Inverse Wave Theory-based Algorithms for the Ultrasonic Inspection of Thin Laminated Materials, based on Reflection Meaurements

Introduction

The inspection of thin laminated materials in aerospace industry is traditionally done by means of transmission measurements. The decrease in the maximum amplitude of a wave that traveled through the material is an indication for the global acoustic properties of the material. A high decrease in amplitude might indicate that a void inclusion is present at the boundary between two layers -- what is called a delamination. No information is given about the depth position of this delamination.
Another disadvantage of the transmission method is that two transducers at opposite sides of the target material are needed. In some cases, it is important that the material has to be accessed from one side only -- e.g. in airplanes standing on the ground.
A solution to both problems can be given by reflection measurements. The transducer that generates the wave also acts as the receiver that measures the resulting waves -- also called the scattered wavefield. When the full time signal is recorded, it is possible to obtain more information about the constituting layers -- and eventually the delaminations.
The processing of reflection measurements using the full time signal is more complex, and also has more requirements on the practical setup.
The methods described in this paper especially apply to Glare. Glare is a laminated material, consisting of alternating layers of aluminium and prepreg. The material properties of Glare make it a good solution for the construction of airplanes.

Plane wave theory

In order to analyze the measured signal, it is necessary to have a good understanding of the behavior of an acoustic wave interacting with a laminated material. In our model, a pressure wave is generated at the transducer. This wave travels towards the target material. Part of the energy will be reflected back to the transducer.

If the frequency spectrum of the emitted source wave is given by $\hat{s}(\omega)$ -- with ω the angular frequency and the hat-symbol indicating that the quantity has to be considered in the frequency-domain -- and the measured resulting pressure is denoted as $\hat{p}(\omega)$, the following relationship holds:

$$\hat{p}(\omega) = \hat{X}(\omega)\hat{s}(\omega) \qquad (1)$$

with $\hat{X}(\omega)$ the response function of the material. This response function is dependent on the acoustic and geometrical parameters of the different layers.
When acoustic waves traveling through a medium encounter a physical boundary -- a change in density and/or particle velocity --, part of the energy in the wave is reflected at the boundary, while another part is transmitted into the material behind the boundary. The ratio between the reflected and incident pressure is denoted by the reflection coefficient *r*.

The configuration

The configuration consists of a number of homogeneous layers. We denote this number by *n*. A layer is characterized by its density ρ, acoustic velocity *v* and the thickness. If the *z*-axis of a Cartesian coordinate system is pointing downwards in the material and the top of layer *i* is at depth z_i, the thickness of layer *i* is given by $z_{i+1} - z_i$ -- as shown in Figure (1).

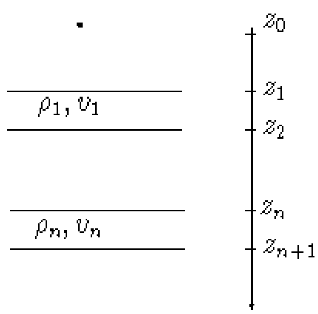


Figure 1: Configuration

The wavefield is defined by the quantities of the pressure and particle velocity in each point. The pressure of the scattered wavefield at the top of layer *i* is denoted as p_i. The total thickness of the material typically is $.4\,\mathrm{mm}$ till $4\,\mathrm{mm}$. A transducer is placed about $50\,\mathrm{mm}$ above the material. Waves traveling from the transducer towards the material can be assumed plane -- the wavefield has only a gradient in time and depth, not in the lateral directions -- when they arrive at the boundary of the material. Therefore, it is reasonable to use the plane wave theory [1].

Recursive model

One way to obtain a model for the total wavefield at the transducer, is to recursively calculate the wavefield at the top of the constituting layers. In this approach, the transducer is first put at the top of the bottom layer -- layer *n* --, and the resulting scattered wavefield is calculated. Next, the transducer is moved upwards to the top of layer *n*-1, and the wavefield at this position is calculated using the results from the previous recursion.

