# Obtaining local reflectivity at two-way travel time by filtering acoustic reflection data

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## SUMMARY

A modified implementation of Marchenko redatuming leads to a filter that removes internal multiples from reflection data. It produces local reflectivity at two-way travel time. The method creates new primary reflections resulting from emitted events that eliminate internal multiples. We call these non-physical primaries and their presence is a disadvantage. The advantage is that the filter is model free. We give the 3D filter and demonstrate with 1D arguments that starting the focusing wavefield with a unit impulse at zero time, while focusing below the bottom reflector, is the choice that leads to a model free implementation. The starting impulse generates the reflection data. Every later emitted pulse eliminates an internal multiple somewhere in the model and helps removing the transmission amplitude effects in a physical primary. We show that the amplitude of the non-physical primaries are a product of three reflections, making them generally smaller than those of the physical primaries. A 2D modeled shotgather at different stages of filtering the data shows that the filter works well.


## INTRODUCTION

Removal of internal multiples has been addressed with various prediction and subtraction schemes with various success (Weglein *et al.* , 1997; Jakubowicz, 1998; ten Kroode, 2002; Berkhout & Verschuur, 2005). Marchenko redatuming started by focusing a wavefield at a specific subsurface location at zero time (Broggini *et al.* , 2012; Wapenaar *et al.* , 2013). The reason was that the focusing wavefield had a time-symmetric duration almost entirely outside the time window of a virtual Green's function that has its source location at the focal point. That Green's function represents the up- and downgoing components of a vertical seismic profile. The upgoing part of this Green's function is the reflection response of subsurface below the focusing depth level to the downgoing part of this Green's function. From such reflection responses at all subsurface locations it is possible to produce a subsurface image without effects of internal multiples that usually contaminate a seismic image (Slob *et al.* , 2014; Behura *et al.* , 2014; Wapenaar *et al.* , 2014). Slob *et al.* (2014) found that in the upgoing part of a modified focusing wavefield the local reflection coefficient of the horizontal reflector at the focusing level is obtained at the one-way travel time if the initial downgoing part of the focusing wavefield is a unit impulse at negative one-way travel time. This means that an image can be constructed without computing the Green's functions. van der Neut & Wapenaar (2016) found that in 3D the projection back to a surface point not only achieves using a unit impulse as initial downgoing modified focusing wavefield, but also let this new wavefield start at t=0. This effectively removes the difficulty of estimating the initial downgoing part of the focusing wavefield and replaces it with finding a two-way travel time curve in the data.

Here we use that idea and use it to focus just below the bottom reflector.We show how that choice leads to being focused at every depth level in the model. We then show in 1D how the projection back to the acquisition surface leads to a modified Marchenko scheme that can be written as a filter that is repeatedly applied to the data and all terms are summed. We show how the filter can be written such that it only consists of the reflection data itself and a truncation to remove intermediate results at non-positive time. This means that the filter is model free. We show why this model free filter inherently produces primary reflections that are undesired, because they arrive at unknown times and with unknown amplitudes without model information. We also show how these are related to the elimination of internal multiples. We finally show with a 2D example how the data obtained from this process leads to a shotgather without multiples, but with non-physical primary reflections at early times.

## OBTAINING LOCAL REFLECTIVITY AT TWO-WAY TRAVEL TIME

Reflection data measured at a surface can be used to eliminate all multiples from the data. Hence the reflection data is its own filter and the filter is model free. The drawback is that, to eliminate all multiples with a model free filter, the filter generates primary events at unknown times and with amplitudes that involve three reflections. We call these the non-physical primaries. An advantage is that these non-physical primaries mostly occur at small travel times. These non-physical primary events are the reflections from the multiple eliminators. Another advantage is that all the physical primary events are recovered with the local reflectivity as amplitude, while the non-physical primaries all consist of a product of three reflection coefficients. The equation is given by

$$R_r(\mathbf{x},\mathbf{x}',t)+NPP(\mathbf{x},\mathbf{x}',t)=\left(\sum_{m=0}^{\infty}(\mathscr{R}_0\Theta_0\mathscr{R}_0^*)^m R_0\right)(\mathbf{x},\mathbf{x}',t). \tag{1}$$

