D. J. Verschuur\*, A. J. Berkhout, and C. P. A. Wapenaar, Delft Univ. of Technology, Netherlands

# **Summary**

It is not always realized that the earth's surface is the main multiple generator. In the method described in this paper the influence of the (perfectly) reflecting free surface is removed from the seismic data by an inversion procedure. A significant advantage of our method is the fact that nothing about the subsurface needs to be known. The seismic data itself is used as multiple prediction operator. Therefore all propagation and reflection effects of the subsurface are automatically taken into account. On the other hand a source wavelet is needed to make a proper prediction of the multiples. By applying the method adaptively this problem can be solved. Hence, application of our multiple elimination method yields an estimate of the source wavelet as well. In addition the multiple free output is perfectly scaled.

## Introduction

Our surface-related multiple elimination method turns out to be a very attractive alternative for the multiple problem, especially in those situations were other methods fail, e.g. in situations with small or difficult to distinguish velocity differences between primaries and multiples, or in complex media where conventional methods do not suffice at all. Moreover, in the case of some strong subbottom reflectors, which give rise to relatively strong surface-related multiples (which are not water layer-related), the surface-related multiple elimination looks very effective as well.

The historical development of this method starts with Anstey (1967), who observed that with the autoconvolution of a trace primary events were transformed into multiples. Kennett (1979) described an inversion scheme in the  $k_x\text{-}\omega$  domain to eliminate multiples for a horizontally layered elastic medium. Berkhout (1982) redefined the multiple problem in laterally varying media by using a wave theory based matrix formulation. An adaptive version of Berkhout's approach has already

been discussed by Verschuur et al. (1989). In this paper the application on field data is examined, demonstrating the absolute necessity to include the adaptive capability.

### Theory

For the underlying equations the matrix notation, as introduced by Berkhout (1982), will be used. Bold symbols represent matrices, each column containing one Fourier component of a common shot gather as a function of offset. The upgoing wave field at the surface without surface-related multiples,  $P_0^-(z_0)$ , can be written as the spatial convolution of a source wave field and the subsurface response, in matrix notation:

$$\mathbf{P}_{0}^{-}(\mathbf{z}_{0}) = \mathbf{X}_{0}(\mathbf{z}_{0}, \mathbf{z}_{0}) \, \mathbf{S}^{+}(\mathbf{z}_{0}) \,, \tag{1}$$

in which one column of  $S^+(z_0)$  describes the downgoing source wave field at the surface level  $z_0$  (including its directivity pattern) and one related column of  $X_0(z_0,z_0)$  contains the subsurface response. Note that the + stands for downgoing and — for upgoing wave fields. Note also that equation (1) is a description for one frequency component, and should be repeated for all other frequency components of interest. Note also that the description in equation (1) is multi-shot, as each column in the matrix  $P_0^-(z_0)$  contains the response for one shot record.

Fig. 1a describes this equation in a diagram. If the reflectivity effect of the surface is included (which is

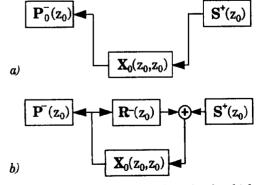


Fig. 1 a) Seismic data without surface-related multiples.
b) Seismic data with surface-related multiples.

shown in Fig. 1b), the total downgoing wave field is not only the source wave field  $S^+(z_0)$  but also includes the upgoing wave field reflected down by the surface, yielding:

$$\mathbf{P}^{-}(\mathbf{z}_{0}) = \mathbf{X}_{0}(\mathbf{z}_{0}, \mathbf{z}_{0}) \left[ \mathbf{S}^{+}(\mathbf{z}_{0}) + \mathbf{R}^{-}(\mathbf{z}_{0}) \mathbf{P}^{-}(\mathbf{z}_{0}) \right], \tag{2}$$

in which  $P^-(z_0)$  contains the upgoing wave field at the surface with all the surface-related multiples included and  $R^-(z_0)$  describes the free surface reflectivity. In the marine case we may assume  $R^-(z_0) = r_0 I = I$ . If equation (2) is rewritten explicitly we arrive at:

$$\mathbf{P}^{-}(\mathbf{z}_{0}) = \left[ \mathbf{I} \cdot \mathbf{r}_{0} \, \mathbf{X}_{0}(\mathbf{z}_{0}, \mathbf{z}_{0}) \, \right]^{-1} \, \mathbf{X}_{0}(\mathbf{z}_{0}, \mathbf{z}_{0}) \, \mathbf{S}^{+}(\mathbf{z}_{0}) \,. \tag{3}$$

