Wavelet Estimation by Prestack Multiple Elimination

D. J. Verschuur, A. J. Berkhout, and C. P. A. Wapenaar, Delft University of Technology, Netherlands

Summary

A prestack, wave equation based method is introduced for eliminating all surface related multiples. For this method the data itself is used as the multiple prediction operator.
The main advantage of this method is the fact that no knowledge about the subsurface is needed, even not about the first reflector in the subsurface (the sea bottom in marine data). On the other hand the reflectivity properties of the pressure free surface and the source wave field characteristics must be known. This last feature may seem a drawback, especially the knowledge about the source wave field including the scale factor in the data, because they will not be known in general.
But the whole procedure can be reformulated in such a way that the multiple elimination becomes an adaptive scheme with the source wave field as the unknown variable. By minimizing the total energy in the data after multiple elimination, the source wavelet can be estimated together with the multiple elimination. It appears that especially the phase spectrum of the wavelet can be well estimated by this method.
In principle this multiple elimination and wavelet estimation procedure can be applied for acoustic data and as well as multi-component data. However, the restriction to acoustic data will be made in this paper.

Introduction

Multiple elimination, especially in laterally inhomogeneous media, is still a significant problem in seismics. Most multiple elimination procedures fail if an inhomogeneous subsurface is present. The nice feature of the proposed method is the fact that no information about the subsurface is needed, which makes it also suitable for complicated geologic subsurfaces. The multiple elimination can be regarded as a processing step in advance to all other inversion and migration procedures.
The multiple elimination method was originally described by Berkhout (1982, chapter 7).

Theory of multiple elimination

Following the matrix notation for seismic data of Berkhout (1982, chapter 6), the full prestack monochromatic seismic data without any surface related multiples (in the two dimensional case given by a matrix in the x-ω domain) can be represented by:

$$\mathbf{P}_0^-(z_0) = \mathbf{Q}^+(z_0)\mathbf{S}^+(z_0), \tag{1}$$

in which $\mathbf{P}_0^-(z_0)$ represents the monochromatic upgoing wave field at the surface ($z=z_0$), $\mathbf{S}^+(z_0)$ represents the downgoing source wave field and $\mathbf{Q}^+(z_0)$ the spatial impulse response of the subsurface without the surface related multiples. In the data matrix $\mathbf{P}_0^-(z_0)$ the columns contain the monochromatic shot records and the diagonal elements the zero offset data. The $\mathbf{Q}^+(z_0)$ matrix can be considered to be the overall reflection matrix of the subsurface. Figure 1a shows equation (1) in a block diagram representation.
In reality the free surface reflects all upgoing energy, which causes a new illuminating downgoing wave field, giving rise to multiple events. This property gives an implicit expression for the total upgoing wave field $\mathbf{P}^-(z_0)$ at the surface containing all multiples:

$$\mathbf{P}^-(z_0) = \mathbf{Q}^+(z_0)\,[\,\mathbf{S}^+(z_0) + \mathbf{R}^-(z_0)\mathbf{P}^-(z_0)\,], \tag{2}$$

with $\mathbf{R}^-(z_0)$ being the reflectivity matrix of the free surface for upgoing waves. Figure 1b shows how the block diagram of figure 1a changes when a reflecting surface is taken into account. Equation (2) can be rewritten to get an explicit expression for the total upgoing wave field $\mathbf{P}^-(z_0)$:

$$\mathbf{P}^-(z_0) = [\,\mathbf{I} - \mathbf{Q}^+(z_0)\mathbf{R}^-(z_0)]^{-1}\mathbf{Q}^+(z_0)\mathbf{S}^+(z_0). \tag{3}$$

