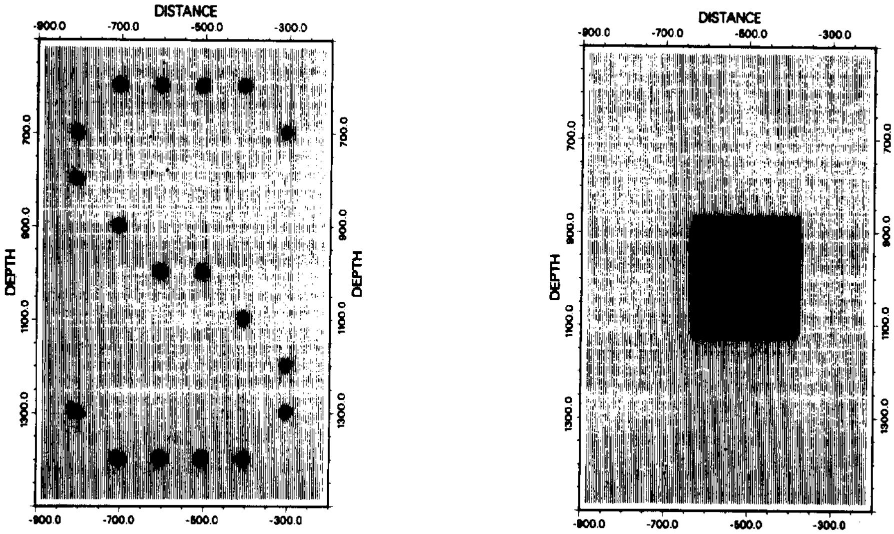

FIG. 4. Images obtained by applying the 2-D analog of equation (11) to synthetic data representing coincident source/receiver experiments in the geometry of Figure 2 for objects consisting of a family of point scatterers and of a homogeneous square block.

accurately recovered by this first term are the (locations of) discontinuities in the function f, in other words, the part of the velocity field normally associated with the reflectivity function that is estimated by migration methods.

For a general inhomogenous reference velocity $c_0(\mathbf{x})$, the weight function W can be determined by ray tracing; however, for constant c_0, W can be determined analytically. In addition, the above analysis can easily be extended to zero-offset or constant-offset seismic experiments. For example, with zero-offset surface experiments, i.e., with $\mathbf{s} = \mathbf{r} = (r_1, r_2, r_3 = 0)$, and with constant background velocity equation (9) becomes

$$\langle f(\mathbf{x})\rangle = \frac{16}{c_0^3}\int_{r_3=0} d^2\mathbf{r}\,\frac{x_3}{\langle \mathbf{x}-\mathbf{r}\rangle}\,u_{sc}(\mathbf{r},\, t = 2|\mathbf{x}-\mathbf{r}|/c_0), \qquad (11)$$

which resembles the classical formula for Kirchhoff migration (Schneider, 1978; see also Norton and Linzer, 1981).

Synthetic examples

Figures 2 and 3 show a synthetic example of the migration algorithm described above, specialized to a two-dimensional geometry. The model consists of a fixed source and point scatterers separated by roughly one wavelength at the central frequency of the source and distributed to form the letter *S*. Receivers were located both on the Earth's surface and in two wells flanking the scatterers (Figure 2). The images in Figure 3 illustrate the resolution obtained by using different subsets of the data. Figure 4 shows reconstructions of the letter *S* and a homogeneous block, based on a synthetic zero-offset experiment using all the receiver positions in Figure 2.

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Principle of Prestack Migration Based on the Two-Way Wave Equation S19.7

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In this paper a brief review of prestack wave field extrapolation based on the two-way wave equation is presented. Based on extrapolation operators for the total wave field, a modeling scheme for one-dimensional media is reviewed. In this modeling scheme, critical angle effects, multiple reflections, and transmission effects are incorporated. Next a prestack migration scheme based on the two-way wave equation for arbitrary inhomogeneous media is introduced. It is concluded that this scheme represents a potential alternative to existing (finite difference) migration schemes, as the square root operator is avoided, while critical angle effects, multiple reflections, transmission effects, and wave conversion may be properly incorporated.

