True Amplitude Inverse Wave-Field Extrapolation

C. G. M. de Bruin, C. P. A. Wapenaar, and A. J. Berkhout, Delft University of Technology, Netherlands

SUMMARY

Generally, the seismic industry has been interested more in correct phase (traveltimes) than in correct amplitude behaviour. For example, in seismic migration techniques one is already satisfied when the events in the data are correctly migrated to their proper positions. Gradually more effort is spent on inverse wave field extrapolation in which also the amplitude is correctly treated. Amplitude errors can be considerable in the presence of large contrasts in the subsurface (salt layers, seabottom etc.). These errors are in the order of neglecting (multiply) reflected waves (~R^2, with R the reflection coefficient of the relevant reflector).

In this paper we propose an iterative scheme for generating true amplitude wave field extrapolation operators taking into account also reflected waves. Any operator, either generated with Finite Differences, Raytracing, Wavenumber modeling or Gaussian beams can be iterated to preserve the amplitude as accurate as is required in the wave field extrapolation process. The iterative approach proves to be convergent and stable. An additional advantage is that at each iteration step a new order of internal multiples can be eliminated. The value of true amplitudes in exploration geophysics is clear: correct angle dependent amplitude information after migration enables us to determine detailed density and velocity in the subsurface.

INTRODUCTION

Wave field extrapolation plays an important role in migration (Berkhout, 1985), inverse scattering (Bleistein, 1984) and redatuming (Berryhill, 1984). We can either simulate (forward extrapolation) or backward-propagate (inverse extrapolation) wave fields at locations where no measurements are available. For the wave field extrapolation we have to apply wave field extrapolation operators which describe the medium in terms of macro structures. These so-called Green's functions represent spatial impulse responses of the medium.

Wapenaar et al. (1989) have shown that only in case of moderate contrasts the Kirchhoff integral with backward-propagating Green's functions describes 'true amplitude' inverse extrapolation of primary waves. The amplitude errors which are made in the primary waves are of the same order as the negligence of multiply reflected waves. We propose a stable and iterative method of generating true amplitude wave field extrapolation operators for media with high contrasts.

INVERSE WAVE FIELD EXTRAPOLATION OF PRIMARY WAVES

The wave field extrapolation technique is based on the Kirchhoff integral, which states that the wave field at any point inside a volume can be computed provided the wave field is known on a surface enclosing this volume. For the seismic situation we consider the configuration of Figure 1 with volume V enclosed by plane acquisition surface S_0, plane surface S_1 and cylindrical surface S_2 with radius r. The wave field is assumed to be radiated by secondary sources in the subsurface below S_1. In the inverse wave field extrapolation process we want to reconstruct the wave field in subsurface point A. The contribution from surface S_2 vanishes when radius r goes to infinity.

According to Wapenaar et al. (1989), in A the upgoing pressure $P^-(\mathbf{r}_A,\omega)$ can be written as (ω is the radial frequency, $\mathbf{r}$ is a short-hand notation for the Cartesian coordinates x,y,z):

$$P^-(\mathbf{r}_A,\omega) = P_0^-(\mathbf{r}_A,\omega) + \Delta P^-(\mathbf{r}_A,\omega), \tag{1a}$$

where $P_0^-(\mathbf{r}_A,\omega)$ is the contribution from the wave field recorded at the surface S_0, given by

$$P_0^-(\mathbf{r}_A,\omega) \approx 2\iint_{-\infty}^{\infty} \Big[\frac{1}{\rho(\mathbf{r})}\Big(\frac{\partial G^-(\mathbf{r},\mathbf{r}_A,\omega)}{\partial z}\Big)^* P^-(\mathbf{r},\omega)\Big]_{z_0} dxdy, \tag{1b}$$

and $\Delta P^-(\mathbf{r}_A,\omega)$ is the contribution from the scattered wave field (denoted by subscript s) at surface S_1, given by

$$\Delta P^-(\mathbf{r}_A,\omega) \approx -2\iint_{-\infty}^{\infty} \Big[\frac{1}{\rho(\mathbf{r})}\Big(\frac{\partial G_s^+(\mathbf{r},\mathbf{r}_A,\omega)}{\partial z}\Big)^* P_s^+(\mathbf{r},\omega)\Big]_{z_1} dxdy. \tag{1c}$$

Here $\rho(\mathbf{r})$ represents the mass density and the asterisk (*) denotes the backward-propagating character of the Green's functions. Equations (1b) and (1c) are one-way Rayleigh-type integrals for inverse extrapolation through inhomogeneous media, assuming that the total wave field at z_0 and z_1 may be split in upgoing (–) and downgoing (+) waves (see Figure 2).