The calculation of the scattered pressure p_i based on the knowledge of p_{i+1} is done in two steps. In the first step, the transducer is moved upwards to the top of layer *i*. Crossing the boundary between layer *i*+1 and layer *i* scales the amplitude of the pressure with $(1-r_i)(1+r_i)$ due to transmission effects. Apart from the energy coming from under the boundary, some energy emitted by the source will be reflected at the boundary and contribute to the total wavefield too.

Propagation effects are taken into account by the propagation operator $\hat{w}_i$.

A reflection-free surface is assumed to be on top of layer *i*. As a consequence, waves traveling upwards in layer *i* will not be reflected back into the configuration. The resulting pressure is therefore called the reflection-free surface pressure, and is denoted by p_i^0.

In the second step, the reflection-free surface is substituted by the real physical boundary, and up-traveling waves will give raise to down-traveling waves when arriving at the top of the configuration. Above the boundary, the material is considered to be a homogeneous halfspace. The resulting pressure is denoted by p_i.

It can be shown ([2]) that the reflection-free surface pressure and the total pressure of the scattered wavefield at the top of layer *i* are given by

$$\hat{p}_i^0 = \hat{w}_i \left(r_i \hat{s} + \left(1 - r_i^2\right) \hat{p}_{i+1} \right) \hat{w}_i \qquad (2)$$

and

$$\hat{p}_i = \frac{\hat{p}_i^0 \hat{s}}{\hat{s} + r_{i-1} \hat{p}_i^0}. \qquad (3)$$

The propagation term $\hat{w}_i$ is given by

$$\hat{w}_i = \exp(-j\omega t_i) \qquad (4)$$

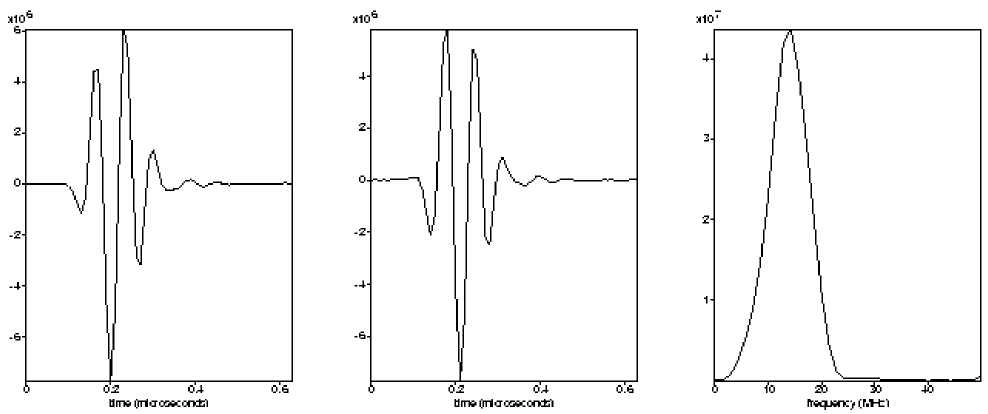
with t_i the one-way traveltime in layer *i*. Since the medium under the laminated material is considered as an acoustic halfspace -- which means that energy that travels through the boundary of the target is never reflected back into the material --, the recursion is started with

$$\hat{p}_n^0 = \hat{w}_n r_n \hat{w}_n. \qquad (5)$$

Using Equation (3), the total scattered pressure at the top of layer *n* can be calculated. One parameter has to be known though: the reflection coefficient r_{n-1}.

Generally, each recursion for the reflection-free surface pressure requires the knowledge of $\hat{w}_i$ and the corresponding calculation of the total scattered pressure requires the knowledge of r_{i-1}. The propagation term $\hat{w}_i$ can be calculated once the two-way traveltime t_i is known.

For a given configuration -- which consists of the acoustic and geometric parameters of each layer -- and a given source signal, the measured wavefield can be calculated. In simulations, we use a measured source signal. This makes the result of the simulation more comparable with the real measurements. The source signal is given in Figure (2(a)). The frequency-content is shown in Figure (2(c)).



