Surface-Related Multiple Elimination in the Presence of Near-Surface Anomalies

D. J. Verschuur, A. J. Berkhout, and C. P. A. Wapenaar, Delft University of Technology, Netherlands*

SUMMARY

In contrast to the "conventional" dereverberation methods, which only remove reverberations within the first layer, the surface related multiple elimination method removes *all* multiples related to the surface by simulating a zero surface reflection coefficient. The main characteristic of this method is the fact that the seismic data itself is used as multiple predicting operator; knowledge about the subsurface is not required.

This paper considers the situation of a complex surface reflectivity, caused by a thin surface layer (e.g. shallow water, weathering). To eliminate the full multiple generating effect of this complex surface, we propose to extend our method as follows. First the thin layer reverberations are removed at source and receiver side with "conventional" layer reverberation elimination by wave field extrapolation. Next, the surface related multiple elimination is applied, taking into account the complex reflectivity of the thin surface layer for upward travelling waves. With this procedure the influence of the first layer is fully removed from the seismic data.

INTRODUCTION

There is no need to introduce the fact that multiples can cause a lot of problems in seismic inversion and should preferably be removed in advance. But it is not always realized that the most important multiple generator is the earth's *surface* instead of the earth's first *layer*. Removing the influence of the free surface will not only remove reverberations in the first layer but also multiples between the surface and deeper interfaces.

A publication on the surface related multiple elimination method can be found in Kennett (1979) for 1-D media and Berkhout (1982) for laterally varying media using the matrix formulation. Verschuur et al (1989) showed some examples of an adaptive version of the 2-D version, simultaneously estimating the source wavelet. Figure 1a shows the effect of the surface related multiple elimination: removing the influence of the free surface and make the free surface fully transparent.

In this paper an extension of this method is made to deal with the situation of a thin surface layer with an anomalous (low or high) velocity. The reverberations are removed in advance before the surface related multiple elimination is applied. The extension to the method as described by Verschuur et al (1989) is the fact that now *all* multiples related to the free surface in combination with the multiples related to the shallow velocity contrast can be removed in stead of the surface related multiples only. Figure 1b shows the effect of this extension: the removal of the first layer influence.

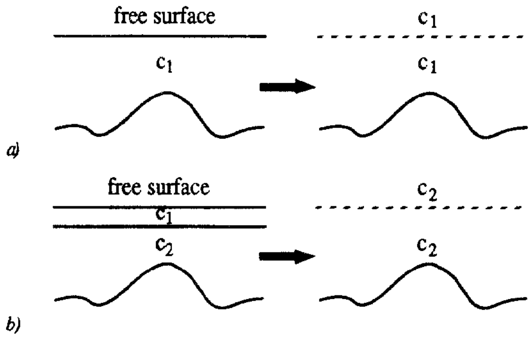

Figure 1

a) Effect of surface related multiple elimination: removal of the influence of the free surface. b) Effect of the extended multiple elimination method: removal of the influence of a thin surface layer.

NOTATION

In this paper the matrix notation will be used as introduced by Berkhout (1982). The traces of all shot records $p(x_i,x_j,z_0,t)$, i.e. wave fields with source position at x_j, receiver position at x_i measured as a function of time t; z_0 is the acquisition depth, are Fourier transformed to the frequency domain, yielding $P(x_i,x_j,z_0,\omega)$. In this domain all seismic prestack data is gathered per frequency component ω_k and put into a matrix, yielding $\mathbf{P}(z_0)$. Each matrix has the monochromatic common shot gathers in the columns and the monochromatic common receiver gathers in the rows. So each element P_{ij} of the data matrix $\mathbf{P}$ (i^{th} row, j^{th} column) defines the response of source position j at receiver position i for the frequency component ω_k.