The problem in practice is that measurements of P_s^+ at surface S_1 cannot be acquired; only registrations at S_0 are available. Note, however, that $\Delta P^-(\mathbf{r}_A,\omega)$ in (1c) is proportional to the product of the <u>scattered</u> wave field P_s^+ at z_1 and the <u>scattered</u> back-propagating Green's function $(G_s^+)^*$ at z_1. Hence, the magnitude of $\Delta P^-(\mathbf{r}_A,\omega)$ is proportional to multiply reflected waves, so it is two orders lower than the amplitude of the total upgoing wave field in A. In case of moderate contrasts $\Delta P^-(\mathbf{r}_A,\omega)$ can thus be neglected.

Since we are always dealing with discrete versions of the wave field in practice, Rayleigh II integral (1b) should be rewritten as a matrix equation:

$$\mathbf{P}^-(z_1) = \underset{\sim}{\mathbf{F}}^-(z_1,z_0)\,\mathbf{P}^-(z_0), \tag{2a}$$

with $\underset{\sim}{\mathbf{F}}^-(z_1,z_0)$ given by

$$\underset{\sim}{\mathbf{F}}^-(z_1,z_0) \approx \frac{2}{\rho}\left[\frac{\partial \underset{\sim}{\mathbf{G}}^-(z=z_0,z_1)}{\partial z}\right]^*. \tag{2b}$$

In matrix $\underset{\sim}{\mathbf{G}}^-(z=z_0,z_1)$ each row contains $G^-(x,z_0;x_A,z_1;\omega_i)$ for one x_A on z_1. Vector $\mathbf{P}^-(z_1)$ contains $P^-(x,z_1,\omega_i)$ and vector $\mathbf{P}^-(z_0)$ contains $P^-(x,z_0,\omega_i)$.

THE ITERATIVE APPROACH

Ideally, inverse extrapolation through an inhomogeneous medium should be performed with equation (1a), with P_0^- and P_1^- given by relations (1b) and (1c) respectively. In matrix notation this equation is transformed into

$$\mathbf{P}^-(z_1) = \underset{\sim}{\mathbf{F}}^-(z_1,z_0)\,\mathbf{P}^-(z_0) + \underset{\sim s}{\mathbf{F}}^+(z_1,z_1)\,\mathbf{P}_s^+(z_1), \tag{3a}$$

with $\underset{\sim}{\mathbf{F}}^-(z_1,z_0)$ given by equation (2b) and with

$$\underset{\sim s}{\mathbf{F}}^+(z_1,z_1) \approx -\frac{2}{\rho}\left[\frac{\partial \underset{\sim s}{\mathbf{G}}^+(z=z_1,z_1)}{\partial z}\right]^*. \tag{3b}$$

In matrix $\underset{\sim s}{\mathbf{G}}^+(z=z_1,z_1)$ each row contains the scattered Green's wave field $G_s^+(x,z_1;x_A,z_1;\omega_i)$ at z_1 for one Green's source at x_A on z_1. The $(G_s^+)^*$ operator describes backward-propagation from z_1 to the interface, reflection and backward-propagation from this interface to z_1. As already mentioned, the problem is that $\mathbf{P}_s^+$ in equation (3a) cannot be measured! The idea to solve this problem is as follows:

Model wave field $\mathbf{P}_{s,0}^+$ by means of the inverse extrapolated wave field $\mathbf{P}_0^-$; then inverse extrapolate $\mathbf{P}_{s,0}^+$ to update the inverse extrapolated $\mathbf{P}_0^-$ .With this newly computed inverse extrapolated wave field $\mathbf{P}_1^-$, we again model scattered wave field $\mathbf{P}_{s,1}^+$, and so on . . .

