. ε_{ijk} is the alternating tensor with $\varepsilon_{123} = \varepsilon_{312} = \varepsilon_{231} = -\varepsilon_{213} = -\varepsilon_{321} = -\varepsilon_{132} = 1$. When we interpret equations 3 and 4 as P- and S-wave Green's functions, we assume that the medium is homogeneous and isotropic locally around the receiver point, $\mathbf{x}_2$. In the following, we will use one Green's function, $G_{\psi_K m}(\mathbf{x}_2,\mathbf{x}_1)$, with K equal to 0, 1, 2, or 3. $K = 0$ denotes P-waves (cf. equation 3) and K = 1, 2, or 3 denotes a shear wave polarized in the plane with normal n_K (cf. equation 4), assuming appropriate P or S velocities are used.

Equations 3 and 4 are weighted sums of the spatial derivatives of point-force responses (and likewise, by reciprocity, we can find similar expressions for the particle displacement due to P- and S-wave sources – see Wapenaar and Fokkema (2006)). Hence, we can use appropriately weighted sums of partial derivatives of equation 1 to represent P- and S-wave source and receiver Green's functions:

$$G^*_{\psi_I \psi_M}(\mathbf{x}_2,\mathbf{x}_1) - G_{\psi_I \psi_M}(\mathbf{x}_2,\mathbf{x}_1) = \int_S \{ G_{\psi_I n}(\mathbf{x}_2,\mathbf{x}) n_j c_{njkl} \partial_k \Psi_{\psi_M l}(\mathbf{x}_1,\mathbf{x}) - n_j c_{njkl} \partial_k G_{\psi_I l}(\mathbf{x}_2,\mathbf{x}) \Psi_{\psi_M n}(\mathbf{x}_1,\mathbf{x}) \}\, dS\,, \quad (5)$$

with

$$\Psi_{\psi_M l}(\mathbf{x}_1,\mathbf{x}) = -\int_{S'} \{ G^*_{n'l}(\mathbf{x}',\mathbf{x}) n_{j'} c_{n'j'k'l'} \partial_{k'} G_{l'\psi_M}(\mathbf{x}',\mathbf{x}_1) - n_{j'} c_{n'j'k'l'} \partial_{k'} G^*_{l'l}(\mathbf{x}',\mathbf{x}) G_{n'\psi_M}(\mathbf{x}',\mathbf{x}_1) \}\, dS'\,. \quad (6)$$

Here, $G_{\psi_I \psi_M}(\mathbf{x}_2,\mathbf{x}_1)$ is the Green's function representing the P- or S-wave component of the wavefield at $\mathbf{x}_2$ due to a P- or S-wave source at $\mathbf{x}_1$. $G_{n'\psi_M}(\mathbf{x}',\mathbf{x}_1)$ is the Green's function representing the n´th component of particle displacement at **x**´ due to a P- or S-wave source at $\mathbf{x}_1$. On the source component, the uppercase subscript M runs from 0 to 3, with 0 denoting a P-wave source, and 1 to 3 denoting a shear-wave source polarized in the plane with normal n_M.

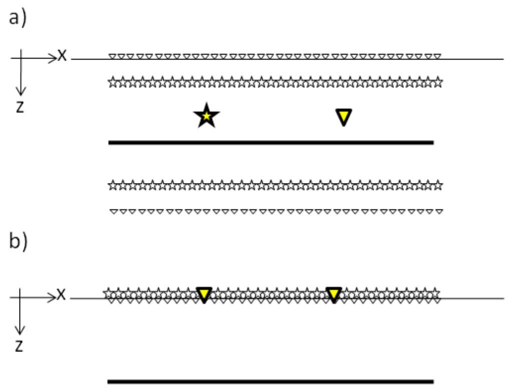

Figure 2: (a) Sketch geometry for an ideal application of equations 8 or 9 to recover PP, PS, or SS responses. (b) Sketch geometry illustrating the configuration used by Grechka and Dewangan (2003). Thin lines indicate the free surface, white triangles and stars indicate boundary receivers and sources respectively, and yellow triangles and stars indicate receivers and sources between which the wavefield is estimated. For illustration purposes, the thick black line indicates a reflector.