For multiple elimination, equation (3) has to be written explicitly for  $X_0(z_0,z_0)$ :

$$X_0(z_0,z_0) = P^-(z_0)\{S^+(z_0)\}^{-1}$$

$$[I + r_0 P^-(z_0) \{S^+(z_0)\}^{-1}]^{-1}$$
. (4)

If we assume that the source directivity is corrected for in advance, the remaining source matrix can be written as an identity matrix with the wavelet:  $S^+(z_0) = I S(\omega)$ . Using this and equation (1), equation (4) can be expanded into the Taylor series:

$$\mathbf{P}_{0}^{-}(\mathbf{z}_{0}) = \mathbf{P}^{-}(\mathbf{z}_{0}) - \mathbf{A}(\omega) \left\{ \mathbf{P}^{-}(\mathbf{z}_{0}) \right\}^{2} + \mathbf{A}^{2}(\omega) \left\{ \mathbf{P}^{-}(\mathbf{z}_{0}) \right\}^{3} \cdot \dots$$
 (5)

with surface factor  $A(\omega) = r_0 \{S(\omega)\}^{-1}$ . The wavelet, and therefore the surface factor  $A(\omega)$ , is not known in general. Therefore, for a successful multiple elimination procedure, equation (5) should be applied adaptively: by minimizing the total energy in the data after multiple elimination the surface factor  $A(\omega)$  is estimated, yielding the inverse of the wavelet together with an appropriate scaling factor. Next, the multiples are removed with the aid of equation (5).

As a final remark to the method we would like to stress the fact that before the multiple elimination procedure is applied, the measured shot records should be decomposed into the *upgoing* pressure wave field at the surface. For marine data the *total* pressure at a depth level below the surface is measured. The decomposition implies a deghosting procedure which can be carried out in the  $k_x$ - $\omega$  domain. For land data the measured particle velocity should be converted into upgoing tractions, which can be accomplished by an appropriate scaling in the  $k_x$ - $\omega$  domain.

## Example on field data

A seismic line from the North Sea has been processed with this multiple elimination procedure. Fig. 2 shows the result for one shot record, with the data before and after multiple elimination and the difference between them. Comparing Fig. 2a and b the enormous reduction of multiples is visible and from the difference plot in Fig. 2c it is clear that the removed events are correlated events indeed. Note also the restored primaries in Fig. 2b, e.g. at 2.4 and 2.7 s (at arrows). Fig. 2 shows the corresponding velocity panel, which shows that primary and multiple events have been effectively separated. However, for the multiple elimination procedure velocity information has not been used, as the method does not require any information about the subsurface!

The wavelet that has been estimated with this adaptive multiple elimination procedure is shown in Fig. 4

Fig. 5 shows a common offset section (600 m offset) of the data before multiple elimination and Fig. 6 the same common offset section extracted from the data after multiple elimination. The reduction of multiples is clearly visible. Note the small synclinal structure around shot position 81 in Fig. 5, which produces a focusing of multiple energy, which has been properly removed in Fig. 6. Note also the remarkable difference of multiple energy density going from left to right in the section. Our multiple elimination method could fully cope with those lateral changes.

We believe that for a successful analysis of *prestack* data, surface-related multiple elimination is a pre-requisite.

## **Acknowledgment**

The authors would like to thank the Dutch Technology Foundation (S.T.W.) and the DELPHI sponsors for their support and SAGA Petroleum A.S. for providing the field data.

#### References

Anctey, N.A. and Newman, P., 1967, Part I: The sectional auto-correlogram and part II: The sectional retro-correlogram, Geophysical Prospecting, Volume 14, 391-426.