Introduction

Many seismic modeling as well as migration schemes are based on forward and inverse extrapolation of the acoustic wave field (Claerbout, 1976; Berkhout, 1982). Generally the acoustic wave equation is split into approximate one-way wave equations which govern independent propagating downgoing and upgoing waves, respectively. Solutions of these one-way wave equations can be described in terms of wave field extrapolation operators (phase shift operator, Kirchhoff summation operator, dip-limited finite difference operator, etc.), while the boundary conditions are given by a downgoing or upgoing wave field at a specific depth level. The interaction between downgoing and upgoing waves need be specified explicitly in terms of reflection operators which are input to modeling schemes, and output of (ideal) migration schemes.

With respect to modeling and migration schemes based on the one way wave equations, the following should be noted: (1) Critical angle effects cannot be handled; whenever vertical velocity variations are present, the assumption that the total wave field may be split into independent downgoing and upgoing waves breaks down for waves with a propagation direction which is nearly horizontal. (2) The implicit square root operator in the one-way wave equations is responsible for the slow convergence of finite difference schemes. (3) Additional effort is required for the incorporation of multiple reflections, transmission effects, and wave conversion.

In seismic literature, several alternative modeling approaches have been proposed, based on the WKBJ-technique, which properly include critical angle events (e.g., Kennett and Illingworth, 1981). Wapenaar and Berkhout (1984) introduced one-way wave field extrapolation operators based on the WKBJ-technique, which can be applied in modeling as well as in migration schemes for critical angle data. Kosloff and Baysal (1983) introduced the application of the two-way wave equation in poststack migration. In their approach a one-way propagation problem (exploding reflector assumption) is handled with the two-way wave equation, so use of the square root operator is evaded. In this paper we review the application of the two-way wave equation in prestack modeling and introduce its application in prestack migration. The method is based on extrapolation operators for the total wave field. It will be shown that critical angle effects can be handled, application of the implicit square root operator is evaded, and multiple reflections, transmission effects, and wave conversion are properly incorporated.

Wave field extrapolation

Our starting point will be the matrix representation of the two-way wave equation,

$$\frac{\partial}{\partial z}\mathbf{B} = \mathbf{A}\mathbf{B}, \tag{1a}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & \rho \\ -\frac{1}{\rho}H_2 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} P \\ \frac{1}{\rho}\frac{\partial p}{\partial z} \end{bmatrix}, \tag{1b,c}$$

$$H_2 = k^2 + \rho\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial}{\partial x}\right) + \rho\frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial}{\partial y}\right), \tag{1d}$$

with $P = P(x,y,z,\omega)$ acoustic pressure, $\rho = \rho(x,y,z,)$ mass density, $k = \omega/c$ wave number, $c = c(x,y,z,)$ propagation velocity, and $\omega/2\pi$ = frequency.

Unlike the implicit square root operator in one-way wave equations, explicit operator **A** can be applied exactly within the seismic bandwidth. The solution of (1a) is given by the following Taylor series summation

$$\mathbf{B}(z_i \pm \Delta z) = \sum_{m=0}^{\infty} \frac{(\pm\Delta z)^m}{m!}\left[\frac{\partial^m \mathbf{B}}{\partial z^m}\right]_{z_i}, \tag{2a}$$

where

$$\partial^m\mathbf{B}/\partial z^m = \partial^{m-1}(\mathbf{AB})/\partial z^{m-1}, \text{ for } m \geq 1. \tag{2b}$$

Note that these relations are exact. In the following, expression (2a) is abbreviated to

$$\mathbf{B}(z_i \pm \Delta z) = \mathbf{W}(z_i \pm \Delta z, z_i)\mathbf{B}(z_i), \tag{2c}$$

where matrix **W** represents the operator for recursive extrapolation of the total wave field **B**. For practical applications some assumptions must be made in order to avoid the infinite summation (2a).