PP + PS = SS

We now show that equation 5 is the exact form of the kinematic equation used by Grechka and Dewangan (2003, equation 5) to recover SS responses from conventional seismic data. We follow Wapenaar and Fokkema (2006) in changing the source and receiver quantities on *S* and *S*´ to be P- and S-wave sources or receivers, respectively. First, we consider the integral in equation 6: from Wapenaar and Fokkema (2006, equations 72 and 73) we can write,

$$\Psi_{\psi_M l}(\mathbf{x}_1,\mathbf{x}) = \frac{2}{\omega^2}\int_{S'} \partial_{j'} G^*_{\psi'_K l}(\mathbf{x}',\mathbf{x}) G_{\psi'_K \psi_M}(\mathbf{x}',\mathbf{x}_1) dS' \cdot \tag{7}$$

Because we use P- and S-wave quantities on the boundary, we are assuming that the medium at and outside the boundary S' is homogeneous and isotropic. Applying the same principles to the integral over *S* such that it consists of only recordings of P- and S-wave sources, we obtain:

$$\begin{aligned} &G^*_{\psi_I\psi_M}(\mathbf{x}_2,\mathbf{x}_1) - G_{\psi_I\psi_M}(\mathbf{x}_2,\mathbf{x}_1) = \\ &\quad \frac{4}{\omega^4}\int_S\int_{S'} \partial_j G_{\psi_I\psi_K}(\mathbf{x}_2,\mathbf{x}) \times \\ &\qquad \partial_{j'} G^*_{\psi'_K\psi_K}(\mathbf{x}',\mathbf{x}) G_{\psi'_K\psi_M}(\mathbf{x}',\mathbf{x}_1) n_{j'} n_j dS' dS \cdot \end{aligned} \tag{8}$$

Again, we assume that the medium at and outside the boundary S is isotropic and homogeneous. Equation 8 is a generalized form of the PP + PS = SS equation used by Grechka and Dewangan (2003). This describes the recovery of any combination of P- and S-wave response. We can split the right side into integrals dependent on PP ($\psi_K = \psi'_K = 0$), PS ($\psi_K = 0$, $\psi'_K = \psi'_k$), SP ($\psi_K = \psi_k$, $\psi'_K = 0$), and SS ($\psi_K = \psi_k$, $\psi'_K = \psi'_k$), and further, if we wish to recover the SS response ($\psi_I = \psi_i$, $\psi_M = \psi_m$), we can write:

$$\begin{aligned} &G^*_{\psi_i\psi_m}(\mathbf{x}_2,\mathbf{x}_1) - G_{\psi_i\psi_m}(\mathbf{x}_2,\mathbf{x}_1) \\ &- \frac{4}{\omega^4}\int_S\int_{S'} \partial_j G_{\psi_i\psi_0}(\mathbf{x}_2,\mathbf{x}) \partial_{j'} G^*_{\psi'_k\psi_0}(\mathbf{x}',\mathbf{x}) G_{\psi'_k\psi_m}(\mathbf{x}',\mathbf{x}_1) n_{j'} n_j dS' dS \\ &- \frac{4}{\omega^4}\int_S\int_{S'} \partial_j G_{\psi_i\psi_k}(\mathbf{x}_2,\mathbf{x}) \partial_{j'} G^*_{\psi_0\psi_k}(\mathbf{x}',\mathbf{x}) G_{\psi_0\psi_m}(\mathbf{x}',\mathbf{x}_1) n_{j'} n_j dS' dS \\ &- \frac{4}{\omega^4}\int_S\int_{S'} \partial_j G_{\psi_i\psi_k}(\mathbf{x}_2,\mathbf{x}) \partial_{j'} G^*_{\psi'_k\psi_k}(\mathbf{x}',\mathbf{x}) G_{\psi'_k\psi_m}(\mathbf{x}',\mathbf{x}_1) n_{j'} n_j dS' dS \\ &= \frac{4}{\omega^4}\int_S\int_{S'} \partial_j G_{\psi_i\psi_0}(\mathbf{x}_2,\mathbf{x}) \partial_{j'} G^*_{\psi_0\psi_0}(\mathbf{x}',\mathbf{x}) G_{\psi_0\psi_m}(\mathbf{x}',\mathbf{x}_1) n_{j'} n_j dS' dS \ . \end{aligned} \tag{9}$$

