Interferometric redatuming with simultaneous and missing sources using sparsity promotion in the curvelet domain

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SUMMARY

Interferometric redatuming is a velocity-independent method to turn downhole receivers into virtual sources. Accurate redatuming involves solving an inverse problem, which can be highly ill-posed, especially in the presence of noise, incomplete data and limited aperture. We address these issues by combining interferometric redatuming with transform-domain sparsity promotion, leading to a formulation that deals with data imperfections. We show that sparsity promotion improves the retrieval of virtual shot records under a salt flank. To reduce acquisition costs, it can be beneficial to reduce the number of sources or shoot them simultaneously. It is shown that sparse inversion can still provide a stable solution in such cases.

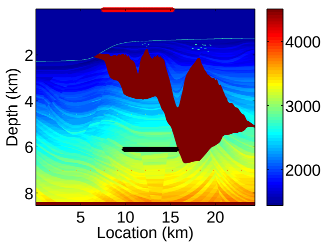

Figure 1: The Sigsbee model with an array of 128 sources at the surface in red and an array of 128 receivers in a horizontal well in black.

INTRODUCTION

With interferometric redatuming, physical receivers can be turned into virtual sources without information about the propagation velocity (Schuster, 2009). One of the many applications of this concept is to redatum seismic sources from the Earth's surface to receiver locations in a horizontal well, which is the key idea behind the virtual source method (Bakulin and Calvert, 2006). Although the theory for interferometry assumes illumination from all directions, interferometric redatuming is generally applied with one-sided illumination from the surface and limited aperture, which can result in the retrieval of spurious events and blurring effects (Snieder et al., 2006; Wapenaar, 2006). To overcome these problems, it is better to solve interferometric redatuming by inversion (multidimensional deconvolution). Wapenaar et al. (2011) do so least-squares inversion. In many cases, the interferometric redatuming problem is poorly conditioned and heavy regularization is required to stabilize the inversion (Minato et al., 2011). If the problem is underdetermined, for instance in case of source undersampling or simultaneous source acquisition, the situation can be especially detrimental. Solving interferometric redatuming with curvelet-domain sparsity promotion (referred to as sparse inversion) can circumvent some of these problems. Sparsity promotion has proven useful in a range of geophysical applications, such as denoising (Hennenfent and Herrmann, 2006), recovery of missing data (Herrmann et al., 2008b), surface-related multiple elimination (van Groenestijn and Verschuur, 2009; Lin and Herrmann, 2011), removing crosstalk of simultaneous source data (Herrmann et al., 2008a; Neelamani et al., 2010; van Groenestijn and Verschuur, 2011) and (least-squares) imaging (Herrmann and Li, 2012; Tu et al., 2012).

THE FORWARD MODEL

In the following representation, sources and receivers are located at the Earth's surface and in a horizontal well, respectively. An example of such configuration is shown in Figure 1. In this case, the receiver array is located below a salt body, distorting the transmitted wavefields significantly. Our aim is to redatum the sources to the receiver array without information about the salt body and to use the obtained signals for local (sub-salt) imaging. By combining multi-component recordings, the downgoing and upgoing constituents of the wavefield at depth can be separated. This is done with a sparsity-promoting decomposition algorithm, developed by van der Neut and Herrmann (2012). Knowing the (flux-normalized) decomposed fields, the following forward model for interferometric redatuming can be derived (Wapenaar et al., 2008):