Assuming that the medium properties c and ρ may be linearized in depth between z_i and $z_i \pm \Delta z$, while the lateral derivatives of the medium properties may be neglected, then a fast converging finite difference operator **W** can be found from (2a), which properly includes critical angle effects. A discussion of this operator is beyond the scope of this paper. The operator is discussed by Berkhout (1982) for the case that critical angle effects may be neglected.

Assuming that the medium properties may be linearized in depth between z_i and $z_i \pm \Delta z$, while they are constant in the lateral directions, then in the k_x, k_y,ω-domain a closed expression for operator $\tilde{\mathbf{W}}$ can be found from (2a), which properly includes critical angle effects. A discussion of this operator is beyond the scope of this paper. For small Δz the operator equals an operator based on Airy functions, as discussed by Kennett and Illingworth (1981).

Assuming a homogeneous layer between z_i and $z_i \pm \Delta z$, then in the k_x,k_y,ω-domain a closed expression for operator $\tilde{\mathbf{W}}$ is found from (2a), which is a special case of above mentioned operator. A discussion of this operator is also beyond the scope of this paper. The same operator is derived by Ursin (1983), by means of eigenvector decomposition applied to operator $\tilde{\mathbf{A}}$.

Modeling

Modeling with the two-way wave equation of prestack data for media with $c = c(z)$, $\rho = \rho(z)$, is based on the following recursive algorithm in the k_x,k_y,ω-domain,

$$\tilde{\mathbf{B}}(z_0) = \tilde{\mathbf{W}}(z_0,z_1) \ldots \tilde{\mathbf{W}}(z_i - \Delta z, z_i) \ldots, \quad \tilde{\mathbf{W}}(z_{N-1},z_N)\tilde{\mathbf{B}}(z_N). \tag{3a}$$

The procedure is as follows. (1) Specify the total field $\tilde{\mathbf{B}}(z_N)$. For instance, in a homogeneous lower half-space for $z \geq z_N$, only downgoing waves are present. (2) Determine $\tilde{\mathbf{B}}(z_0)$ by means of relation (3a). (3) Apply decomposition into down and upgoing waves $\tilde{P}^+$ and $\tilde{P}^-$ at $z = z_0$ (Ursin, 1983). Assuming a homogeneous upper half-space for $z \leq z_0$, then

$$\tilde{R}(z_0) = \tilde{P}^-(z_0)/\tilde{P}^+(z_0) \tag{3b}$$

defines the reflectivity of the layered medium. (4) A common shot gather is now modeled by

$$\tilde{P}_{CSG}(z_0) = \tilde{D}(z_0)\tilde{R}(z_0)\tilde{S}(z_0), \tag{3c}$$

followed by inverse Fourier transforms. Here $\tilde{S}$ and $\tilde{D}$ define the source and detector signature, respectively. Note that critical angle effects may be included, and multiple reflections and transmission effects are incorporated.

Migration

Migration with the two-wave equation of prestack data for media with $c = c(x,y,z)$, $\rho = \rho(x,y,z)$, is based on the following recursive algorithm in the x,y,ω-domain,

$$\mathbf{B}(z_M) = \mathbf{W}(z_M,z_{M-1}) \ldots \mathbf{W}(z_i + \Delta z, z_i) \ldots \mathbf{W}(z_1,z_0)\mathbf{B}(z_0). \tag{4a}$$

The procedure for each common shot gather is as follows. (1) Specify the total field $\mathbf{B}(z_0)$. Note that both the source and the detector data should be incorporated. (2) Determine the downward continued total field $\mathbf{B}(z_M)$ by means of relation