FORWARD MODEL OF SEISMIC DATA

With the matrix notation as described in the previous section the seismic data without surface related multiples can be described as:

$$\mathbf{P}_0^-(z_0) = \mathbf{X}_0(z_0)\mathbf{S}^+(z_0)\,, \tag{1}$$

in which $\mathbf{P}_0^-(z_0)$ is the upgoing wave field at the free surface ($z=z_0$), $\mathbf{X}_0(z_0)$ is the response matrix of the subsurface without surface related multiples (so primaries and internal multiples only) and $\mathbf{S}^+(z_0)$ describes the downgoing source wave field. For ideal sources $\mathbf{S}^+(z_0)=S(\omega)\mathbf{I}$ with $S(\omega)$ the source wavelet. If we include the influence of the free surface any upgoing event will reflect against the free surface and will go down into the subsurface again. This reflection at z_0 from below is described by the reflection matrix $\mathbf{R}^-(z_0)$. For an acoustic free surface $\mathbf{R}^-(z_0)=-\mathbf{I}$. The reflected events against the free surface generate an additional downgoing "source" wave field. This gives for the total upgoing wave field $\mathbf{P}^-(z_0)$ (including surface related multiples):

$$\mathbf{P}^-(z_0) = \mathbf{X}_0(z_0)\,[\,\mathbf{S}^+(z_0) + \mathbf{R}^-(z_0)\mathbf{P}^-(z_0)\,]. \tag{2}$$

Rewriting this expression explicitly for $\mathbf{P}^-(z_0)$ yields

$$\mathbf{P}^-(z_0) = [\mathbf{I} - \mathbf{X}_0(z_0)\mathbf{R}^-(z_0)]^{-1}\,\mathbf{X}_0(z_0)\mathbf{S}^+(z_0), \tag{3a}$$

or, by expanding the inverse term as a Taylor series,

$$\mathbf{P}^-(z_0) = [\sum_{n=0}^{\infty}\{\mathbf{X}_0(z_0)\mathbf{R}^-(z_0)\}^n]\,\mathbf{X}_0(z_0)\mathbf{S}^+(z_0). \tag{3b}$$

Defining

$$\mathbf{X}(z_0) = [\mathbf{I} - \mathbf{X}_0(z_0)\mathbf{R}^-(z_0)]^{-1}\,\mathbf{X}_0(z_0)\,, \tag{4}$$

we arrive at an expression similar to equation (1):

$$\mathbf{P}^-(z_0) = \mathbf{X}(z_0)\mathbf{S}^+(z_0). \tag{5}$$

The difference between equation (1) and equation (5) is the influence of the free surface. If the surface is fully absorbing ($\mathbf{R}^-(z_0)=\mathbf{0}$) equation (5) reduces to equation (1) again.

Now a strong velocity contrast is introduced at the bottom of the first layer at $z=z_1(x,y)$, which does not need to be flat. Each propagation through the thin layer between z_0 and z_1 will add a train of reverberations to the wave field. The downgoing source wave field just below this thin layer can be described by (Figure 2a):

$$\mathbf{S}^+(z_1) = \mathbf{T}^+(z_1)\mathbf{W}^+(z_1,z_0)[\sum_{n=0}^{\infty}\{\mathbf{Q}^+(z_0)\}^n]\mathbf{S}^+(z_0), \tag{6a}$$

or

$$\mathbf{S}^+(z_1) = \mathbf{T}^+(z_1)\mathbf{W}^+(z_1,z_0)[\mathbf{I}-\mathbf{Q}^+(z_0)]^{-1}\,\mathbf{S}^+(z_0), \tag{6b}$$

with $\mathbf{T}^+(z_1)$ the downward transmission matrix for the interface at $z=z_1$, $\mathbf{W}^+(z_1,z_0)$ describing the downward propagation from depth level $z=z_0$ to $z=z_1$ and $\mathbf{Q}^+(z_0)$ describing one downward reverberation loop within this layer:

$$\mathbf{Q}^+(z_0) = \mathbf{R}^-(z_0)\mathbf{W}^-(z_0,z_1)\mathbf{R}^+(z_1)\mathbf{W}^+(z_1,z_0). \tag{7}$$

$\mathbf{R}^+(z_1)$ describes the reflection at $z=z_1$ from above and $\mathbf{W}^-(z_0,z_1)$ the upward propagation from depth level $z=z_1$ to $z=z_0$.