In this approach the zeroth order approximation is given by

$$\mathbf{P}_0^-(z_1) = \underset{\sim}{\mathbf{F}}^-(z_1,z_0)\,\mathbf{P}^-(z_0)\,. \tag{4}$$

Note that this equation is equal to equation (2). Simulating the reflected (scattered) wave field $\mathbf{P}_{s,0}^+$ with the zeroth order result is described by

$$\mathbf{P}_{s,0}^+(z_1) = \underset{\sim s}{\mathbf{W}}^+(z_1,z_1)\,\mathbf{P}_0^-(z_1)\,, \tag{5a}$$

with

$$\underset{\sim s}{\mathbf{W}}^+(z_1,z_1) \approx -\frac{2}{\rho}\left[\frac{\partial \underset{\sim s}{\mathbf{G}}^+(z=z_1,z_1)}{\partial z}\right]. \tag{5b}$$

Here forward operator $\underset{\sim s}{\mathbf{W}}^+(z_1,z_1)$ contains both propagation <u>and</u> reflection! The first order approximation (denoted by the subscript) is given by

$$\mathbf{P}_1^-(z_1) = \underset{\sim}{\mathbf{F}}^-(z_1,z_0)\,\mathbf{P}^-(z_0) + \underset{\sim s}{\mathbf{F}}^+(z_1,z_1)\,\mathbf{P}_{s,0}^+(z_1), \tag{6a}$$

or, subsequently substituting equation (4) in (5a) and the result in (6a), yields

$$\mathbf{P}_1^-(z_1) = \left[\,\underset{\sim}{\mathbf{I}} + \underset{\sim s}{\mathbf{F}}^+(z_1,z_1)\,\underset{\sim s}{\mathbf{W}}^+(z_1,z_1)\,\right]\underset{\sim}{\mathbf{F}}^-(z_1,z_0)\,\mathbf{P}^-(z_0). \tag{6b}$$

and this is equal to

$$\mathbf{P}_1^-(z_1) = \left[\,\underset{\sim}{\mathbf{I}} + \underset{\sim s}{\mathbf{F}}^+(z_1,z_1)\,\underset{\sim s}{\mathbf{W}}^+(z_1,z_1)\,\right]\mathbf{P}_0^-(z_1). \tag{6c}$$

As $\underset{\sim s}{\mathbf{F}}^+(z_1,z_1)$ and $\underset{\sim s}{\mathbf{W}}^+(z_1,z_1)$ are complex conjugates their product is real, positive and in the order of R^2. Hence, the first order approximation involves an amplitude increase with respect to the zeroth order approximation. In analogy with equation (6a) the second order approximation is given by

$$P_2^-(z_1) = \underset{\sim}{F}^-(z_1,z_0)\, P^-(z_0) + \underset{\sim s}{F}^+(z_1,z_1)\, P_{s,1}^+(z_1), \tag{7a}$$

where

$$P_{s,1}^+(z_1) = \underset{\sim s}{W}^+(z_1,z_1)\, P_1^-(z_1). \tag{7b}$$

The first order approximation which we obtained is now used for the second order approximation.

Summarizing, the iterative approach of obtaining the primary upgoing wave by taking also into account the scattered energy can for the n^{th} iteration be written as

$$P_n^-(z_1) = \underset{\sim}{F}_{TRUE}^-(z_1,z_0)\, P^-(z_0), \tag{8a}$$

with

$$\underset{\sim}{F}_{TRUE}^-(z_1,z_0) = \Big[\sum_{n=0}^{\infty} \big(\underset{\sim s}{F}^+(z_1,z_1)\, \underset{\sim s}{W}^+(z_1,z_1) \big)^n \Big]\, \underset{\sim}{F}^-(z_1,z_0). \tag{8b}$$

This operator preserves <u>true</u> amplitude information, even in the presence of large contrasts (evanescent energy excluded). The iterations are based on a series expansion of the inverse wave field extrapolation operator. It is very important to realize that in equation (8) the operator is iteratively obtained, <u>not</u> the extrapolation step itself!

The amplitude of the upgoing wave field in the n^{th} iteration is related to the exact amplitude at depth z_1, according to

$$|P_n^-(z_1)| = (1 - R^{2n+2})\, |P^-(z_1)|. \tag{9}$$

In case $|R| << 1$ one can still put up with the expression as in equation (2). But, in case there are strong contrasts ($|R| > 1/4$) like when encountering salt layers, seabottoms and limestone layers, this iterative approach gets very important. With this iterative scheme also internal multiples can be iteratively eliminated (provided that the macro model is correct!).

EXAMPLE

The validity of the iterative approach is demonstrated with a simple example. A two-layer medium was modeled with very strong contrasts including internal multiples (Figure 3). Figure 4a shows the upgoing wave field at z_0 to be inverse extrapolated back to z_1. The inverse extrapolated result at z_1 without applying iterations is depicted in Figure 4b; applying 5 iterations gives the result of Figure 4c. In Figure 5a-d the amplitude cross sections are depicted, showing the amplitude of the inverse extrapolated upgoing wave field (dotted line) and the amplitude of the exact upgoing wave field (solid line) at z_1, as a function of the trace number. In case we do not carry out any iterations, the discrepancy in the amplitude is considerable. With 5 iterations a true amplitude for the inversely extrapolated result has been achieved.

