Angle-Dependent Reflectivity by Shot Record Migration

C. G. M. de Bruin, C. P. A. Wapenaar, and A. J. Berkhout, Delft University of Technology, Netherlands

SUMMARY

Angle dependent reflectivity can be obtained in the pre-stack migration process based on shot record inversion. Therefore a new imaging technique has been developed which recovers the complete reflectivity matrix from downward extrapolated shot records. The reflectivity matrix is an operator which transforms the incident wave field into a reflected wave field, taking into account angle dependency.

All present-day seismic migration schemes determine only the diagonal of the reflectivity matrix, each diagonal element representing the zero-offset reflectivity at one lateral position on a reflector. In this simple diagonal form, however, angle dependent reflectivity is not accounted for. By considering the rows of this matrix the angle dependent reflection coefficient can be retrieved. The first tests with this shot record based imaging technique show good results upto high angles, both in the acoustic and in the elastic case. With angle dependent reflectivity results it becomes feasible to obtain density and velocity information.

INTRODUCTION

The physical model for one-way wave propagation can simply be represented in terms of matrix multiplications: by a downward propagation matrix, a reflection matrix and an upward propagation matrix. The reflectivity properties of a reflector are described by reflectivity matrix **R**. In seismic migration it is common practice to represent reflectivity by a single reflection coefficient. In this case the reflectivity matrix is a diagonal matrix, each element representing the reflection coefficient at a lateral position at a constant depth level: $R(x,z_i)$ in the 2-D case. As a consequence only information about the impedance can be retrieved.

It is shown in this paper that more information can be extracted from the reflectivity matrix. This information is in the form of space-variant, angle dependent reflectivity information $R(x,z_i;\alpha)$. With this additional information the medium parameters (ρ,c_p,c_s) can be determined. The retrieval of angle dependent reflectivity from seismic data is based on shot record migration as proposed by Berkhout (1982). Therefore, in contrast with amplitude versus offset techniques, complicated subsurfaces can be handled.

REFLECTIVITY IN THE FORWARD MODEL

Following Berkhout (1982), the two-dimensional model of seismic shot records, which describes the monochromatic primary response of one CSP-record, can be written as follows (in matrix notation):

$$\vec{P}^-_{CSP}(z_0) = \mathbf{D}(z_0)\left[\sum_i \mathbf{W}(z_0,z_i)\,\mathbf{R}(z_i)\,\mathbf{W}(z_i,z_0)\right]\vec{S}^+(z_0), \quad (1)$$

see also Figure 1. Here $\vec{S}^+(z_0)$ describes the downgoing source wave field at z_0. Vector $\vec{P}^-_{CSP}(z_0)$ represents the total pressure of all reflected wave fields, arriving back at the surface z_0. Detector matrix $\mathbf{D}(z_0)$ defines the field patterns. Matrices $\mathbf{W}(z_i,z_0)$ and $\mathbf{W}(z_0,z_i)$ describe the propagation of the source and the reflected wave field, respectively.

Reflectivity matrix $\mathbf{R}(z_i)$ defines the relationship between the downward and upward travelling pressure fields at z_i:

$$\vec{P}^-(z_i) = \mathbf{R}(z_i)\,\vec{S}^+(z_i). \quad (2)$$

The determination of matrix **R** is generally complicated for a given subsurface model. Therefore, often **R** is taken as a diagonal matrix. The inhomogeneities at z_i are considered 'locally reacting', i.e. one point of the incident wave field contributes to one point of the reflected wave field at the same lateral position. Therefore we call this diagonal matrix the zero-offset (ZO) reflectivity matrix. In this case the rows of $\mathbf{R}(z_i)$ consist of only one non-zero sample.

Ideally, one point of the reflected wave field $P^-(x_j,z_i,\omega)$ at the reflector is computed from a spatially-weighted average of the total incident wave field $S^+(x,z_i,\omega)$ at the reflector (Figure 2). We call this spatially-weighted average at x_j the *reflectivity convolution operator* $R_j(x,z_i,\omega)$. For a laterally invariant reflector, equation (2) can also be written by means of a spatial convolution:

$$P^-(x,z_i,\omega) = R(x,z_i,\omega) * S^+(x,z_i,\omega), \quad (3)$$

where the asterisk denotes convolution along the x-axis. A reflectivity convolution operator defines angle dependent reflection. If we spatially Fourier transform relation (3) to the k_x-ω domain (k_x is the horizontal wavenumber) we obtain

$$\tilde{P}^-(k_x,z_i,\omega) = \tilde{R}(k_x,z_i,\omega)\, \tilde{S}^+(k_x,z_i,\omega). \tag{4}$$