The left-hand side contains the desired primary reflection events $R_r(\mathbf{x},\mathbf{x}',t)$ with local reflectivity values, and non-physical primaries $NPP(\mathbf{x},\mathbf{x}',t)$. The source is located at $\mathbf{x}'$ and the receiver at $\mathbf{x}$, both at the acquisition plane $\mathbb{D}_0$ at depth level $z_0$. The right-hand side contains the data $R_0$, two operators and one truncation. The first operator, including the truncation, acts on a function $p$ and the result is the causal function $q$. It is given by

$$\begin{aligned} q(\mathbf{x},\mathbf{x}',t) &= (\Theta_0\mathscr{R}_0^* p)(\mathbf{x},\mathbf{x}',t) \\ &= \Theta_0\int_{t'}\int_{\mathbb{D}_0} R_0(\mathbf{x},\mathbf{x}_0,t')p(\mathbf{x}_0,\mathbf{x}',t+t')d\mathbf{x}_0dt'. \end{aligned} \tag{2}$$

The truncation is given by $\Theta_0=(0,1)$ for $(t\le 0, t>0)$. The second operator performs a time-convolution and does not require truncation. It acts on the function $q$ and is given by

$$(\mathscr{R}_0 q)(\mathbf{x},\mathbf{x}',t)=\int_{t'}\int_{\mathbb{D}_0} R_0(\mathbf{x},\mathbf{x}_0,t')q(\mathbf{x}_0,\mathbf{x}',t-t')d\mathbf{x}_0dt'. \tag{3}$$

In both operators $t'\in\mathbb{R}$. These operators are defined in analogy to the ones given in van der Neut & Wapenaar (2016). It is important to note that the first term in the right-hand side of equation 1 is the data and the second requires correlating the data with itself, truncating the result by keeping only the values at positive times, and a convolution with the data. Following terms repeat these operations and all results are summed. In the following sections we show in two steps how this works in 1D and then give a 2D numerical example.

## THE TWO BOUNDARIES PROBLEM

Let us start with the simplest 1D example of a model containing two reflectors characterised by local reflection coefficients $r_0$ and $r_1$, located at depth levels $z_0$ and $z_1$, respectively. The travel time from $z_0$ to $z_1$ is $t_1$. The measured reflection response at but just above $z_0$ is then given by

$$R_0(t) = [r\delta(t) + r_1\delta(t-2t_1)] \overset{t}{*} \sum_{m=0}^{\infty}(-r_0r_1)^m\delta(t-2mt_1), \quad (4)$$

where $\overset{t}{*}$ denotes time-convolution. It can be seen that an operator exists that acts on all primary events. Hence finding the inverse of this operator provides the desired operator that can be applied to the reflection response and produces the desired primary events of local reflectivity. Because the operator in the right-hand side of equation 4 is an infinite summed sequence its inverse is known and given by

$$g_1^+(t) = \delta(t) + r_0r_1\delta(t-2t_1). \quad (5)$$

At this point it is important to note that $g_1^+(t)$ is closely related to the focusing wavefield that focuses just below $z_1$ and the relation is given by

$$g_1^+(t) = f_1^+(t) \overset{t}{*} \delta(t-t_1)(1+r_0)(1+r_1), \quad (6)$$

which is the downgoing focusing wavefield compensated for the local transmission effects from just above $z_0$ to just below $z_1$ and delayed such that the initial wavefield starts at $t=0$. This is the direct part of the transmission response just below $z_1$, or, equivalently, the inverse of $f_{1d}^+$ being the direct part of the downgoing focusing wavefield. This is the unknown part of the downgoing focusing wavefield and by removing it from the formulation this unknown is eliminated. Now we only need to know that it exists. Compensation for transmission effects was already used by Slob *et al.* (2014) to show that then the local reflection coefficient is obtained in the upgoing focusing wavefield. As shown by van der Neut & Wapenaar (2016) in 3D it is useful to back-project the focusing wavefield to the acquisition surface, because the amplitude and time curve of the direct part of transmission response are unknown, but required to start the Marchenko scheme. We show here that we only need to focus just below the bottom reflector to obtain all local reflection coefficients at two-way travel time. By projecting the focusing point back to the surface we obtain a function $g_1^+(t)$ that still focuses just below $z_1$, hence the depth information is retained in the function $g_1^+$. If we send $g_1^+$, using equation 5, into the medium instead of only a single impulse the earth response is

