Wave Propagation in Finely Layered Media: A Parametric Approach

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Summary

When a wavefield propagates through the earth's subsurface its initial waveform gradually changes due to the presence fine-layering. This change in shape is not captured by the *conventional* macro model parametrizations which are based on the trend (compaction properties) in and throughout the stratigraphic sections. The main objective of this paper is to account for these *dispersive* propagation effects in a *global* way, i.e. defining an extended macro model parametrization.

Introduction

One of the main objectives in the study of wave propagation in finely layered media is the definition of a *replacement* medium that characterizes the propagation properties due to the fine-layering in a *global* way, i.e. without requiring *specific* information on the individual layers. The different techniques to compose such a replacement medium all have in common that they are based on a certain *spatial* averaging process of the medium fluctuations. This notion can be understood by reconciling the fact that wave propagation in finely layered media can be seen as the superposition of two driving mechanisms. These mechanisms comprise of a *spatial* averaging process acting on the *signature*, i.e. the *coherent* part of the *generalized* primary [Resnick 1985, Herrmann 1992, Burridge 1989] and a gradual shift of primary energy into the *coda*, i.e. the irregular part of the response with all the long delayed reverberations (peg-leg multiples). The latter process reflects the apparent randomness of the fine-layering sequence, i.e. the *specific* medium properties.

Finding a quantitative description for the distortion of the signature of the probing seismic signal is of primary importance in our research efforts. It was shown by Herrmann (1992) that the dispersive effects occurring in an *acoustic* medium can be accounted for by an extended macro model parametrization and this paper forms the first step in the generalization of the proposed concept to the *elastic non normal incidence* case.

The two main classes of averaging techniques will be discussed first, followed by a proposal for the *stochastic* parametrization of an *elastic* finely layered medium. Subsequently a preliminary analysis is performed to evaluate the sensitivity of the generalized primary to changes in the proposed *stochastic* parameters in which the *elastic* medium is defined.

Equivalent medium theory versus parametric approach

Two different theories, describing wave propagation in highly discontinuous media, have emerged during the last three decades. The first approach is based on the assumption that the layer thicknesses are infinitely small compared to the wavelength of the seismic signal. This implies that the phenomena are studied in the *static limit,* i.e. $\omega \rightarrow 0$. With such an approach an equivalent TI (transversely isotropic) medium can be composed which captures the elastic effects due to the fine-layering (when one adheres to the "empirical" restriction of at least $\pm$ 10 TI layers per wavelength). These TI parameters are computed by means of a comprehensive and fast *spatial* averaging technique [Folstad 1992] resulting in a reduction of the number of layers.

In the second approach the cross-/autocovariance function of the elastic medium contrasts plays a central role. This formulation was first initiated by O'Doherty-Anstey (197l), followed by Resnick [1986] and Burridge [1993] and provides a description for the generalized primary, in terms of the *sample* cross-/autocovariance function of the reflection coefficients.

The primary aim of this research project is to construct a *parametric* medium representation defined by only a few *global* parameters characterizing the *effects* of the fine-layering, i.e. the definition of an extended macro model parametrization [Herrmann 1992].

Both the equivalent medium theory and the O'Doherty-Anstey equation can be applied to define the required *parametric* replacement medium. For the effective medium this implies that the *actual* medium has to be replaced by only a few homogeneous TI layers (macro layers) and this is obviously violating the restriction of $\pm$ 10 layers per wavelength. This violation manifests itself by the fact that the *anisotropic* dispersion *(angle-* and *frequency* dependent time delay and amplitude attenuation) of the propagating wavefield and the presence of the coda gradually disappear as the number of TI layers decreases.

For the O'Doherty-Anstey equation, on the other hand, the *parametric* approach amounts to the parametrization of the cross-covariance function of the elastic medium contrasts, i.e. replacing the *sample* covariance function by a parametrization of the *spatial* averaged covariance function. This replacement is allowed because of the *spatial* averaging process which acts on the signature. The advantage of this approach is that it is valid beyond the *static* limit so that the *dispersion* is described accurately.

