Wavefield Extrapolation in the Wavenumber Domain, Taking Lateral Variations into Account

F. J. Dessing" and C.P.A. Wapenaar, Delft University of Technology, Netherlands

Summary

The bandwidth of wave field extrapolation operators in the double wavenumber domain is directly related to the complexity of the medium through which the propagation is described. For simple or moderately complex macro models (generally with lateral variations) this implies that non-recursive forward or inverse extrapolation can be done much more efficiently in the wavenumber domain than in the space domain, without loosing accuracy.

Introduction

The forward model of seismic data that is used in our seismic research project DELPHI is based on a number of matrix multiplications (space-frequency domain formulation). In its simplest form (i.e. after pre-processing) it is given by

$$\mathbf{P}^- = [\mathbf{W}^-\mathbf{R}\mathbf{W}^+]\mathbf{P}^+ \tag{1}$$

where $\mathbf{P}^+$ contains the downgoing (source) wave fields at the surface, $\mathbf{W}^+$ describes downward propagation into the subsurface, R represents reflection in the subsurface, W describes upward propagation to the surface and, finally, $\mathbf{P}^-$ contains the upgoing wave fields at the surface.

Equation (1) is a monochromatic representation of the seismic data (i.e., one frequency component); the different elements in the matrices correspond to different spatial positions. In this paper we investigate the possibility of processing in the *double wavenumber* domain taking horizontal as well as vertical variations of the subsurface parameters into account (for reference, Wapenaar 1992). We will show that without loss of generality equation (1) (or its extended version) may be transformed to the wavenumber-frequency domain, yielding

$$\tilde{\mathbf{P}}^- = [\tilde{\mathbf{W}}^-\tilde{\mathbf{R}}\tilde{\mathbf{W}}^+]\tilde{\mathbf{P}}^+ \tag{2}$$

where the tilde denotes the transformed domain.

Equation (2) is again a monochromatic representation of the seismic data; the different elements in the matrices correspond to different wavenumbers. The theory underlying this equation will be briefly reviewed. It will be shown that the structure of the matrices depends largely on the complexity of the subsurface. In the special case of a laterally invariant subsurface the matrices are diagonal matrices which means that the processing (based on the inversion of this equation) can be done very efficiently. In the case of moderate lateral variations the matrices exhibit a narrow band structure and in the case of strong lateral variations the matrices are full

matrices. In other words, the "bandwidth" of the matrices in equation (2) is proportional to the magnitude of the lateral variations of the subsurface parameters.

With the help of an example for a moderately complex macro model it will be shown that processing (here: non-recursive inverse wave field extrapolation) is much more efficient in the double wavenumber domain than in the space domain.

Extrapolation in the space domain

Consider a 2-D monochromatic downgoing acoustic wave field $P^+(x, z_0, \omega)$, registered as a function of the horizontal coordinate x at depth level z_0 and frequency ω. Downward extrapolation from depth level z_0 to depth level z_m is mathematically described by the generalized convolution integral (Berkhout, 1985)

$$P^+(x', z_m, \omega) = \int_{-\infty}^{\infty} W^+(x', z_m | x, z_0, \omega) P^+(x, z_0, \omega)\, dx . \tag{3}$$

Here $P^+(x', z_m, \omega)$ represents the extrapolated downgoing wave field at (x', z_m). The extrapolation operator $W^+(x', z_m | x, z_0, \omega)$ may be seen as the downgoing response at (x', z_m) of a dipole at (x, z_0).

For discretized wave fields (discretization interval Ax) equation (3) can be rewritten as a matrix equation, according to

$$\vec{\mathbf{P}}^+(z_m) = \mathbf{W}^+(z_m | z_0)\, \vec{\mathbf{P}}^+(z_0) . \tag{4}$$

Here $\vec{\mathbf{P}}^+(z_0)$ and $\vec{\mathbf{P}}^+(z_m)$ are column vectors, containing the (monochromatic) discretized wave fields at depth levels z_0 and z_m, respectively. Matrix $\mathbf{W}^+$ is explained in Fig. 1.