$$g_1^-(t) = R_0(t) \overset{t}{*} g_1^+(t) = R_0(t) + r_0r_1R_0(t-2t_1), \quad (7)$$
$$= r_0\delta(t) + r_1\delta(t-2t_1). \quad (8)$$

Equation 7 tells us that the new focusing function gives a response that is equal to the data plus a weighted and delayed version of the data. Equation 8 tells us that $g_1^- = R_r(t)$. When all we have is the measured reflection data the problem is of course that we don't know $g_1^+$, but Marchenko theory tells us how to get it from the data. If we send in $g_1^-(-t)$ we obtain

$$g_1^+(-t) = R_0(t) \overset{t}{*} g_1^-(-t) + T(t+t_1), \quad (9)$$

in which the last term in the right-hand side is a modified transmission response time advanced by the one-way travel time. This equation contains a correlation that we can reverse in time according to

$$g_1^+(t) = R_0(-t) \overset{t}{*} g_1^-(t) + T(-(t+t_1)), \quad (10)$$

It can now be seen that here the truncation needs to be performed at $t=0$ because $T(-(t+t_1)) = 0$ only for $t>0$. This has the consequence that the initial impulse in $g_1^+$ cannot be recovered from these equations. The advantage is that we know it is an impulse and we can write $g_1^+(t) = \delta(t) + g_{1m}^+(t)$ with the coda $g_{1m}^+(t) = 0$ for $t \le 0$. Hence we can rewrite these equations as

$$g_1^-(t) = R_0(t) \overset{t}{*} \left(\delta(t) + g_{1m}^+(t)\right), \quad (11)$$
$$g_{1m}^+(t) = \Theta_0 R_0(-t) \overset{t}{*} g_1^-(t), \quad (12)$$

Substituting the expression for $g_{1m}^+$ of equation 12 in 11 gives

$$\left[1 - R_0(t) \overset{t}{*} \Theta_0 \left(R_0(-t) \overset{t}{*}\right)\right] g_1^-(t) = R_0(t), \quad (13)$$

where it should be noted that in this case $R_r(t) = g_1^-$. The solution of this equation is the the 1D version of equation 1. In this simplest example there is only one multiple to be eliminated. The downgoing event that eliminates the multiple has a reflection that arrives at the arrival time of the second primary reflection such that it corrects for the two-way transmission effects and no non-physical primary reflection events exist.

## THE *N* BOUNDARIES PROBLEM

We show here that equation 13 is valid for a layered model with an arbitrary number of reflectors. To show this we first add two more reflecting boundaries to demonstrate the effect of eliminating multiples as a general principle. We keep the two reflectors and introduce two deeper reflectors at depth levels $z_3$ and $z_4$ with local reflection coefficients $r_3$ and $r_4$ and one-way travel times $t_2$ and $t_3$ as shown in Figure 1. We focus just below the bottom reflector at $z_4$ and use the same equations

$$g_1^-(t) = R_0(t) \overset{t}{*} \left(\delta(t) + g_{1m}^+(t)\right), \quad (14)$$
$$g_{1m}^+(t) = R_0(-t) \overset{t}{*} g_1^-(t) + T(-(t+t_1+t_2+t_3)) - \delta(t), \quad (15)$$

where the time-reversed time-advanced modified transmission response of the whole medium is $T(-(t+t_1+t_2+t_3))$ and it is again zero for $t>0$. By truncating all events at and before $t=0$ in equation 15 and substituting the result in equation 14 we find equation 13, but it now includes non-physical primaries

$$\left[1 - R_0(t) \overset{t}{*} \Theta_0 \left(R_0(-t) \overset{t}{*}\right)\right] (R_r(t) + NPP(t)) = R_0(t). \quad (16)$$