The first term at the right hand side of equation (3) generates all surface related multiples. So equation (3) gives us the forward model of acoustic data with all multiples included. Note the fact that $\mathbf{Q}^+(z_0)$ contains <u>all</u> upgoing events that are reflected in the subsurface and not at the free surface. In fact this $\mathbf{Q}^+(z_0)$ is the data we are interested in for further seismic processing.
This means that for eliminating the surface related multiples, equation (3) has to be rewritten to get an expression for $\mathbf{Q}^+(z_0)$:

$$\mathbf{Q}^+(z_0) = \mathbf{P}^-(z_0)\,[\,\mathbf{S}^+(z_0) + \mathbf{R}^-(z_0)\,\mathbf{P}^-(z_0)\,]^{-1}, \tag{4}$$

or

$$\mathbf{Q}^+(z_0) = \mathbf{P}^-(z_0)\mathbf{S}^+(z_0)^{-1}[\,\mathbf{I} + \mathbf{R}^-(z_0)\mathbf{P}^-(z_0)\mathbf{S}^+(z_0)^{-1}]^{-1}, \tag{5}$$

in which $\mathbf{P}^-(z_0)\mathbf{S}^+(z_0)^{-1}$ represents the data deconvolved for the source wave field. If we define

$$\mathbf{X}^+(z_0) = \mathbf{P}^-(z_0)\mathbf{S}^+(z_0)^{-1}, \tag{6}$$

equation (5) transforms into

$$\mathbf{Q}^+(z_0) = \mathbf{X}^+(z_0)[\,\mathbf{I} + \mathbf{R}^-(z_0)\mathbf{X}^+(z_0)]^{-1}. \tag{7}$$

$\mathbf{X}^+(z_0)$ can be considered as the spatial impulse response matrix

of the subsurface with the free surface included.
Equation (7) defines the multiple elimination procedure that is investigated in this paper. But applying equation (7) will give serious problems on inverting the term on the right hand side. To show this, the inversion term of equation (7) is expanded into a Taylor series giving:

$$\mathbf{Q}^+(z_0) = \mathbf{X}^+(z_0)[\,\mathbf{I} - \mathbf{R}^-(z_0)\mathbf{X}^+(z_0) + \{\mathbf{R}^-(z_0)\mathbf{X}^+(z_0)\}^2 - \{\mathbf{R}^-(z_0)\mathbf{X}^+(z_0)\}^3 + \dots\,] \quad (8)$$

Each term in this Taylor series eliminates one higher order of surface related multiples. For strong multiples this series will not converge rapidly and the inversion in equation (7) is not very stable stable. Stabilization of this inversion will have a negative influence on the result. Therefore equation (8) is taken as the multiple elimination procedure because it is always stable as only a restricted number of terms is taken into account.
Note the fact that apart from the source wave field and the reflection characteristics of the free surface no other external information is used in this elimination procedure. Furthermore, in the acoustic case the free surface will be totally reflecting for all angles of incidence, which means that the reflectivity matrix $\mathbf{R}^-(z_0)$ can be replaced by a single reflection coefficient r_0=-1, yielding:

$$\mathbf{Q}^+(z_0) = \mathbf{X}^+(z_0) - r_0\mathbf{X}^+(z_0)^2 + r_0^2\mathbf{X}^+(z_0)^3 - \dots \quad (9)$$

The procedure mainly consists of matrix multiplications in the x–ω domain which makes the algorithm very suitable for vector machines.

Estimation of the wavelet by multiple elimination

As mentioned before, for the proposed procedure we need the source wave field, including the directivity of the sources, the frequency dependent wavelet and the data scale factor. This last requirement comes from the fact that the data itself is used as multiple prediction operator, so true amplitudes are needed to get a good amplitude match between the predicted multiples and the multiples present in the data. All this information will never be available in real seismic data. Therefore the whole method will only work if it is data adaptive. So the source characteristics must be estimated in order to get rid of the multiples completely.
If we assume that the sources are vibrators (flat k_x-spectrum) the source wave field matrix can be written as:

$$\mathbf{S}^+(z_0) = S(\omega)\,\mathbf{I}, \quad (10)$$

with S(ω) the frequency dependent wavelet. Combining equation (1), (6), (9) and (10) the result is:

$$\mathbf{P}_0^-(z_0) = \mathbf{P}^-(z_0) - R_0(\omega)\mathbf{P}^-(z_0)^2 + R_0^2(\omega)\mathbf{P}^-(z_0)^3 - \dots \quad (11)$$