Spatial Fourier transforms

We define the spatial Fourier transform (the plane wave decomposition) of the wave field $P^+(x, z_0, \omega)$ by

$$\tilde{P}^+(k_x, z_0, \omega) = \int_{-\infty}^{\infty} e^{jk_x x} P^+(x, z_0, \omega)\, dx \tag{5a}$$

and its inverse by

$$P^+(x, z_0, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-jk_x x} \tilde{P}^+(k_x, z_0, \omega)\, dk_x . \tag{5b}$$

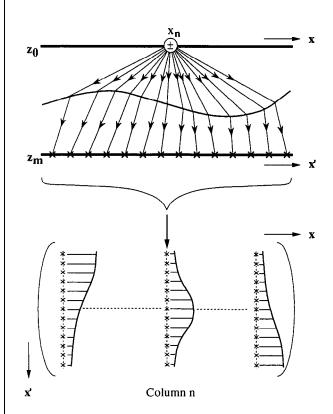Fig.1 The extrapolation matrix $\mathbf{W}^+\left(z_m|z_0\right)$; column n contains the (monochromatic) discretized response $\Delta x\, W^+\left(x', z_m|\, x_n, z_0, \omega\right)$ as a function of x' at depth z_m for a dipole source at $\left(x_n, z_0\right)$.

For discretized wave fields we may use a similar matrix notation as in the previous section:

$$\tilde{\vec{P}}^+ (z_0) = \Gamma \vec{P}^+ (z_0) \text{ and } \vec{P}^+ (z_0) = \Gamma^{-1}\tilde{\vec{P}}^+ (z_0).\qquad(6)$$

Here $\tilde{\vec{P}}^+ (z_0)$ is a column vector, containing the discretized wave field as a function of k_x, Γ is the discretized Fourier transform operator. It can be scaled such that $\Gamma^{-1} = \Gamma^H$.

Extrapolation in the wavenumber domain

The operator for wave field extrapolation in the double wavenumber domain is now easily derived. Applying the Fourier transform operator Γ to both sides of equation (4) gives

$$\Gamma\vec{P}^+ (z_m) = \Gamma \mathbf{W}^+ (z_m|z_0)\, \vec{P}^+ (z_0)\qquad(7)$$

Using equations (6) yields

$$\tilde{\vec{P}}^+ (z_m) = \tilde{\mathbf{W}}^+ (z_m|z_0)\, \tilde{\vec{P}}^+ (z_0),\qquad(8a)$$

where the transformed extrapolation operator reads

$$\tilde{\mathbf{W}}^+ (z_m|z_0) = \Gamma\, \mathbf{W}^+ (z_m|z_0)\, \Gamma^H.\qquad(8b)$$

The interpretation of matrix $\tilde{\mathbf{W}}^+ (z_m|z_0)$ is shown in Fig. 2. The "bandwidth" of this matrix is proportional to the complexity of the macro model. In general it is smaller than in the space domain (Fig.1). For the special case of a homogeneous macro model this matrix reduces to a diagonal matrix, with the phase shift operator of Gazdag (1978) on its diagonal.

Application

Introduction

Now that we have the theoretical framework we can investigate the advantages of processing in the double wavenumber domain. This will be done by inversely extrapolating the upgoing response at z_0 of a plane wave source at z_m (the upgoing response
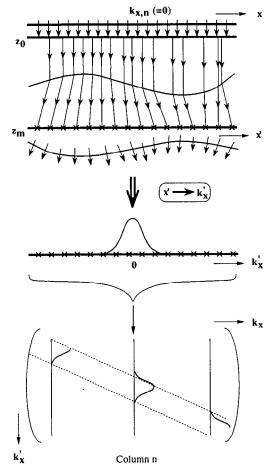