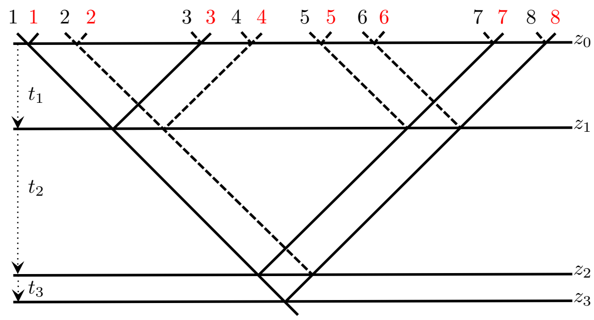


Figure 1: The down- (black numbers) and upgoing (red numbers) events in a medium with four reflectors; direct (soiid lines) and non-physical (dashed lines) primaries; depths and one-way travel times are indicated.

Table 1: Event numbers with amplitudes in down- and upgoing events and their arrival times

| *event* | $g_1^+$ | $g_1^-$ | *time* |
|---|---|---|---|
| 1 | 1 | $r_0$ | $\delta(t)$ |
| 2 | $r_2r_3$ | $r_0r_2r_3$ | $\delta(t-2t_3)$ |
| 3 | $r_0r_1$ | $r_1$ | $\delta(t-2t_1)$ |
| 4 | $r_0r_1r_2r_3$ | $r_1r_2r_3$ | $\delta(t-2(t_1+t_3))$ |
| 5 | $r_1r_2$ | $r_0r_1r_2$ | $\delta(t-2t_2)$ |
| 6 | $r_1r_3$ | $r_0r_1r_3$ | $\delta(t-2(t_2+t_3))$ |
| 7 | $r_0r_2$ | $r_2$ | $\delta(t-2(t_1+t_2))$ |
| 8 | $r_0r_3$ | $r_3$ | $\delta(t-2(t_1+t_2+t_3))$ |

Let us have a closer look at this medium with four reflectors. Figure 1 shows all final events numbered 1 to 8. The red numbers correspond to $g_1^-(t) = R_r(t) + NPP(t)$ and the black numbers correspond to $g_1^+(t)$ that generated them. Because $g_{1m}^+(t)$ in equation 14 was expressed in terms of $g_1^-(t)$ with the aid of equation 15 it is not visible in equation 16, but of course these events are implicitly used. Table 1 shows the amplitudes of all events and their arrival times.

Two important conclusions can be drawn based on observations from this result. The first observation is that events 1 and 3 were already found when we focused just below $z_1$ when there were only two boundaries, events 5 and 7 are added when we would have focused just below $z_2$ in case there were three boundaries, and events 2, 4, 6 and 8 belong to focusing just below $z_3$. From this observation we can conclude that when we focus just below the bottom reflector we focus at every depth level. The second observation is that if the first order multiple is eliminated, all multiples are eliminated. All multiples start with one extra bounce and this has to be included in the downgoing wavefield. We conclude that all non-physical primaries consist of a product of three reflection coefficients, whereas the amplitude of each physical primary is the local reflection coefficient of the reflector where it originates from.

Equation 16 is the general expression that can be used for a medium with $N$ reflecting boundaries and the two conclusions above hold for any layered medium. The number of retrieved events from a medium with $N$ reflectors is $2^{N-1}$ of which there are $N$ desired primary events and all the others are undesired events. These are also all primary events but they are generated by downgoing waves at unknown times for which reason we call them non-physical primaries. These non-physical primary events can be eliminated from the data by introducing time-truncations after both correlation and convolution and both at zero-time and at time $t$ at which time we want to find $R_r(t)$. Details for this procedure can be found in Zhang & Slob (2017).