Similarly, each upgoing wave field $\mathbf{P}^-(z_1)$ reaching level $z=z_1$ from below will arrive at the free surface ($z=z_0$) as follows (Figure 2b):

$$\mathbf{P}^-(z_0) = [\sum_{n=0}^{\infty}\{\mathbf{Q}^-(z_0)\}^n]\mathbf{W}^-(z_0,z_1)\mathbf{T}^-(z_1)\mathbf{P}^-(z_1), \tag{8a}$$

or

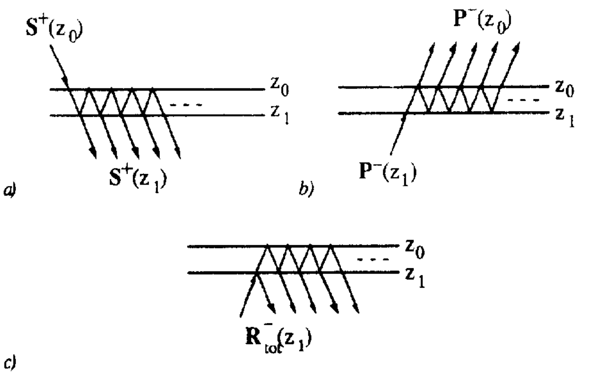

Figure 2
a) The downgoing source wave field below the first layer consists of the original source wave field with a train of first layer reverberations. b) The upgoing wave field at the free surface consists of the upgoing wave field below the first layer with a train of first layer reverberations. c) A wave reflected at the bottom of a thin layer from below will consist of a train of reverberations.

$$\mathbf{P}^-(z_0) = [\mathbf{I}-\mathbf{Q}^-(z_0)]^{-1}\,\mathbf{W}^-(z_0,z_1)\mathbf{T}^-(z_1)\mathbf{P}^-(z_1), \tag{8b}$$

with $\mathbf{Q}^-(z_0)$ describing one upward reverberation loop within this layer:

$$\mathbf{Q}^-(z_0) = \mathbf{W}^-(z_0,z_1)\mathbf{R}^+(z_1)\mathbf{W}^+(z_1,z_0)\mathbf{R}^-(z_0). \tag{9}$$

The total forward model of the data including the effects of this thin layer becomes

$$\mathbf{P}^-(z_0) = [\mathbf{I}-\mathbf{Q}^-(z_0)]^{-1}\,\mathbf{W}^-(z_0,z_1)\mathbf{T}^-(z_1)\,\mathbf{X}(z_1)\,\mathbf{T}^+(z_1)\mathbf{W}^+(z_1,z_0)[\mathbf{I}-\mathbf{Q}^+(z_0)]^{-1}\mathbf{S}^+(z_0), \tag{10}$$

with the response matrix related to level $z=z_1$, $\mathbf{X}(z_1)$, defined in analogy with (4) as

$$\mathbf{X}(z_1) = \{\mathbf{I} - \mathbf{X}_0(z_1)\mathbf{R}^-_{tot}(z_1)\}^{-1}\,\mathbf{X}_0(z_1)\,. \tag{11}$$

In this expression $\mathbf{X}_0(z_1)$ is the response matrix related to depth level $z=z_1$ without the influence of the thin layer and the free surface, and $\mathbf{R}^-_{tot}(z_1)$ is the *total* reflection at z_1 from below (see Figure 2c) which is defined as

$$\mathbf{R}^-_{tot}(z_1) = \mathbf{R}^-(z_1) + \mathbf{T}^+(z_1)\mathbf{W}^+(z_1,z_0)\mathbf{R}^-(z_0)[\mathbf{I}-\mathbf{Q}^-(z_0)]^{-1}\mathbf{W}^-(z_0,z_1)\mathbf{T}^-(z_1). \tag{12}$$

In forward model (10) the direct wave (including the reverberations in the first layer) is not included.

ELIMINATION OF SURFACE RELATED MULTIPLES

Both forward models (5) and (10) as described in the previous section can be inverted to get a description of the multiple elimination in the situation without and with a thin surface layer.

The inversion of the forward model as described in equations (4) and (5) yields:

$$X(z_0) = P^-(z_0)\{S^+(z_0)\}^{-1}, \quad (13)$$

$$X_0(z_0) = [\,I + X(z_0)R^-(z_0)\,]^{-1}\,X(z_0)\,, \quad (14a)$$

or by using the Taylor series expansion:

$$X_0(z_0) = \sum_{n=0}^{\infty}[-X(z_0)R^-(z_0)]^n\,X(z_0)\,. \quad (14b)$$

With this procedure all surface related multiples are eliminated. The only quantity we have to know to apply this method, besides the free surface reflection matrix $R^-(z_0)$, is the inverse source wave field matrix $\{S^+(z_0)\}^{-1}$ and nothing about the subsurface! As the inverse source wave field is not known in general this multiple elimination method can be applied in an adaptive way by minimizing the energy, estimating the inverse source wave field simultaneously! The advantage of writing this multiple elimination as a Taylor series expansion in equation (14b) is the fact that in this way a stabilization is guaranteed by taking only a limited number of terms into account.