CONCLUSIONS

We have shown that it is possible to perform <u>true</u> amplitude inverse wave field extrapolation by modeling the extrapolation operators iteratively. The iterative procedure is stable and convergent. It has also been demonstrated that internal multiples can be optionally eliminated with each iteration step. Due to the one-way character of the wave field extrapolation process, the method is perfectly suited for inverse wave field extrapolation through high contrast media in practical situations.

Acknowledgement

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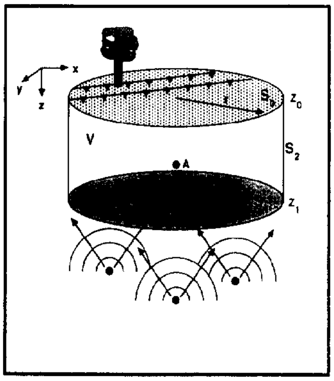

FIG. 1. Geometry for Kirchhoff integral in seismic situation; pressure in A is computed from measurements on surface of volume V.

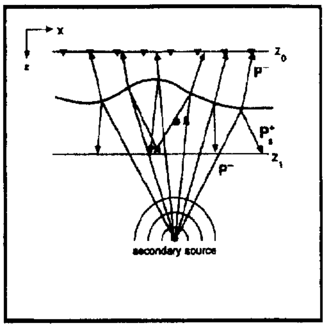

FIG. 2. Since scattered wave-field P_s^+ cannot be measured in practice, it is always neglected, thereby introducing second-order amplitude errors.

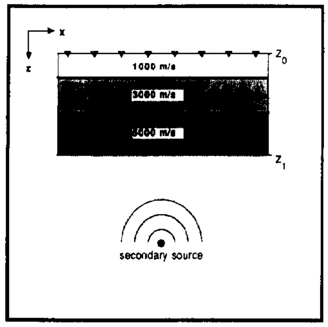

FIG. 3. Example shows principle of iterative approach; acoustic subsurface model containing two reflectors with strong velocity contrasts (density constant).

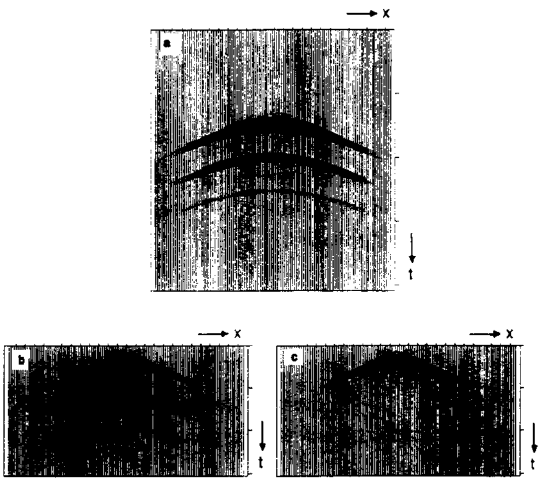

FIG. 4. (a) Wave field at z_0 to be inverse extrapolated; (b) inverse extrapolated wave field at z_1 applying 0 iterations; and (c) inverse extrapolated wave field at z_1 applying 5 iterations.

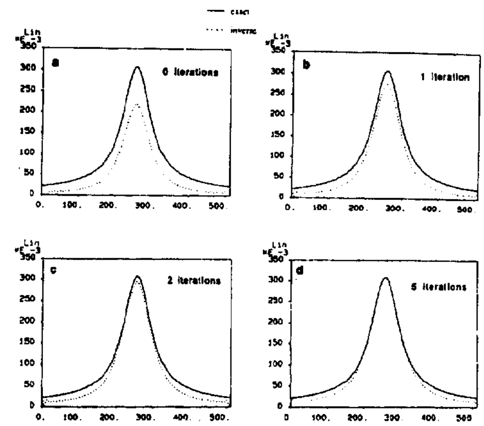

FIG. 5. Amplitude cross-sections as a function of trace number for inverse extrapolated result (dotted line) and the exact result (solid line). (a) With O iterations, amplitude shows a considerable discrepancy (30%); (b) with 1 iteration, discrepancy has largely been reduced; (c) with 2 iterations, amplitude has improved again; and (d) with 5 iterations, there is almost a perfect match. Discrepancy at large offsets is due to negligence of evanescent waves.