We have moved all terms dependent on SS responses to the left-hand side. Grechka and Dewangan (2003) consider only PP and PS responses, and from equation 9, we see this is equivalent to assuming that the surface integrals on the left-hand side of equation 9 are equal to zero. This is equivalent to assuming that P-wave quantities dominate on both boundaries. Further, if both boundaries S' and S are spheres with very large radius such that energy to (from) location $\mathbf{x}_1$ ($\mathbf{x}_2$) leaves (arrives) at the boundary approximately perpendicularly, then the spatial derivatives in equation 9 can be approximated by, $\partial_j = -j\,\omega / c_K$, where, $c_K = c_P$ for K = 0, and $c_K = c_S$ for K = 1, 2, or 3. Equation 9 can then be written as:

$$\begin{aligned} &G^*_{\psi_i\psi_m}(\mathbf{x}_2,\mathbf{x}_1) - G_{\psi_i\psi_m}(\mathbf{x}_2,\mathbf{x}_1) \approx \\ &\frac{-4}{c_P c_{P'} \omega^2}\int_S\int_{S'} G_{\psi_i\psi_0}(\mathbf{x}_2,\mathbf{x}) G^*_{\psi_0\psi_0}(\mathbf{x}',\mathbf{x}) G_{\psi_0\psi_m}(\mathbf{x}',\mathbf{x}_1) dS' dS \cdot \end{aligned} \tag{10}$$

Finally, Grechka and Dewangan (2003) also assume that one of the boundaries has collocated source and receivers, and the locations $\mathbf{x}_2$ and $\mathbf{x}_1$ can be receiver locations. While this introduces zero-offset Green's functions, we can avoid complications by assuming that we are only interested in the reflected and/or scattered part of the wavefield.

Equations 8 to 10 require that both surface integrals are closed. This is never the case in exploration seismology. However, if source and receiver lines are long enough, we can treat these as pseudo-infinite boundaries (i.e., we assume that the integrand approaches zero as we approach the ends of the lines). Unfortunately, we then also require that lines of sources and receivers are located in the subsurface, such that our pseudo-infinite lines enclose the medium of interest on both sides. This ideal scenario is illustrated in Figure 2a, which shows two receiver lines (white triangles) forming the boundary S', and two source lines (white stars) forming the boundary S, with a separate source (yellow star) and receiver (yellow triangle) forming a geometry similar to that in Figure 1a. The thick black line indicates a single plane reflector, but the medium in our equations may be arbitrarily complex. Now consider the configuration used in equation 10. Grechka and Dewangan (2003) consider surface seismic data; therefore, our configuration in Figure 2a changes to that shown in Figure 2b. We do not have source and receiver boundaries in the subsurface, and all sources and receivers are located on the same datum. Therefore, we no longer have a geometry corresponding to that in Figure 1a. Despite this, Grechka and Dewangan (2003) show that pseudo-shear-wave data can be recovered using such geometry, but due to the approximations mentioned above, the correct amplitudes are not recovered.

Discussion

We have derived a generalized form of the PP + PS = SS equation used by Grechka and Dewangan (2003) that correctly describes the recovery of PP, PS, SP, and SS wavefields between two points. This new approach identifies that, for an exact recovery of the SS response, we require:

- A closed acquisition surface (with sources and receivers on independent surfaces)
- All SS responses between $\mathbf{x}_1$, *S*, and *S*', and between $\mathbf{x}_2$, *S*, and *S*'. Hence, some SS responses must be known to recover the exact SS response between $\mathbf{x}_1$ and $\mathbf{x}_2$ using this method.