$$\mathbf{u} = \mathbf{D}\mathbf{g_0}. \tag{1}$$

Here, $\mathbf{u}$ is the flux-normalized upgoing wavefield at the horizontal receiver array in vectorized form in the time-space donmain. $\mathbf{D}$ is a matrix that involves forward Fourier transformation, multidimensional convolution with the downgoing field and inverse Fourier transformation. The vector $\mathbf{g_0}$ is the unknown Green's function as if there were virtual sources in the well, also vectorized in the time-space domain. To reduce acquisition costs, it can be beneficial to sub-sample the source array by removing random shot locations or to shoot sources simultaneously with random time delays as in blended acquistion (Berkhout, 2008). For these scenarios, the upgoing wavefield $\mathbf{u}$ and matrix $\mathbf{D}$ can be written as $\mathbf{u} = \mathbf{RMu_0}$ and $\mathbf{D} = \mathbf{RMD_0}$, where $\mathbf{u_0}$ and $\mathbf{D_0}$ are the upgoing wavefield and 'operator-matrix' for full recording geometry, $\mathbf{M}$ is a mixing matrix and $\mathbf{R}$ a restriction matrix. For missing sources, $\mathbf{R}$ is an identity matrix with a number of rows deleted (corresponding to the missing sources) and $\mathbf{M}$ is an identity matrix. For simultaneous sources, $\mathbf{M}$ introduces the source mixing and phase encoding (i.e. the individual source emission times) (Wason et al., 2011) and $\mathbf{R}$ is an identity matrix .

CROSSCORRELATION

Interferometric redatuming by crosscorrelation can be implemented by applying the adjoint (indicated by superscript *) of $\mathbf{D}$ to $\mathbf{u}$, yielding the correlation function $\mathbf{c}$:

$$\mathbf{c} = \mathbf{D}^*\mathbf{u}. \tag{2}$$

Note that equation 2 involves crosscorrelation and summation over sources, as typically applied in interferometry by crosscorrelation (Schuster, 2009) and the virtual source method (Bakulin and Calvert, 2006). From equations 1 and 2, it follows that $\mathbf{c} = \mathbf{D}^*\mathbf{D}\mathbf{g_0}$, being the normal equation. In simple media with perfect acquisition conditions, $\mathbf{D}^*\mathbf{D}$ (referred to as the point-spread function) will be close to a bandlimited identity matrix and, consequently, $\mathbf{c}$ can be interpreted as a bandlimited representation of $\mathbf{g_0}$ (Wapenaar et al., 2011). In complex settings with shadow zones, or when sources are missing or shooting simultaneously, $\mathbf{D}^*\mathbf{D}$ can be defocused or contain crosstalk. In such cases, equation 1 should be inverted to optimize the estimate of $\mathbf{g_0}$.

LEAST-SQUARES INVERSION

One approach is to solve equation 1 by regularized least-squares inversion (Wapenaar et al., 2008). In this case the following minimization scheme is implemented:

$$\underset{\mathbf{g_0}}{\text{minimize}}\ \frac{1}{2}\,||\mathbf{u} - \mathbf{D}\mathbf{g_0}||_2^2 + \lambda^2\,||\mathbf{g_0}||_2^2\,. \tag{3}$$

Here, subscript 2 denotes the ℓ_2-norm and λ is a regularization parameter to constrain the solution length $||\mathbf{g_0}||_2^2$. Regularization also smoothens the solution, which can result in the loss of high frequency information.

SPARSE INVERSION

In many cases the forward problem of equation 1 is poorly conditioned (Minato et al., 2011) and it can be beneficial to impose additional constraints on the sparsity of the solution. It has been shown that seismic data is generally sparse in the curvelet domain (Herrmann et al., 2008b). For this reason, we define the transform $\mathbf{S} = \mathbf{C_2} \otimes \mathbf{W}$, where $\mathbf{C_2}$ is the two-dimensional curvelet transform along the virtual source and receiver coordinates, $\mathbf{W}$ is the discrete wavelet transform and $\otimes$ denotes a Kronecker product. The desired Green's function $\mathbf{g_0}$ can be written in terms of its transform-domain coefficients $\mathbf{x_0}$, according to

$$\mathbf{g_0} = \mathbf{S}^*\mathbf{x_0}. \tag{4}$$

We assume that the representation of the Green's function in terms of $\mathbf{x_0}$ is sparse, which is exploited by solving the following (convex) optimization problem (Herrmann et al., 2008b):

$$\underset{\mathbf{x_0}}{\text{minimize}}\ ||\mathbf{x_0}||_1 \text{ subject to } ||\mathbf{u} - \mathbf{D}\mathbf{S}^*\mathbf{x_0}||_2 \leq \sigma. \tag{5}$$

Here, σ is a user-defined noise level quantifying the tolerated mismatch in the residual of the forward problem and subscript 1 denotes the ℓ_1-norm. Once the coefficients $\mathbf{x_0}$ are found, the desired Green's function $\mathbf{g_0}$ can be constructed by evaluation of equation 4. An additional debiasing step can be applied for amplitude balancing, by minimizing the ℓ_2-norm residual of equation 1 using only the non-zero entries of $\mathbf{g_0}$ found by sparse inversion (Figueiredo et al., 2008).