In the previous work conducted in this field [Herrmann 1992] it can be observed that the effects of the fine-layering are substantial (even for the acoustic normal incidence case) and comprise of both *angle* and *frequency* dependent attenuation and time delay. It was shown that these effects (for the normal incidence case) can be accounted for by the introduction of an *extended* macro model, defined in *global* stochastic parameters describing the *acoustic* medium for each *litho-stratigraphic* interval. In fact, these stochastic parameters correspond to the quantities that characterize fractal Brownian motion (fBm, medium fluctuations) or its derivative fractal Gaussian noise (fGn, medium contrasts) [Mandelbrot 19821. A similar approach will be followed for the ***elastic*** situation.

The elastic model

The transition from the ***acoustic*** stochastic subsurface model to an ***elastic*** model introduces an extra complication due to the fact that the problem becomes multivariate. With other words, the correlation between the *compressional* and the *shear* wave speeds and the correlation of these wave speeds with the density has to be taken into account. An ***elastic*** medium can be characterized by the vector

$$\vec{\kappa}(z) = \begin{bmatrix} \rho(z) \\ c_p(z) \\ c_s(z) \end{bmatrix}, \qquad (1)$$

where p(z) is the depth dependent density, $c_p(z)$ the compressional wave speed and c,(z) the shear wave speed. Fluctuations in these parameters are defined by $\Delta\vec{\kappa}(z) = \vec{\kappa}(z) - \vec{\mu}_\kappa$ with $\vec{\mu}_\kappa$ the mean of ?(z) . The cross-covariance function of these fluctuations - in which entry C_{ij} denotes the cross-covariance between the i^{th} and j^{th} element of the $\vec{\kappa}$ vector - can be constructed in a simple way, extending the work of Kerner (1992). This approach states that the fluctuations in the elastic medium properties $\Delta\vec{\kappa}(z)$ can be modelled by a multivariate stochastic process

$$\begin{bmatrix} \Delta\rho(z) \\ \Delta c_p(z) \\ \Delta c_s(z) \end{bmatrix} = \begin{bmatrix} 1 & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & 1 & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & 1 \end{bmatrix} \begin{bmatrix} \chi_1(z) \\ \chi_2(z) \\ \chi_3(z) \end{bmatrix} \qquad (2)$$

or

$$\Delta\vec{\kappa}(z) = \Lambda\vec{\chi}(z) \qquad (3)$$

in which the h's denote the cross correlation coefficients and the $\chi_i(z)$'s are *independent* stochastic processes. From equation (2) or (3) one can obtain the following expression for the cross-covariance matrix of $\Delta\vec{\kappa}(z)$:

$$c_{\kappa\kappa}(\zeta) = \begin{bmatrix} (& & & \\ \Lambda & 0 & c_2(\zeta) & 0 \\ & 0 & 0 & c_3(\zeta) \end{bmatrix} \Lambda^T, \qquad (4)$$

where ζ denotes the lag parameter, the $c_i(\zeta)$'s constitute the autocovariance functions $c_{\chi\chi}(\zeta)$ of $\chi_i(z)$ and Λ^T represents the transposed of the matrix A.

This proposed model implies that the entries (c_{ij}) in the covariance matrix consist of *weighted* contributions of the autocovariance functions (c_i) of the parameters. Expressions for the cross power spectra of the medium contrasts can be acquired in a similar fashion. Summarizing, the presented stochastic model allows for a predetermination of the cross-correlations between the ***elastic*** parameters as well as a free choice for the random processes χ_i.

From a previous analysis [Herrmann 1992] it became clear that stochastic fractals constitute a family of stochastic processes the properties of which are very similar to the medium properties evidenced from well-log measurements. Application of this stochastic model implies for the power spectra of the driving stochastic processes χ_i:

$$S_i(k) = \nu_i |k|^{\alpha - 2}. \qquad (5)$$

where ν_i denotes the intercept with the vertical axis, k the frequency and a the slope of the log-log power spectrum $(-1<\alpha<1)$.