## NUMERICAL EXAMPLES

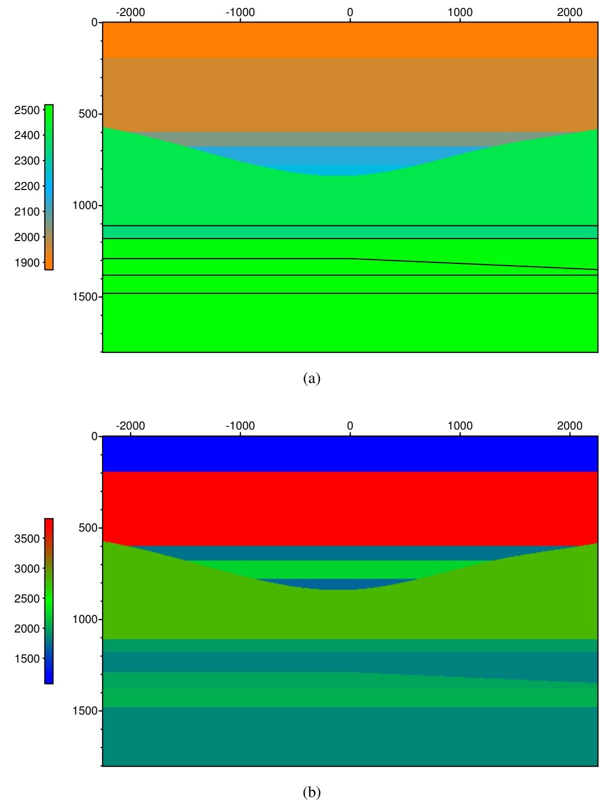


Figure 2: (a) Velocity model, (b) density model.

For the 2D numerical example we use the model of Wapenaar *et al.* (2014). The acoustic velocities and densities in the different layers in the model are depicted in Figure 2. It consists of a layered model with generally increasing velocities and variable densities with a smooth syncline, below which one interface shows a dip in the right-hand side of the model. We have computed surface reflection data with a 20 Hz Ricker wavelet as the source signature. The central shotgather is shown in Figure 3. Reflection responses for all shots are used as data and for the operators in equation 1. After one iteration (m=1 in equation 1) all events that reflect from the multiple eliminators are already constructed, but not yet with the correct amplitude. The following iterations improve the amplitudes and reduce the strength of the multiples. After 20 iterations the shotgather does not contain any multiples and instead contains primary reflections from the multiple eliminators. All physical primaries are preserved but their amplitudes are now the local reflectivity. This can be seen in Figure 4 where the result after the first iteration is shown in Figure 4a and the result after 20 iterations in Figure 4b. All events visible in Figure 4b are also visible in 4a. By comparing Figures 3 and 4b we can see the strong imprint of the syncline in the modeled shotgather. This would make it hard to determine an accurate velocity model and obtain a high-quality image. A velocity model can be more easily made for the first second from the reflection data and after 1s from the retrieved primary reflection data. We suggest that with the improved velocity model a high-quality image can be obtained from the retrieved primaries data.

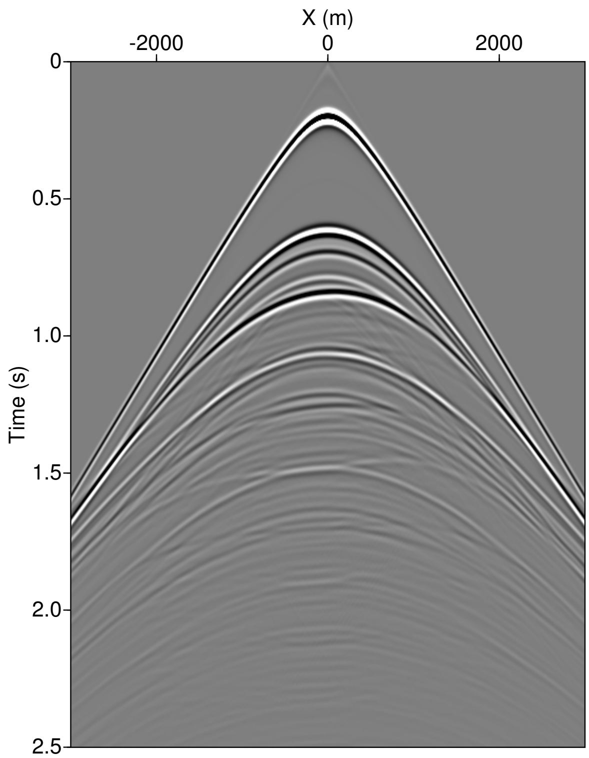


Figure 3: The shotgather with source in the middle of the model.