Problems arise when we want to apply this multiple elimination method in the case of a shallow velocity contrast, e.g. a shallow water bottom. Because of missing near offsets too much angle information of the first layer primary is missing. Keep in mind that with the surface related multiple elimination the data itself is used to predict the multiples. Secondly, several important multiples (e.g. bounces within the second layer) are not removed with the free surface related processing only. Therefore a two-step approach is followed to handle this situation. First the ringing effects of the near surface layer at source and receiver side are removed and after that the surface related multiple elimination as described is applied.

The first step is inverting equation (10) to get rid of the thin layer reverberations:

$$X(z_1) = \{W^-(z_0,z_1)T^-(z_1)\}^{-1}[I-Q^-(z_0)]\,P^-(z_0)\{S^+(z_0)\}^{-1}$$
$$[I-Q^+(z_0)]\{T^+(z_1)W^+(z_1,z_0)\}^{-1}. \quad (15)$$

In fact equation (15) describes a redatuming from depth level $z=z_0$ to $z=z_1$. The elimination of the layer related reverberations in equation (15) is a similar approach as described by Berryhill and Kim (1986).

After this "preprocessing" step a surface related multiple elimination with respect to depth level $z=z_1$ can be applied like described in equation (14a) and (14b)

$$X_0(z_1) = [\,I + X(z_1)R^-_{tot}(z_1)\,]^{-1}\,X(z_1)\,, \quad (16a)$$

or

$$X_0(z_1) = \sum_{n=0}^{\infty}[-X(z_1)R^-_{tot}(z_1)\,]^n\,X(z_1). \quad (16b)$$

Note that for the "free surface" reflection matrix now the total reflectivity matrix of the first layer $R^-_{tot}(z_1)$ has to be used. The response of the thin layer itself should be removed from the data before applying this two step method (by k-f filtering or muting).

EXAMPLES

Consider the subsurface model of Figure 3a. Figure 3b shows a shot record of this subsurface model, modeled with an acoustic finite difference program. There are surface multiples blurring the primaries of the lowest reflectors. After applying the surface related multiple elimination, as described by equation (14b) the result is Figure 3c, which shows that the primaries of the lowest reflectors have been restored from interference with the multiples. Figure 4a shows the same model as in Figure 3a, but with a shallow water layer included. Figure 4b shows a modeled shot record. Note the large influence of this thin layer to the data, compared with Figure 3b, caused by the reverberations in this thin layer. Figure 4c shows the same shot record after the first multiple elimination step: the ringing effects of the small layer have been removed, but multiples related to the water bottom in combination with the free surface are still present. After applying the second step, using the interface related multiple elimination with a reflection matrix as defined in equation (12) all surface related multiples are removed (Figure 4d).

CONCLUSIONS

The surface related multiple elimination has already been proved to be a very good method. In the case of a shallow strong velocity contrast at the bottom of the first layer poor results can be expected in practice because of missing near offsets by which multiples cannot be predicted correctly, and by the fact that surface related multiple elimination would not eliminate enough multiples. Therefore the influence of the thin layer (ringing) is removed in advance, which means a redatuming to the thin layer bottom. After that the surface related multiple elimination can be applied to remove all multiples related to the combination of free surface/thin layer bottom. The examples show that this two-step approach is a very nice solution to this shallow velocity anomaly problem.