Application to well-log measurements

Fig. 1 and Fig. 2 depict the cross- covariance functions and power spectra computed from a *real* well-log (ρ, c_p) measurement (courtesy Rijks Geologische dienst). The analysis is performed under the assumption of stochastic stationarity over the length of the well-log. The successive plots of the *sample* cross-covariances for the medium contrasts are depicted in Fig. 1 (a,b,c) respectively (use has been made of the relation between the medium fluctuations and contrasts ($c_{\chi'\chi'}(\zeta) = -c''_{\chi\chi}(\zeta)$). Inspection of these plots shows that the contrasts in the medium fluctuations belong to the *anticorrelated* category of fractal Gaussian noise fGn [Mandelbrot 1982], i.e. $0<\alpha<1$ (see Fig. 2). This observation is very important because of its predominant effect on the *trans-*

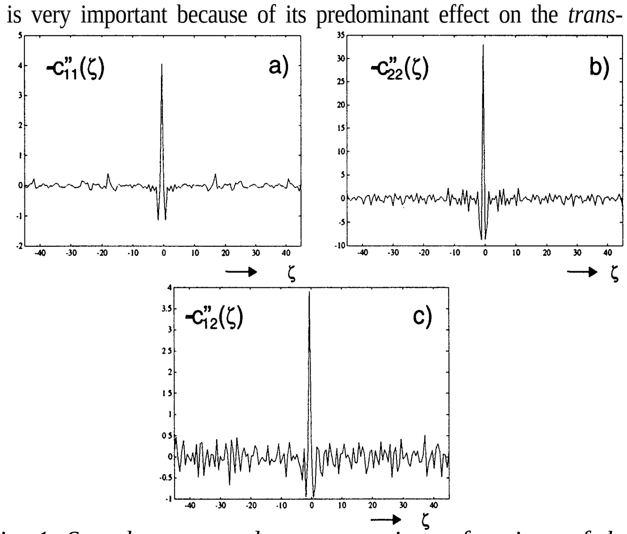

Fig. 1. Sample auto- and cross-coveriance functions of the medium contrasts, computed from a real well-log.
a0. autocovariance for the contrasts in p, -c"11 93)
b0 autocovariance for the contrasts cp, -c"22 (#)
c0 cross-covariance for the contrasts in p, c, -c"12 (#)p

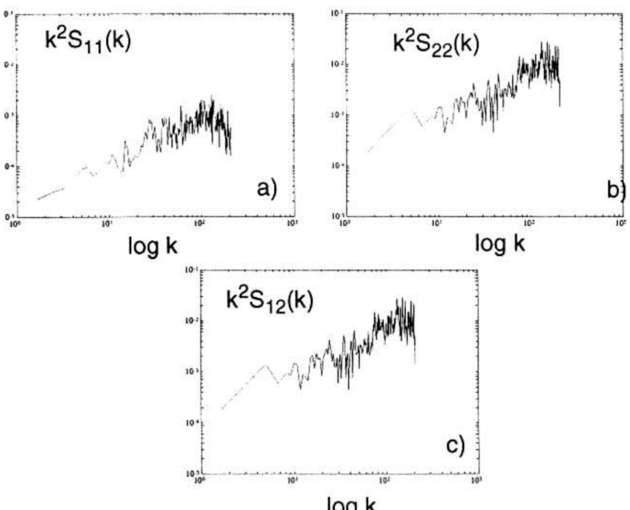

Fig. 2 *Power spectra of the contrasts in the medium parameters, computedfrom a real well-log, spectral window 512 pnts.*
a) log-log plot of the power spectrum of ρ, $k^2S_{11}(k)$
b) log-log plot of the power spectrum of c_p, $k^2S_{22}(k)$
c) log-log plot of the cross power spectrum ρ, c_p, $k^2S_{12}(k)$

parency of the earth's subsurface for seismic energy The notion anticorrelation is substantiated by the log-log power spectra of the medium contrasts which clearly display a *blue* power law behaviour (positive slope α), see Fig. 2.