with

$$R_0(\omega) = r_0 S^{-1}(\omega), \quad (12)$$

being a frequency dependent reflection function which scales the data $\mathbf{P}^-(z_0)$ in such a way that it can be used as a proper prediction operator. By letting $R_0(\omega)$ vary in such a way that the multiples are eliminated in an optimal way, equation (11) can be used to eliminate the surface multiples and estimate the inverse source wavelet at the same time!
One problem left is how to decide whether the multiples have been perfectly eliminated. For this the total energy in the section after multiple elimination, $P_0^-(z_0)$, is used as measure. This can be intuitively understood by the fact that the free surface will return energy in the subsurface. If the free surface has been transformed in a reflection-free surface, all energy will vanish after being detected once, and a minimum energy is reached.
The wavelet is parametrized by a number of complex frequency points, through which the wavelet is splined in the frequency domain. In general about 5 to 10 definition points, giving twice as much variables (real and imaginary parts), is enough. By a standard optimization procedure the optimum wavelet can be found and the multiples are removed from the data.
It appears that this method is especially suited for estimating the phase of the (inverse) wavelet, because timing errors have a larger effect on the multiple residues than amplitude errors. Note the fact that the wavelet is necessarily estimated together with the scaling factor in the data.

Examples

Two synthetic examples will be shown. First consider the single reflector subsurface as shown in figure 2a. Data is modeled with a wavelet as shown in figure 2b giving the synthetic shot records as the one shown in figure 2c. After adaptive prestack multiple elimination with 9 definition points between 10 and 50 Hz. the estimated wavelet looks as shown in figure 3a with the multiple free shot record in figure 3b. The results are very good; note especially that the phase spectrum has been well estimated.
Figure 4a shows a laterally varying subsurface model. Finite difference data was simulated on this model with a 0 to 40 Hz zero phase square cosine wavelet, which is shown in figure 4b.

Figure 4c shows a synthetic shot record, with strong multiples hiding primary events. After adaptive multiple elimination with 5 definition points between 10 and 30 Hz. , the wavelet has been very well estimated (figure 5a) and the multiples have been well suppressed restoring the primary events (figure 5b).
More examples will be discussed during the presentation.

Remarks

The whole procedure has been derived for the acoustic case. But in principle the same procedure can be applied for the full elastic case. Then the data matrix $\mathbf{P}^{-}(z_0)$ contains the multi-component dataset after decomposition into P and S wave responses and the reflectivity matrix $\mathbf{R}^{-}(z_0)$ should contain the elastic reflectivity operators at the free surface. In that case both surface related multiples and conversions can be eliminated as was shown by Verschuur et al. (1988).
In the presence of noise the method can never 'explode' as the minimum energy criterion after multiple elimination will guarantee stabilization.

Conclusions

A method has been introduced to eliminate all surface related multiples without requiring any knowledge about the subsurface structure. As the source wavelet is needed, the scheme has been transformed into an adaptive one, estimating the wavelet. By minimizing the total energy of the upgoing wave field, a parametrized version of the wavelet can be found and the multiples are eliminated simultaneously.
The method shows excellent results on realistically simulated data..

References

Berkhout, A.J., 1982, Seismic migration: Imaging of acoustic energy by wave field extrapolation. A. Theoretical aspects, second edition: Elsevier Science Publ. Co., Inc.

Verschuur, D.J., Herrmann, P., Kinneging, N.A., Wapenaar, C.P.A. and Berkhout, A.J., 1988, Elimination of surface related multiple reflected and converted waves: presented at the 58th Ann. Internat. Mtg., Soc. Explor. Geophys.

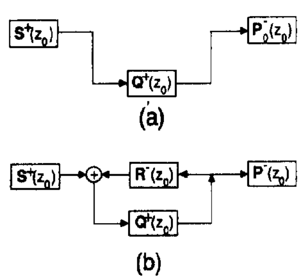

FIG. 1. (a) Block diagram representation of seismic data without surface-related multiples, and (b) with all surface-related multiples included.