Thus, we have identified the two key requirements for the exact application of the method proposed by Grechka and Tsvankin (2002) and Grechka and Dewangan (2003). In practice, it is unlikely that these requirements will be satisfied. Despite this, Gechka and Dewangan (2003) show that the method can still be used to give useful results. This is similar to many applications of seismic interferometry, where approximations and assumptions are made to allow the method to be applied to real seismic data (for approximations and assumptions, see Wapenaar and Fokkema, (2006); for an example application of interferometry to real data, see Bakulin and Calvert, (2006)).

Using the generalized relationship in equation 8, we can also derive other relationships between P- and S-wave responses. For example, by following the same steps used to reach equation 9, but with $I = 0$, $M = 0$ we can write:

$$
\begin{aligned}
&\frac{-4}{\omega^4}\int_S\int_{S'}\partial_j G_{\psi_0\psi_k}(\mathbf{x}_2,\mathbf{x})\partial_{j'}G^*_{\psi'_k\psi_k}(\mathbf{x}',\mathbf{x})G_{\psi'_k\psi_0}(\mathbf{x}',\mathbf{x}_1)n_{j'}n_j\,dS'dS \\
&= G^*_{\psi_0\psi_0}(\mathbf{x}_2,\mathbf{x}_1) - G_{\psi_0\psi_0}(\mathbf{x}_2,\mathbf{x}_1) \\
&+\frac{4}{\omega^4}\int_S\int_{S'}\partial_j G_{\psi_0\psi_0}(\mathbf{x}_2,\mathbf{x})\partial_{j'}G^*_{\psi_0\psi_0}(\mathbf{x}',\mathbf{x})G_{\psi_0\psi_0}(\mathbf{x}',\mathbf{x}_1)n_{j'}n_j\,dS'dS \\
&+\frac{4}{\omega^4}\int_S\int_{S'}\partial_j G_{\psi_0\psi_k}(\mathbf{x}_2,\mathbf{x})\partial_{j'}G^*_{\psi_0\psi_k}(\mathbf{x}',\mathbf{x})G_{\psi_0\psi_0}(\mathbf{x}',\mathbf{x}_1)n_{j'}n_j\,dS'dS \\
&+\frac{4}{\omega^4}\int_S\int_{S'}\partial_j G_{\psi_0\psi_0}(\mathbf{x}_2,\mathbf{x})\partial_{j'}G^*_{\psi'_k\psi_0}(\mathbf{x}',\mathbf{x})G_{\psi'_k\psi_0}(\mathbf{x}',\mathbf{x}_1)n_{j'}n_j\,dS'dS\,.
\end{aligned}
\tag{11}
$$

As in equation 9, all terms dependent on SS have been moved to the left-hand side. Note, with I = 0, and M = 0, only one surface integral is dependent on SS. Equation 11 could, therefore, be used to formulate an inverse problem to find $G_{\psi_{k'}\psi_k}(\mathbf{x}',\mathbf{x})$ given all combinations of PP and PS. Such an approach may be related to the multidimensional deconvolution approach to interferometry (e.g., Wapenaar et al., 2008).

As well as reformulating PP + PS = SS, it is likely that further applications of equation 8 could include novel approaches to P- and S-wave acquisition and processing. For example Curtis and Halliday (2010) propose various applications of the point-force version of these interferometric representations, including the combination of active and passive recordings, balancing directionally biased active or passive wavefields, and replacing missing or dead traces in land seismic surveys. Hence, it is likely that similar applications may be found using the P- and S-wave version of the integral. Applications may also be found in imaging and inversion. Halliday and Curtis (2010) have shown that the crosscorrelation imaging condition of Oristaglio (1989) is a special case of the source-receiver interferometric representation of scattered waves. Thus, expressions such as equation 9 may find applications in imaging and inversion of P- and S-wave data.

Conclusions

We have derived generalized source-receiver interferometric integrals for P- and S-waves. We have shown that the fully dynamic PP + PS = SS method is a special case of these integrals. This general form includes all components of any source and receiver type, and allows derivation of new relationships between P- and S-wave responses. We have shown that the new relationships may be used to formulate an inverse problem to estimate SS responses from PP and PS data. The new relationships may find further applications in acquisition and processing, and in imaging and inversion, of P- and S-wave data.

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