Step transmission response of a real well-log

In order to elucidate the *offset* related effects due to the fine-layering a plot of the PP time integrated impulse transmission response is included (Fig. 3) for the same configuration. It is clear that the distinct step function (corresponding to a delta shaped impulse response) is more profoundly smoothed for increasing angles of incidence. This implies an increasing *dispersion*. frequency dependent time delay and amplitude attenuation, for increasing angles of incidence.

Elastic response of fGn

In this section experiments are conducted aimed at clarifying the significance of the introduced macro parameters (α,λ_{ij}), defining the smoothness of the medium (degree of anticorrelation) and the *mutual* correlation between the different elastic medium parameten. The other statistical medium parameters. like the variances, are kept *fixed,* yielding an isolation of the *specifc* effects on the *signature* due to changes in either α or λ_{ij}.

Tau-p gathers of the convolved PP transmission responses are depicted in Fig. 4 (a,b) for media with α = 0.25 and α = 0.75 respectively. The number of layers is 15,000 and the contrasts are taken to be high leading to a negligible amplitude for the primary.

From Fig. 4 it can be deduced that the dispersive effects become more pronounced for increasing horizontal slowness and the overall *transparency* is enhanced for increasing degree of *anti-correlation* (increasing α) in the medium contrasts. This latter notion is consistent with the observations for the *acoustic case* [Herrmann 1992]. Note that the *signature* of the responses almost completely consists of multiple *scattered* and *converted* waves. This means that the amplitude of the generalized primary has been *reinforced* due to the *positive* interaction of the first peg-leg multiples. However. there is still a *dispersion* compared to the solution in a *smooth* varying medium. From the amplitude cross-sections (Fig. 5) computed from Fig. 4 it can be observed that the overall trend in the angle dependence of the effects (the anisotopy) stays approximately the same. This notion leads to the folowing preliminary conclusion that a change in *anticorrelation* :a) does not change the overall angle dependence of the signature significantly. It nearly only leads to an increase in attenuation by a constant factor (not dependent on the angle).

The second point of interest in this quantitative study is the sensitivity of the generalized primary to variations in the *mutual* correlations between the *elastic* medium properties. For this purpose mother random medium realization is generated (for a = 0.75) with the cross-correlation coefficients set to zero. i.e. no *mutual* correlation. In Fig. 6 an amplitude cross-section of the PP transmission response is depicted to illustrate the effects of changing the *mutual* correlations (λ_{ij}). A preliminary conclusion drawn from this amplitude cross-section is that the *anisotropic* amplitude effects tend to be more pronounced for increasing mutual correlations, a notion already observed by Kemer (1992) for *traveltimes* in random media with varying Poisson ratios, i.e correlations between c_p and c_s

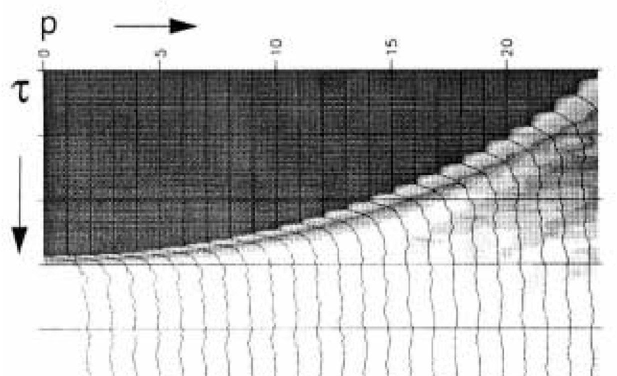

Fig. 3 Time integrated broadband PP transmission response (horizontal particle displacement) for a medium defined in terms of a well-log measurement. Notice the increased "smoothing" of the step function as p increases.