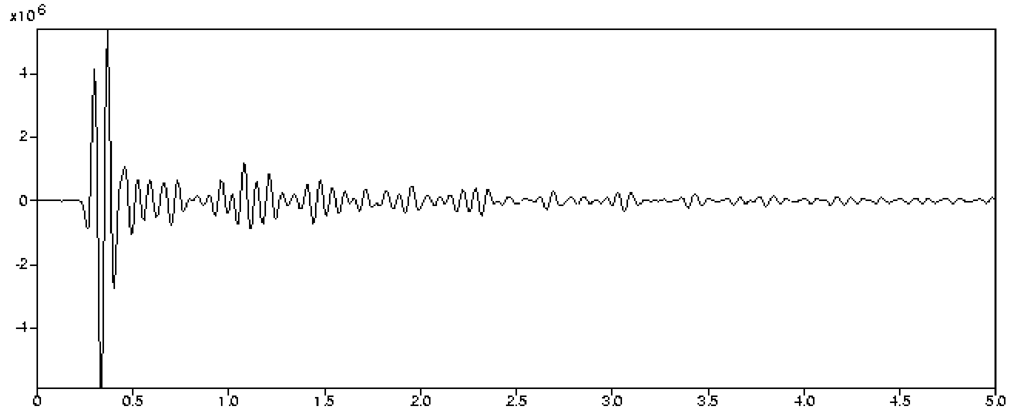
(a) original source (b) modified source (c) frequency content

Figure 2: Time- and frequency behavior of the actual source wavelet

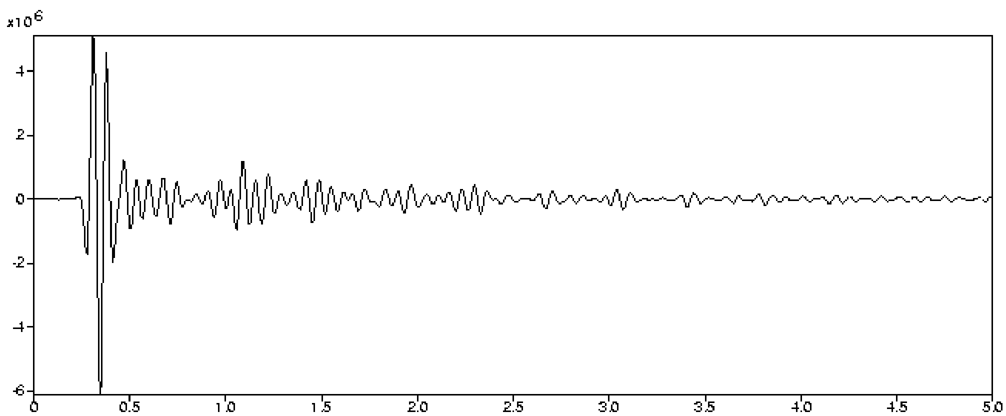
Using this source signal, the resulting pressure of the scattered wavefield for the configuration given in Table (1) -- a typical Glare-example -- is shown in Figure (3(a)).

acoustic parameters of the configuration			
Layer	thickness (m)	velocity (m/sec)	density (kg/m^3)
Water	10^{-4}	1480	1000
Aluminium	4.10^{-4}	6300	2740
Prepreg	$3,75.10^{-4}$	3100	1600
Aluminium	4.10^{-4}	6300	2740

Table 1: Acoustic parameters of a Glare configuration



(d) using original source



(e) using modified source

Figure 3: Simulated glare scan

Inversion

Now we have established a forward model for our reflection measurements, we are going to use this in an inversion scheme. The problem is to retrieve the material parameters for each layer from these measurements. To achieve this, two inversion approaches are compared in this paper.

Direct inversion

The forward model requires the acoustic parameters of the individual layers in order to calculate the total scattered pressure. The direct inversion process starts with this scattered pressure, and recursively gives the acoustic parameters of the individual layers. This procedure is also used in seismic processing [3].