Fig. 2 The propagation matrix $\tilde{\mathbf{W}}^+ (z_m|z_0)$. Column n contains the (monochromatic) Fourier transformed response as a function of k_x' at depth z_m for a plane wave source with. wave number $k_{x,n}$ at depth z_0.

at z_0 will be called the test data). inverse extrapolation is described by

$$\vec{\tilde{P}}^- (z_m) = \mathbf{F}^- (z_m| z_0) \vec{\tilde{P}}^- (z_0) \text{ (in the space domain) and by (9)}$$

$$\tilde{\vec{P}}^- (z_m) = \tilde{\mathbf{F}}^- (z_m| z_0) \tilde{\vec{P}}^- (z_0) \text{ (in the wavenumber domain),(10)}$$

where the modified matched filter approach has been used such that

$$\mathbf{F}^- = [\mathbf{W}^+]^* \qquad \text{and} \qquad \tilde{\mathbf{F}}^- = \mathbf{\Gamma} [\mathbf{W}^+]^* \mathbf{\Gamma}^H . \qquad (11)$$

The model

A moderately complex macro model has been used (Fig. 3). developed by Rietveld et al (1992). In this model a plane wave source has been placed at $z_m = 800$ m.
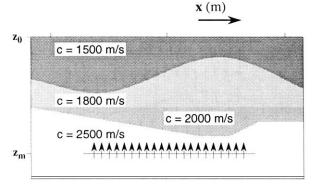

x (m)

z_0

c = 1500 m/s

c = 1800 m/s

c = 2000 m/s

c = 2500 m/s

z_m

Fig. 3 The moderately complex macro model together with the plane wave source at $z_m = 800$ m. The response at z_0 is shown in Fig. 6

The propagation operators

With a one-way recursive forward modeling scheme (Blacquiere 1989) the non-recursive $\mathbf{W}^+$ operator has been created for the model of Fig. 3. $\mathbf{W}^+$ describes the forward propagation from $z_0 = 0$ m to $z_m = 800$ m. $\mathbf{W}^+$ and one column of W+ are displayed in Fig. 4. By applying equation (8b) the operator $\tilde{\mathbf{W}}^+$ has been compiled. From Fig. 5 it is clear that the bandwidth of $\tilde{\mathbf{W}}^+$ is much smaller than that of $\mathbf{W}^+$ (Fig. 4).

Inverse extrapolation (no filtering)

The test data (the response at $z_0 = 0$ m of a finite plane wave source at $z_m = 800$ m) have been modeled by a forward finite-difference modeling of a horizontal plane wave source at z_m (Fig. 3). In Fig. 6 (left) the (x, t)-registration at z_0 is displayed. inverse extrapolation with $\tilde{\mathbf{F}}^-$ (based on the Fourier transformed modified matched filter operator according to equation (11)) results in the (x. t)-reg. istration of Fig. 6 (right). Inverse extrapolation with $\mathbf{F}^-$ yields exactly the same results (these are therefore not displayed). The
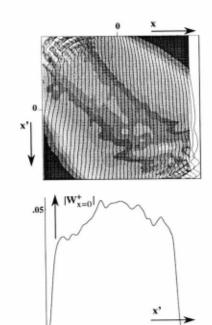

$|\mathbf{W}^+_{x=0}|$

.05

x'

-762   0   762

Fig. 4 The propagation operator $\mathbf{W}^+ (z_m| z_0)$ for the moderately complex macro model of Fig. 3. The dipole source response for a single source at x = 0 is extracted from the matrix.