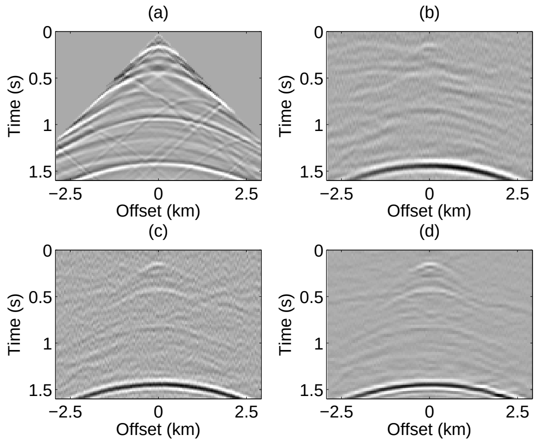

Figure 2: Virtual shot records retrieved from all sources by a) direct modeling, b) crosscorrelation, c) least-squares inversion and d) sparse inversion.

EXAMPLES

In the following example, 128 source locations are redatumed from the Earth's surface to a receiver array in a horizontal well below a salt body (Figure 1). Prior to redatuming, Gaussian noise is added to the data with $SNR = 5$ (Signal-to-Noise Ratio). The upgoing and downgoing fields are separated with a sparsity promoting inversion scheme (van der Neut and Herrmann, 2012). In Figure 2a a shot record from the desired Green's function is shown, obtained by direct modeling. Early arrival times have been muted to remove strong reflections from heterogeneties at the receiver level. In Figure 2b we show the redatumed shot record obtained by crosscorrelation, using all 128 sequential sources. We observe the strong reference reflector at $t \approx 1.5s$ and the weaker reflections earlier in the section. The response also contains many artefacts. Least-squares inversion can improve the results considerably (Figure 2c), but the retrieved response is rather noisy, despite severe regularization. The results of sparse inversion is accurate and relatively free of noise, see Figure 2d.

Next we make 32 supershots, each containing of 4 randomly selected sources encoded with random time shifts between $0s$ and $1.6s$. One supershot is shown in Figure 3a for illustration, where it is clear that various events are interfering. Applying interferometric redatuming by crosscorrelation results in severe crosstalk, see Figure 3b. Although least-squares inversion can remove some crosstalk, the response is noticably noisy, see Figure 3c. Sparse inversion allows us to recover the Green's function with a relatively low noise floor, as shown in Figure 3d.

In another test, we use seqential sources but a random selection of 50% of the shots are removed. An example of a common receiver gather of the input data is shown in Figure 4a. The undersampling causes additional noise when crosscorrelation is used for redatuming (Figure 4b). Once again a noisy result is obtained with least-squares inversion (Figure 4c), whereas the result of sparse inversion is relatively clean (Figure 4d).

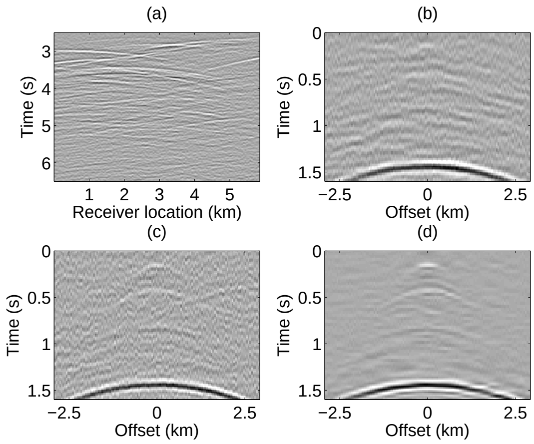

Figure 3: a) Example of the pressure recordings of one supershot. Virtual shot records retrieved from simultaneous sources by b) crosscorrelation, c) least-squares inversion and d) sparse inversion.