The inverse algorithm

From Equations (2) and (3), the inverse recursive equations can be calculated, resulting in

$$\hat{p}_i^0 = \frac{\hat{p}_i \hat{s}}{\hat{s} - \hat{p}_i r_{i-1}} \tag{6}$$

and

$$\hat{p}_{i+1} = \frac{\hat{p}_i^0 - \hat{w}_i r_i \hat{s} \hat{w}_i}{\hat{w}_i \left(1 - r_i^2\right) \hat{w}_i} \tag{7}$$

Since the distance between the transducer and the material is large compared with the thickness of the material, no events due to multiple reflections between the transducer and the top of the target will occur in the time-window of interest. As a consequence, the reflection-free surface pressure at the transducer is also the total pressure. This pressure, denoted by $\hat{p}_0^0$, is the input for the direct inversion. Starting from $\hat{p}_0^0$ and recursively using Equations (6) and (7), the pressure $\hat{p}_n$ at the bottom layer can be calculated.

Iteration step *i* means that the effects of layer *i* are eliminated, and that the result is the wavefield that would exist if the transducer was moved from the top of layer *i* towards the top of layer *i*-1.

Acoustic information

The time passed before the arrival of the first event is the two-way travel time. This quantity allows the reconstruction of the propagation factor $\hat{w}$. The ratio of the amplitude of the source wavelet and the amplitude of the first event in the considered signal is an indication for the reflection coefficient *r*. Under the assumption that the first event can be distinguished from other events, it is possible to obtain the acoustic parameters of the upper layer.

Due to interference of events, it is generally not possible to obtain the acoustic parameters of each layer without any processing techniques. Especially multiple events -- waves that propagate in the target (downward and upward) and have bounced at least twice before detection -- make it hard to distinguish the primary events -- waves that propagate in the subsurface (downward and upward) and have bounced only once.

After obtaining the parameters in the upper layer, one recursion step is performed in order to eliminate the effects of the considered layer. These effects also include the multiple events caused by back-reflection in the upper layer. In the next step, the acoustic parameters of the under-laying layer are obtained.

Applying this procedure recursively for all layers gives the acoustic parameters of the different layers constituting the material.

Example

We wrote an algorithm that performed all of the above-mentioned steps. Applying this algorithm to the signal shown in Figure (3) gives an estimated travel-time in each layer, and a reflection coefficient for each boundary between two layers. These parameters are compared with the exact parameters -- the ones used for obtaining the signal -- in Table (2).

Exact and estimated acoustic parameters								
	$t_0(10^{-8}s)$	$t_1(10^{-8}s)$	$t_2(10^{-8}s)$	$t_3(10^{-8}s)$	r_0	r_1	r_2	r_3
exact	6,757	6,349	12,090	6,349	0,842	-0,554	0,554	-0,842
estimated	6,765	6,327	12,616	6,409	0,844	-0,550	0,550	-0,846
modified	7,313	1,591	-	-	0,936	0,904	-	-

Table 2: The results of the inverse algorithm

Applying a constant phase-shift to the original source-signal results in a modified source-signal, shown in Figure (2(b)). Using this source signal, the resulting pressure of the scattered wavefield for the configuration given in Table (1) is shown in Figure (3(b)). Using the original source-signal for performing the inverse algorithm will give erroneous results. The estimated acoustic parameters are given in the third row of Table (2). This situation occurs in practice when the source-signal is not repeatable, and the processing is performed with a source-signal that is different from the original one.

Parametric optimization

Procedure

The measured wavefield is the result of an operator acting on a set of parameters. These parameters include the source signal and the acoustic and geometric properties of each layer. Two different sets will result in two different measurements. The difference between the measurements can learn something about the differences between the sets of parameters.

In parametric optimization, we start with an initial set of parameters -- i.e. the initial model -- that is a reasonable approximation for the real set -- the set we want to find. Using the forward algorithm, the resulting wavefield can be calculated. If the chosen parameters exactly match the real parameters, the calculated wavefield will match the measured wavefield -- assuming no noise is introduced. Since this will generally not be the case, there will be a difference between the calculated and the measured wavefield.