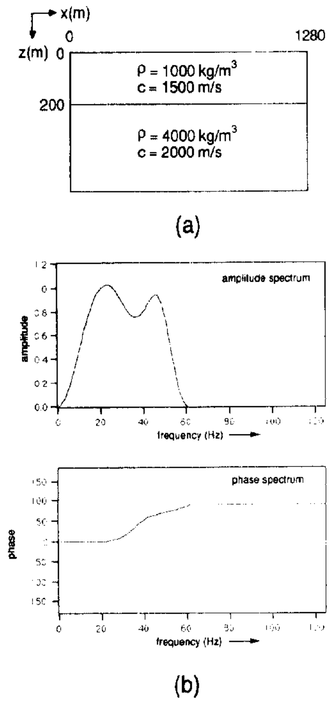

FIG. 2. (a) Subsurface model for synthetic shot record modeling. (b) Wavelet used for modeling simulated data.

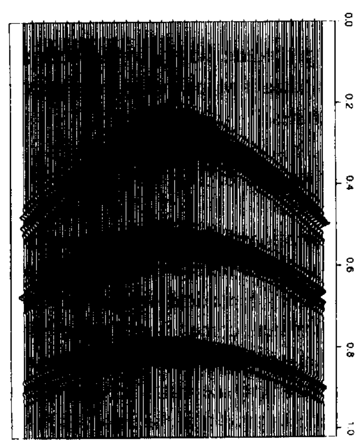

FIG. 2c. Simulated shot record.

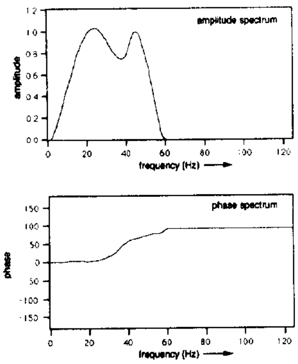

FIG. 3a. Wavelet after estimation procedure with 9 definition points.

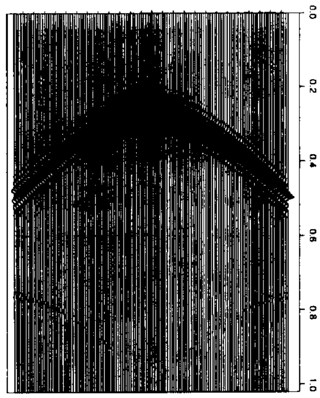

FIG. 3b. Shot record of Figure 2c after adaptive multiple elimination procedure.

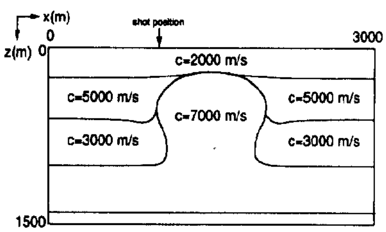

FIG. 4a. Subsurface model for finite-difference shot record modeling.

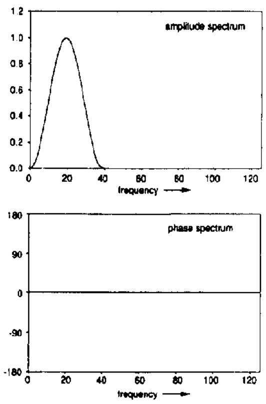

FIG. 4b. Wavelet used for modeling simulated data.

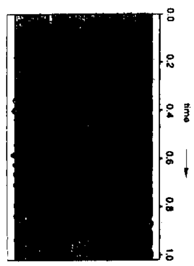

FIG. 4c. Shot record modeled with a finite-differrence modeling program.

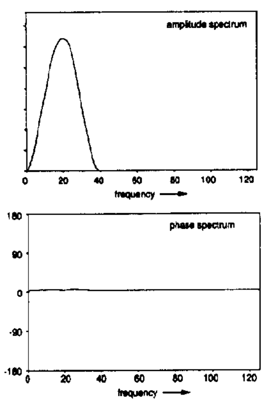

FIG. 5a. Wavelet after estimation procedure with 5 definition points.

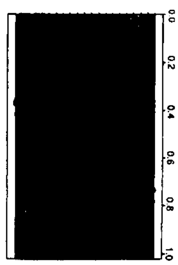

FIG. 5b. Shot record of Figure 4c after adaptive prestack multiple elimination procedure.