$\tilde{R}(k_x,z_i,\omega)$ is named the (angle dependent) *reflection function* at level z_i and is given by

$$\tilde{R}(k_x,z_i,\omega) = \frac{\rho_2\sqrt{k_1^2-k_x^2} - \rho_1\sqrt{k_2^2-k_x^2}}{\rho_2\sqrt{k_1^2-k_x^2} + \rho_1\sqrt{k_2^2-k_x^2}} \tag{5}$$

with ρ_1 and ρ_2 the densities of the upper and lower layer. By substituting $k_1=\omega/c_1$, $k_2=\omega/c_2$ and $k_x=k_1\sin\alpha$ the well-known angle dependent reflection coefficient $R(\alpha)$ is obtained (c_1 and c_2 are the velocities in the upper and lower layer).

So, if a reflector is locally reacting then **R** is a diagonal matrix, hence $R(x,z_i,\omega) = R_0\delta(x)$, $\tilde{R}(k_x,z_i,\omega) = R_0$, so the reflection function does not vary with the angle of incidence (Figure 3a). Unfortunately, the locally reacting assumption is not valid in many practical situations, hence reflectivity matrix **R** will have a band-structure and the reflection function will show angle dependence as can be seen in Figure 3b. Hence, if we are interested in finding the angle dependent reflection by seismic migration, we should not only compute the diagonal elements but the full reflectivity matrix **R**.

REFLECTIVITY IN THE PRE-STACK MIGRATION

The forward model for one seismic experiment, formulated in equation (1), can be rewritten into (we assume detector matrix D to be an identity matrix)

$$\vec{P}^-_{CSP}(z_0) = \mathbf{X}(z_0)\, \vec{S}^+(z_0) \tag{6a}$$

with

$$\mathbf{X}(z_0) = \sum_i \left[\mathbf{W}(z_0,z_i)\, \mathbf{R}(z_i)\, \mathbf{W}(z_i,z_0)\right]. \tag{6b}$$

In the inverse problem we have to determine **R**, given the seismic measurements $\vec{P}^-_{CSP}(z_0)$, the source $\vec{S}^+(z_0)$ and a macro subsurface model. Compensation for the propagation effects between the surface and depth level z_i can be described by

$$\vec{S}^+(z_i) = \mathbf{W}(z_i,z_0)\, \vec{S}^+(z_0) \tag{7a}$$

and

$$\vec{P}^-(z_i) = \mathbf{F}(z_i,z_0)\, \vec{P}^-_{CSP}(z_0). \tag{7b}$$

The inverse wave field extrapolation operator is given by

$$\mathbf{F}(z_i,z_0) = \left[\mathbf{W}(z_0,z_i)\right]^{-1} \approx \mathbf{W}^*(z_i,z_0) \tag{8}$$

where * denotes complex conjugation (matched filter approach). After this downward extrapolation we thus have at z_i:

$$\vec{P}^-(z_i) = \mathbf{X}(z_i)\, \vec{S}^+(z_i) \tag{9a}$$

with

$$\mathbf{X}(z_i) = \mathbf{R}(z_i) + \sum_{m\neq i} \left[\mathbf{F}(z_i,z_0)\, \mathbf{W}(z_0,z_m)\, \mathbf{R}(z_m)\, \mathbf{W}(z_m,z_0)\, \mathbf{F}(z_0,z_i)\right]. \tag{9b}$$

Now we have to retrieve reflectivity information $\mathbf{R}(z_i)$ from the extrapolated data. For the moment we consider a one-reflector model, hence,

$$\mathbf{X}(z_i) = \mathbf{R}(z_i). \tag{9c}$$

Equation (9) formulates an underdetermined system because the number of unknowns is greater than the number of equations. A particular solution of (9a) and (9c) reads

$$< \mathbf{R}(z_i) >_n = \frac{1}{\|\vec{S}^+(z_i)\|^2}\, \vec{P}^-(z_i) \left(\vec{S}^+(z_i)\right)^{*T} \tag{10a}$$

where $<..>_n$ denotes that this non-unique solution is obtained from shot record n and where T denotes a transposition. By adding all individual shot record results according to

$$\mathbf{R}(z_i) = \sum_{n=1}^{N} < \mathbf{R}(z_i) >_n \tag{10b}$$

a unique solution for the full reflectivity matrix is obtained (Wapenaar and Berkhout, 1987).