Berkhout, A.J., 1982, Seismic migration: Imaging of acoustic energy by wave field extrapolation. A. Theoretical aspects, second edition: Elsevier Science Publ. Co., Inc.

Kennett, B.L.N., 1979, The suppression of surface multiples on seismic records, Geophysical Prospecting, Volume 27, 584-600.

Verschuur, D.J., Berkhout, A.J., and Wapenaar, C.P.A., 1989, Wavelet estimation by prestack multiple elimination: presented in Dallas at the 59th Ann. Internat. Mtg., Soc. Explor. Geophys.

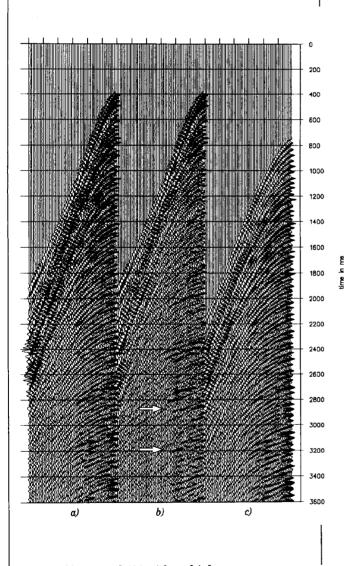


Fig. 2 a) Shot record 180 with multiples.

b) Shot record 180 after surface-related multiple elimination.
c) Difference between a) and b) i.e. the eliminated multiples only.
(Courtesy SAGA Petroleum A.S.).

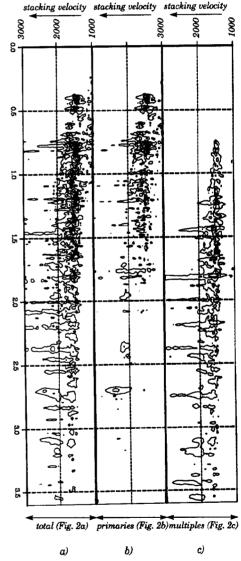
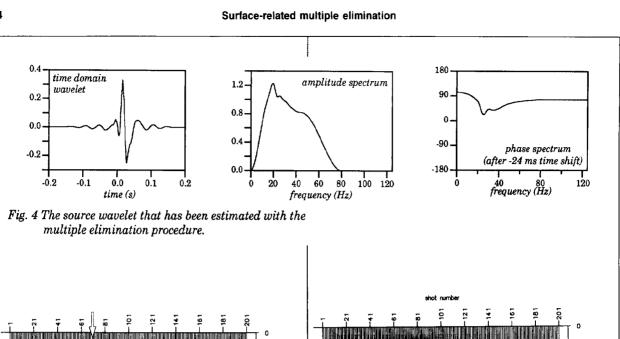


Fig. 3 Velocity panels corresponding with the CMP gathers of the shot record as shown in Fig. 2.

a) Data with multiples.

- b) Data after multiple elimination.
- c) Difference between data before and after multiple elimination.



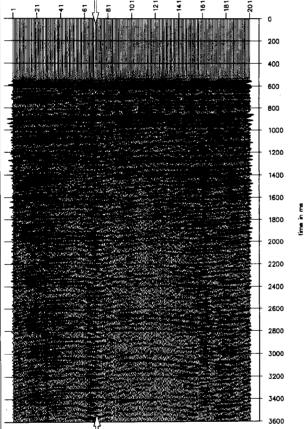


Fig. 5 Common offset section (600 m) with multiples. Note the significant lateral changes in multiple energy, e.g. at shot number 70.

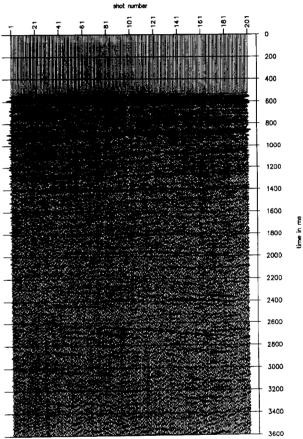


Fig. 6 Common offset section (600 m) after multiple elimination. Note that despite the significant lateral changes in multiple eenergy, the method is very effective.