(4a). (3) Apply decomposition into down and upgoing waves P^+ and P^- at $z = z_M$. Note that errors made in this decomposition do not contribute to deeper z_M-levels, since the total field is downward continued independently in step 2. (4) Retrieve the direct source wave S from the downgoing wave P^+ by means of a "first arrival time window". (5) Apply zero-offset imaging (Berkhout, 1982, chapter 7.7; de Graaff, 1984) according to

$$\langle R(z_m)\rangle = \frac{1}{2\pi}\int_\omega S^*(z_M)P^-(z_M)d\omega. \tag{4b}$$

Repeat steps 2–5 for all z_M. Repeat the total procedure for all common shot gathers and sum the individual migration results. Note that the square root operator is avoided, critical angle effects may be incorporated, multiple reflections do not contribute to the imaged result, and transmission effects are included.

Discussion

From the foregoing it follows that the two-way wave equation migration scheme is more general than the two-way wave equation modeling scheme (3-D media versus 1-D media). This stems from the fact than the aim of recursive migration [reflectivity at zero offset and zero time, relation (4b)] is less complicated than the aim of recursive modeling [reflectivity for all offsets and all times, relation (3b)]. The 1-D modeling scheme finds its application in linearized multidimensional inversion techniques for the generation of the background medium response.

Both the modeling and migration schemes can be extended for the incorporation of shear waves when equation (1) is replaced by the two-way elastic wave equation. In this case matrices **A** and **W** become 4 × 4-matrices, while **B** contains horizontal and vertical particle displacements as well as normal and shear stresses. Decomposition of **B** yields down and upgoing longitudinal waves as well as down and upgoing transversal waves (Ursin, 1983). A further discussion of elastic wave modeling and migration is beyond the scope of this paper.

A final remark should be made concerning the migration scheme. Like all multiple elimination schemes, accurate knowledge of the velocity model is required for an optimum performance with respect to the multiples.

Conclusions

Application of the two-way wave equation in prestack migration allows proper handling of critical angle effects, multiple reflections, transmission effects, and wave conversion. For arbitrary inhomogeneous media, the absence of the square root operator is most advantageous with respect to the convergence speed of finite difference schemes. The principle introduced in this paper represents a potential alternative to existing (finite difference) migration schemes.

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Computational and Asymptotic Aspects of Velocity Inversion S19.8

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This talk discusses computational and asymptotic aspects of the Born inversion method. We show how asymptotic analysis is exploited to reduce the number of integrations in a $k - f$ like solution formula for the velocity variation. The output of this alternative algorithm produces the *reflectivity function* of the surface. This is an array of singular functions—Dirac delta functions which peak on the reflecting surfaces—each scaled by the normal incidence reflection strength at the surface. Thus, imaging of a reflector is achieved by construction of its singular function and estimation of the reflection strength is deduced from the peak value of that function. By asymptotic analysis of the application of the algorithm to the Kirchhoff representation of the backscattered field, we show that the peak value of the output estimates the reflection strength even when the condition of small variation in velocity, an assumption of the original derivation, is violated. Furthermore, this analysis demonstrates that the method provides a migration algorithm when the amplitude has not been preserved in the data. The design of the computer algorithm is discussed, including such aspects as constraints due to causality and spatial aliasing. We also provide O-estimates of computer time. This algorithm has been successfully implemented on both synthetic data and CMP stacked field data.

The purpose of this talk is to discuss certain computational features and their basis in asymptotic methods of the result known as *Born inversion* (Cohen and Bleistein, 1979a). In that paper, a method was proposed for inverting backscattered (CMP stacked) seismic data for sound speed. This inverse problem was modeled by the acoustic wave equation with a sound speed which was written as a perturbation from a reference or background sound speed. A nonlinear integral equation, involving the product of the unknown perturbation and the unknown wave field in the earth was derived. The known quantities in this equation are the backscattered (CMP stacked) observations at the surface. Linearization of this integral equation led to a Fredholm integral equation of the first kind for the sound speed perturbation. The kernel of this integral equation is not square integrable in all of variables. Thus, the theory for such bounded (compact) kernels (which predicts ill-posedness) does not apply to this