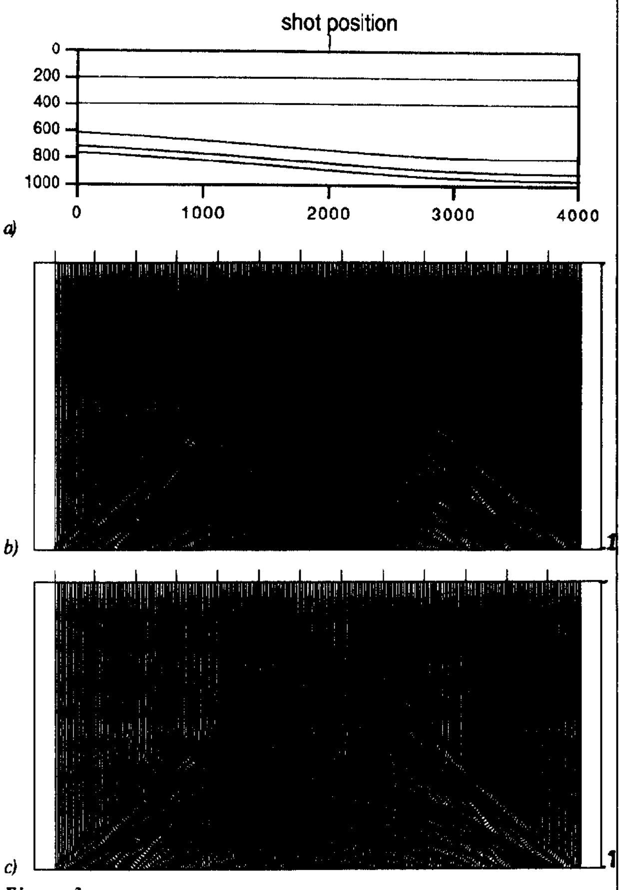

Figure 3

a) The subsurface model used for modeling simulated data with a finite difference modeling scheme. b) Shot record modeled with the model of a) with the source position indicated with the arrow. c) Shot record of b) after surface related multiple elimination.

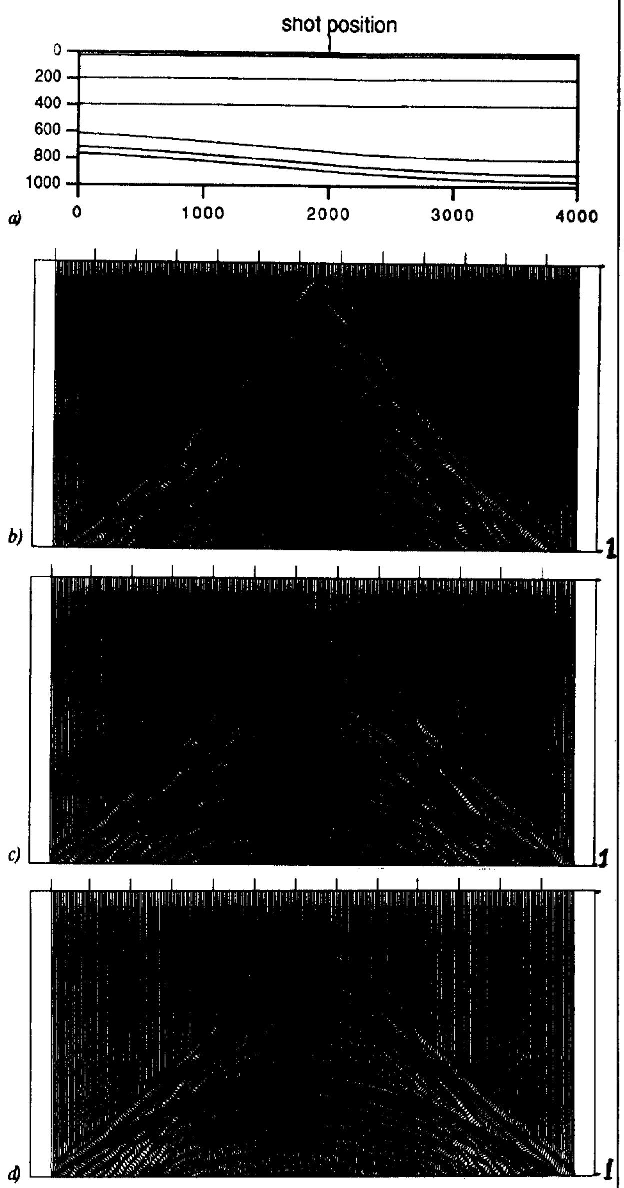

Figure 4

a) The same subsurface model as in Figure 3a), but with a shallow water layer inserted. b) Shot record modeled with the model of a) with the source position indicated with the arrow. c) Shot record of b) after the first multiple elimination step removing the water layer reverberations. d) Shot record after the second multiple elimination step removing all multiples related to the free surface in combination with the thin water layer.

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