## CONCLUSIONS

We have given the equation for retrieving primary reflections from reflection data. The operator is the reflection data itself. One iteration involves a spatial integration and time-correlation, time truncation at $t = 0$ and a spatial integration and time-convolution. After one iteration all primary events already exist, but not yet with the correct amplitude and the multiples have been reduced but not completely. Following iterations modify the amplitudes of the primaries and eliminate all internal multiples. We have shown how this works with 1D arguments that only require the essential time-correlation, time truncation and time-convolution. We have demonstrated the effectiveness of the scheme with a 2D numerical example from the literature. The retrieved primary reflection data is a better starting point for velocity analysis than the data itself. We suggested that with an improved velocity model a high-quality image of the target area can be made with the retrieved primary reflection data.

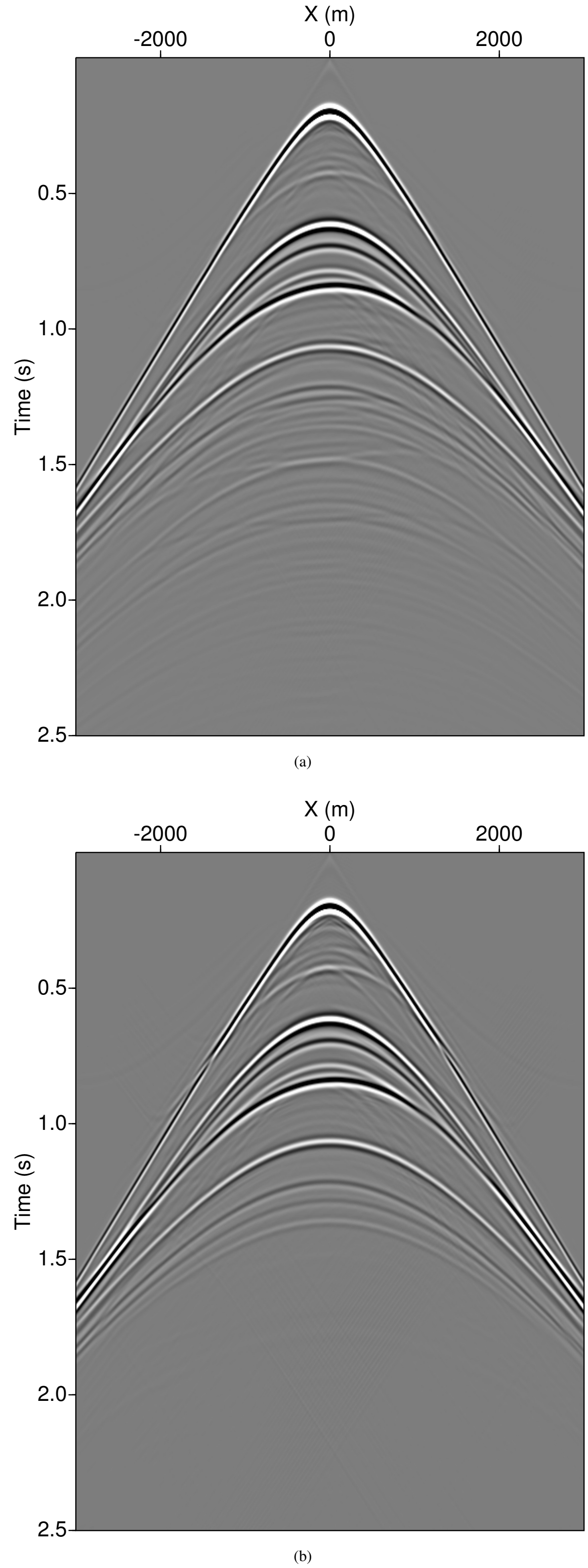


Figure 4: Retrieved primaries after one (a) and 20 (b) iterations.