OBTAINING ANGLE DEPENDENT REFLECTIVITY

Thus far in the practice of seismic migration, only the diagonal elements of matrix $\mathbf{R}(z_i)$ used to be selected. As argued in a previous section, in order to obtain angle dependent reflectivity information the rows of the **R** matrix have to be computed. In the following we restrict ourselves to interpreting just the j^{th} row of the reflectivity matrix $\mathbf{R}(z_i)$, as defined by (10b). This is the reflectivity convolution operator $R_j(x,z_i,\omega)$ at lateral position x_j. If we spatially Fourier transform this function to the k_x-ω domain the angle dependent reflection function $\tilde{R}_j(k_x,z_i,\omega)$ for lateral position x_j at the reflector is obtained.

So far we considered the retrieval of angle dependent reflectivity for one frequency component only. In the broad-band case the imaging step must be taken into account. Straightforwardly applying the conventional imaging step

$$R_j(x,z_i,t=0) = \frac{\Delta\omega}{2\pi}\sum_{\omega} R_j(x,z_i,\omega) \tag{11}$$

would result in losing the angle dependent reflectivity information. Therefore another imaging operator has to be contrived which must be based on the fact that the reflection functions must be summed along lines of constant angle in the k_x-ω domain, that is, along lines of constant

$$\frac{k_x}{\omega} = \frac{\sin\alpha}{c_1} = \frac{1}{c_x}.$$

In seismic literature c_x^{-1} is generally referred to as the rayparameter p. So, if we replace the wavenumber variable k_x by the rayparameter p:

$$\tilde{R}_j(k_x,z_i,\omega) \rightarrow \tilde{R}_j(p,z_i,\omega), \tag{12}$$

then imaging should be carried out along lines of constant p:

$$\tilde{R}_j(p,z_i,\tau=0) = \frac{1}{N_\omega}\sum_{\omega}\tilde{R}_j(p,z_i,\omega). \tag{13a}$$

(N_ω is a weighting factor equal to the integral of the source spectrum.)

The above described procedure can be repeated for each extrapolation depth level. With the imaging technique we are not restricted to the 'one-reflector case' any more. Hence, if (9b) applies instead of (9c), then in equations (10) and (12) R should be replaced by X, hence (13a) should be replaced by

$$\tilde{R}_j(p,z_i,\tau=0) = \frac{1}{N_\omega}\sum_{\omega}\tilde{X}_j(p,z_i,\omega). \tag{13b}$$

EXAMPLE

As an example we consider the configuration of Figure 4a (see de Bruin, 1988). This full elastic subsurface model consists of an overburden and a target zone, consisting of several thin layers. The seismic PP-response from one shot record is shown in Figure 4b. The results on the elastic angle dependent reflection coefficient R_{pp}, retrieved from the dataset of Figure 4b by means of our shot record based migration algorithm, are presented in Figure 5. Upto at least α=30°, the migration results (dashed lines) show a high degree of similarity with the theoretical results (solid lines).

CONCLUSIONS

We have shown theoretically that it is possible to retrieve angle dependent reflectivity $R(x,z_i;\alpha)$ by means of shot record migration and stacking for any subsurface. Once this function is found it is in principle feasible to determine the detailed density and P- and S-wave velocities in a medium. In an example we have demonstrated the validity of our approach for a 1-D full elastic subsurface. More examples will be presented during the presentation.

REFERENCES

Berkhout, A.J., 1982, Seismic migration, Volume A, Theoretical aspects: Elsevier, Amsterdam - New York.

de Bruin, C.G.M., 1988, Angle-dependent reflectivity by means of shot record migration: M.Sc. thesis, Delft University of Technology.

Wapenaar, C.P.A., and Berkhout, A.J., 1987, Full pre-stack versus shot record migration: Presented at the 57th SEG Ann. Internat. Mtg., New Orleans.

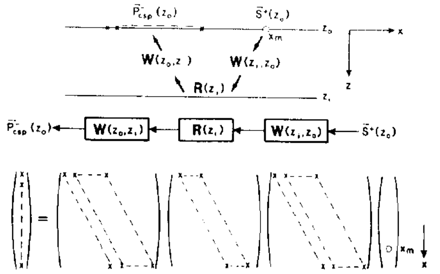


FIG. 1. Basic model for the seismic response from depth level z_i (no field patterns).