Conclusion

From the analysis presented in this paper it can be concluded that fine-layering can have serious consequences on the amplitude versus frequency *(dispersion)* and on the amplitude versus horizontal slowness (p) behaviour *(anisotropic dispersion)* of the probing seismic signal. An apparent amplitude restoration is found for media of which the contrasts display an *anticorrelated* behaviour ($0<\alpha<1$). The *transparency* of the earth's subsurface for seismic energy - the earth acts as a low pass filter - strongly depends on the *a* of the *elastic* medium contrasts and to a smaller extent on the *mutual* correlation (λ_{ij}) between these contrasts.

The applications of the extended macro model , which is defined in the *global* paramters α and λ_{ij}, are twofold. In the first place this model facilitates the construction of *forward* and *inverse* wavefield extrapolation operators [Wapenaar and Herrmann 1993] which account for the quantitative *elastic* effects evidenced from the fine-layering. Secondly, an inversion scheme can be developed with which one can estimate *the fine-layering* parameters.

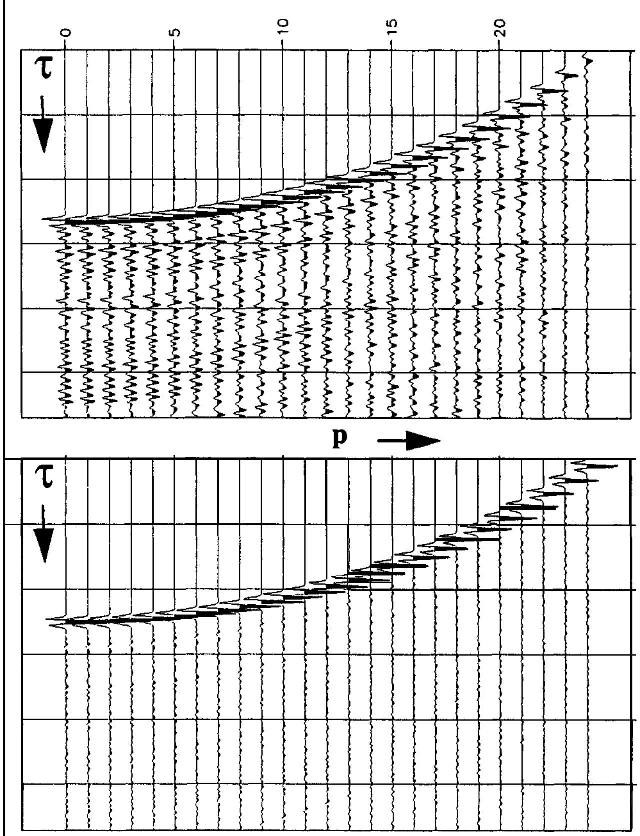

Fig. 4 τ-p gathers of the PP transmission response for random media with α = 0.25 and a = 0.75 resp. In both media $\lambda_{ij} \neq 0$. The distortion is less for a= 0.75 than for a = 0.25

Acknowledgments

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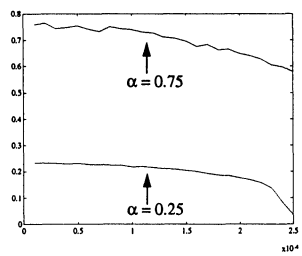

Fig. 5 Amplitude cross-section for a = 0.25 and α = 0.75

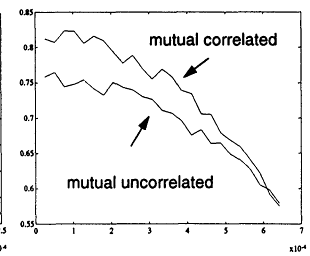

Fig. 6 Amplitude cross-section for cross-correlated and uncorrelated (α = 0.75)

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