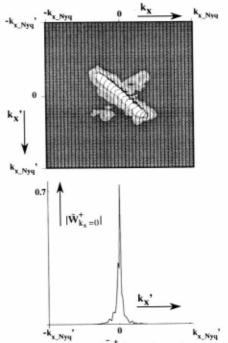$-k_{x\_Nyq}'$ $-k_{x\_Nyq}$ 0 k_x k_{x_Nyq}

k_x'

k_{x_Nyq}

0.7

$|\tilde{\mathbf{W}}^+_{k_x=0}|$

$-k_{x\_Nyq}$ 0 k_{x_Nyq}'

Fig. 5 The propagation matrix $\tilde{\mathbf{W}}^+ (z_m| z_0)$ for the model of Fig. 3. The transformed plane wave response for a plane wave source with $k_x=0$ (one column of the matrix) clearly shows the small bandwidth.
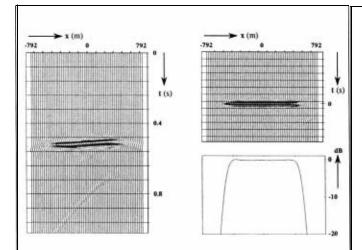
Fig. 6 *Left: The response at z_0 of a plane wave source at $z_m = 800$ m in the model of Fig. 3; Right: the inversely extrapolated wave field with the full $\tilde{F}$. The alignment at t = 0 s is very good and the amplitude cross section is almost perfectly flat.*
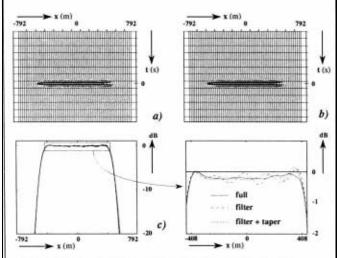

Fig. 7 *a) The inversely extrapolated wave field of Fig. 6 obtained by applying a band filtered version of $\tilde{F}$, leaving only 15 diagonals symmetric around the main diagonal; b) As in a), but after applying a taper to the outer diagonals of those 15 diagonals. c) The amplitude cross section of the extrapolated wave fields with the full $\tilde{F}$ operator (as in Fig. 6) and the band filtered versions $\tilde{F}_{bf}$ confirm that the useful information in the operators is centered around the main diagonals of $\tilde{F}$.*

information is aligned very well at t = 0 s and the amplitude cross section shows a constant amplitude along the plane wave source and **a** fast decay to zero outside the source area.

Filtering

Application of $\mathbf{F}_{bf}$, a band filtered version of the $\tilde{\mathbf{F}}$ operator, with only 7 diagonals left on both sides of the main diagonal (i.e.. in total 15 diagonals left), leads to the results of Fig. la. Applying a taper to 3 diagonals on both sides gives the inversely extrapolated wave field of Fig. 7b. The alignment at t = 0 s is perfect; this means that there is no phase error in the wavenumber domain. There is a slight error in the amplitudes (Fig. 7c). The tapered version gives smaller amplitude errors than the one without taper. From this experiment we can conclude that, for this heterogeneous macro model (Fig. 3). we roughly need a tenth of the operator matrix in the wavenumber domain (opposed to the full operator matrix in the space domain).

Conclusion

We have shown that non-recursive wave field extrapolation in moderately complex macro models is ideally done in the wavenumber domain instead of in the space domain. This is because of the fact that the double Fourier transformed operator for wave field extrapolation exhibits a narrow band structure which makes processing very efficiently, without loosing accuracy.

Acknowledgment

References

Berkhout, A.J., 1985, Seismic Migration: Imaging of acoustic energy by wave field extrapolation. A. Theoretical Aspects, Elsevier Science Publ. Co., 3rd edition.

Blacquiere, G., 1989. 3D Wave Field Extrapolation in Seismic Depth Migration, Ph. D. Thesis. Delft University of Technology

Gazdag, J.. 1978, Wave equation migration with the phase-shift method. Geophysics 43. 1342-1351

Rietveld. W.E.A.. Berkhout, A.J.. and Wapenaar, C.P.A., 1992, Optimum seismic illumination of hydrocarbon reservoirs. Geophysics 57, 1334-1345

Wapenaar, C.P.A., 1992, Wave equation based seismic processing: in which domain?. 54th EAEG meeting, Paris, Expanded Abstracts, 116-117