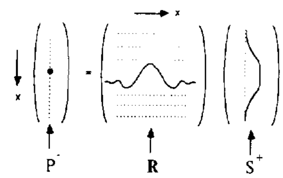


FIG. 2. Each row of the reflectivity matrix **R** represents a reflectivity convolution operator.

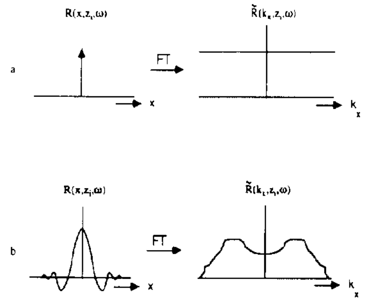
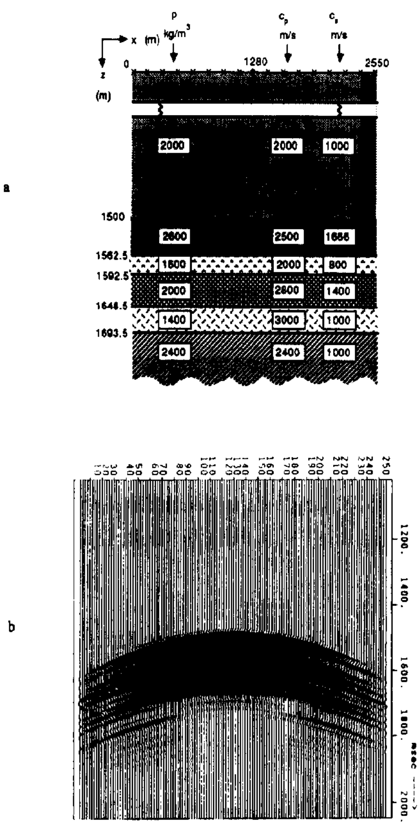


FIG. 3. (a) A locally reacting reflectivity operator transforms into an angle independent reflection function. (b) A non-locally reacting reflectivity operator transforms into an angle dependent reflection function.



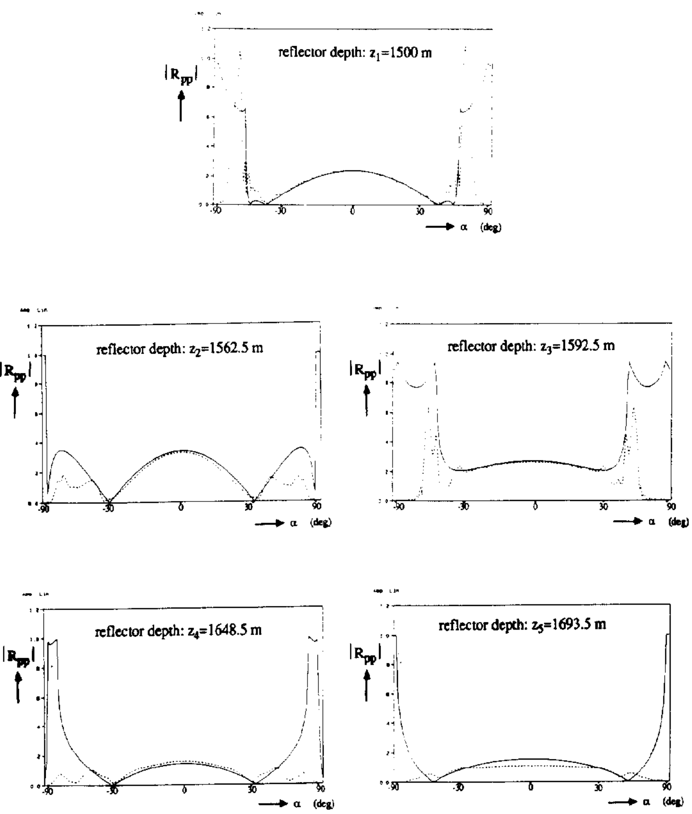


FIG. 4. (a) Subsurface model of a horizontally layered full elastic medium. (b) Seismic *(PP-)* response from the elastic model.

FIG. 5. Full elastic angle-dependent reflectivity results (dashed lines) from the model of Figure 4 together with the theoretically computed results (solid lines).