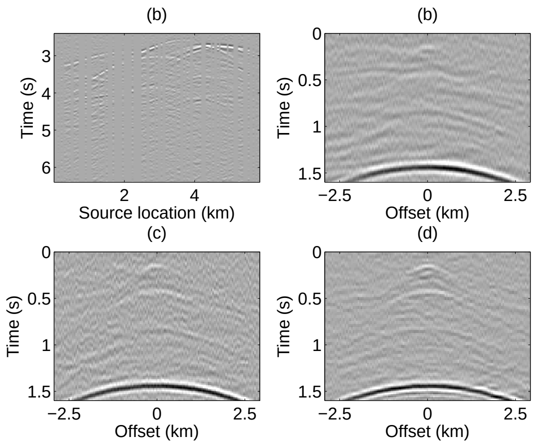

Figure 4: a) Example of a common receiver gather with 50% of the sources missing. Virtual shot records retrieved with missing sources by b) crosscorrelation, c) least-squares inversion and d) sparse inversion.

INTERFEROMETRIC IMAGING

Interferometric redatuming is often followed by local imaging in a target area below the well. This strategy requires a background velocity model of the medium below the well but no information of the medium above the well. For accurate imaging, the following forward model can be derived under the Born approximation (i.e. internal scattering is not taken into account) (Plessix and Mulder, 2004):

$$\mathbf{g_0} \approx \mathbf{K}\delta\mathbf{m_0}. \tag{6}$$

Here the vector $\delta\mathbf{m}_0$ describes the perturbations of the background medium in vectorized form and $\mathbf{K}$ is the Born scattering operator describing the propagation from the virtual sources to the perturbations and from there to the receivers. Reverse-Time-Migration (RTM) can be implemented by applying the adjoint of $\mathbf{K}$ to the retrieved Green's functions. In this way, an image $\delta\mathbf{m}$ of the medium perturbations can be obtained:

$$\delta\mathbf{m} = \mathbf{K}^*\mathbf{g_0}. \tag{7}$$

In Figure 5a we show the true perturbations $\delta\mathbf{m}_0$ in the target area below the receivers that we aim to image. In Figure 5b, we show the RTM image obtained from the Green's functions as retrieved by crosscorrelation. Note the artefact that is indicated by the white arrow (which might be caused by a reflection from the salt flank). Redatuming by least-squares inversion and RTM can remove this artefact and improve the resolution drastically but the result is slightly noisy, see Figure 5c. Using Green's functions obtained by sparse inversion produces a relatively clean image, as shown in Figure 5d. Similar conclusions hold for the case with simultaneous sources (Figure 6) and missing sources (Figure 7), although these images tend to appear slightly more noisy.

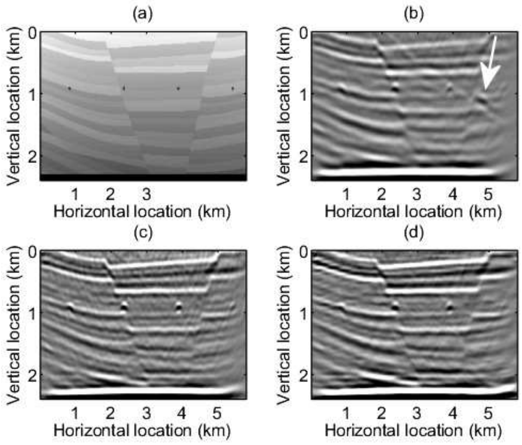

Figure 5: a) True perturbations in the target area and RTM images from Green's functions retrieved by b) crosscorrelation, c) least-squares inversion and d) sparse inversion of sequential source data.

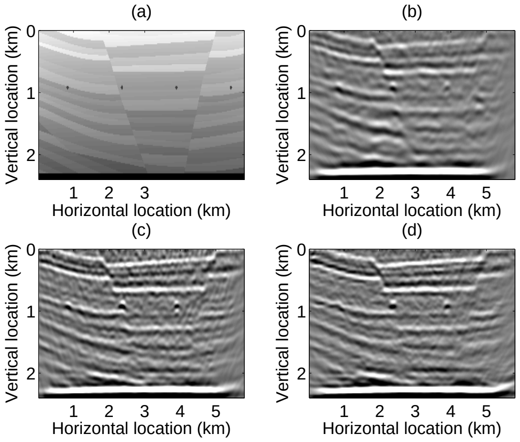

Figure 6: a) True perturbations in the target area and RTM images from Green's functions retrieved by b) crosscorrelation, c) least-squares inversion and d) sparse inversion of simultaneous source data.

DISCUSSION

By substituting equation 6 into equation 7, it follows that $\delta\mathbf{m} = \mathbf{K}^*\mathbf{K}\delta\mathbf{m_0}$. The image $\delta\mathbf{m}$ can thus be interpreted as a representation of the true medium perturbations $\delta\mathbf{m_0}$ blurred with the so-called image resolution function $\mathbf{K}^*\mathbf{K}$ (Schuster and Hu, 2000). With least-squares migration (Nemeth et al., 1999; Plessix and Mulder, 2004) or migration deconvolution (Yu et al., 2006) we aim to remove the imprint $\mathbf{K}^*\mathbf{K}$ from the image. Herrmann and Li (2012) shows how this can effectively be done with sparsity promotion in the image domain. Instead of solving interferometric redatuming and imaging as sequential steps, they can also be combined. This can be shown by substituting equation 1 into equation 7, yielding the following forward problem:

$$\mathbf{u} \approx \mathbf{D}\mathbf{K}\delta\mathbf{m_0}. \tag{8}$$

The upgoing field $\mathbf{u}$ is now described in terms of data-driven operator $\mathbf{D}$, describing the propagation above the receivers and model-driven operator $\mathbf{K}$, describing the propagation below the receivers. Although $\mathbf{K}$ is derived under the Born approximation, operator $\mathbf{D}$ correctly accounts for multiple scattering in the overburden (Wapenaar et al., 2008). By defining an appropriate transform domain $\mathbf{S}$ (for instance the curvelet domain) and expressing the medium perturbations in terms of their transform-domain coefficients $\delta\mathbf{x}_0 = \mathbf{S}^*\delta\mathbf{m_0}$, the following optimization problem can be introduced:

$$\underset{\delta\mathbf{x}_0}{\text{minimize}}\, ||\delta\mathbf{x}_0||_1 \text{ subject to } ||\mathbf{u} - \mathbf{D}\mathbf{K}\mathbf{S}^*\delta\mathbf{x_0}||_2 \leq \sigma. \tag{9}$$

To speed up computation time, it might be possible to replace the computationally expensive multidimensional convolution operation $\mathbf{DK}$ by a single Born scattering operator $\mathbf{K}$ that includes the downgoing field in the source function. A comparable strategy has been aplied by Tu et al. (2012) to include surface-related multiples in the source function of $\mathbf{K}$ for least-squares migration of full wavefields.

CONCLUSIONS

Sparsity promotion in the curvelet domain has been applied succesfully for robust interferometric redatuming with simultaneous and missing sources, producing virtual shot records with relatively low noise levels. Results depend strongly on a good choice of σ, governing the balance between sparsity promotion and signal preservation.

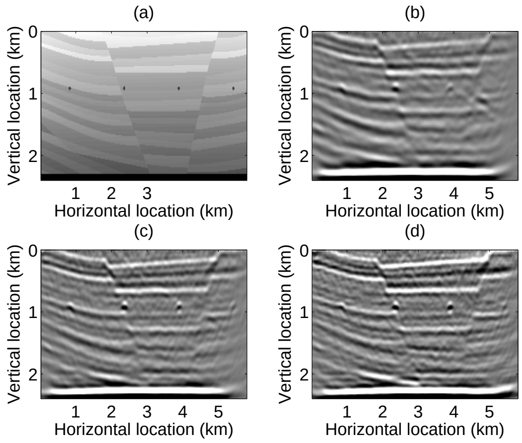

Figure 7: a) True perturbations in the target area and RTM images from Green's functions retrieved by b) crosscorrelation, c) least-squares inversion and d) sparse inversion of sequential source data with missing sources.

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