Changing one parameter will result in a change in the difference, from which the influence of that particular parameter can be obtained. Combination of these influence-data results in a new set of parameters that is a better approximation than the first set -- assuming that a smaller difference is due to a better approximation.

Repeating this procedure gives a series of sets that converge towards the real set measurements and the true acoustic parameters.

Implementation topics

Parametric optimization problems are encountered in lots of areas [4]. Some choices have to be made before implementing an algorithm that performs a parametric optimization. At first, a criterium for qualifying the approximation has to be chosen -- an error-function has to be defined. The sum of the quadratic errors over the signal is one possibility.
Also, a set of parameters has to be determined. Better results are obtained when these parameters are independent from each other. When the contour-lines of the error-function are concentric circles, a faster convergence is obtained. Scaling the parameters can make these contour-lines more or less circular.
Another issue is the improvement of the set of parameters, based on the difference between the measured signal and the calculated signal. Different methods exists, under which the steepest descent and the conjugate gradient.
One possibility is to use the traveltime in each layer and the reflection coefficient at each boundary for the set of parameters. These parameters have to be scaled in order to give circular contour-lines. Typical examples of these contour plots are given in Figure (4).

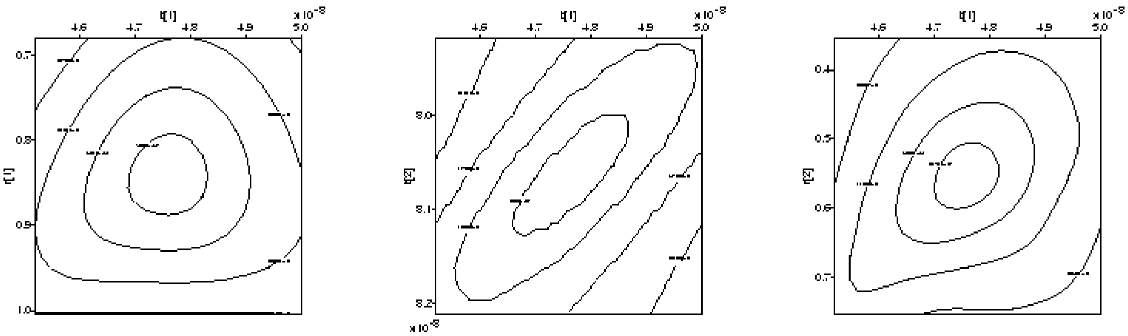


Figure 4: contour-plots of the error-function in parametric optimization

As long as the initial model is close enough to the real model, the iteration will converge to the global minimum of the error-function. It might occur however that the initial set is located close to a local minimum. In this case, the global minimum will not be obtained.

Comparison of the two inversion approaches

Both methods have advantages and disadvantages. The Direct Inversion method is very sensitive to small perturbations on the source signal. Since the first reflection generally is a strong one, the denominator in Equation (7) can become small - resulting in numerical instabilities. On the other hand, a delamination can be clearly seen using this method. A delamination between layers *i* and *i*+1 will result in a strong reflection coefficient r_i, and this coefficient is almost directly obtained after *i* recursion steps.
In cases where the source signal is very repeatable, this method is a fast solution for the detection of delaminations.

The method of the Parametric Optimization is less sensitive to changes in the source signal. It is very valuable when a good approximation of the parameters is available. This typically is the case in C-scans.
The resulting parameters from a measurement on a given location, can serve as the first set of input parameters for the processing of the signal obtained in the next lateral location. The procedure of Parametric Optimization searches for a minimum value of the quadratic error.
In the case the material consists of many layers, the method might give a local minimum instead of a global minimum, especially when the first set of parameters is not a good approximation for the real set.

Conclusion

The use of reflection measurements for the inspection of thin layered materials is very valuable, both from the practical side -- e.g. single-side access -- as from the theoretical side -- more detailed results can be obtained.
In this paper two approaches for applying an inversion to the acoustic reflection measurements have been described, which both aim at the retrieval of the acoustic parameters of the inspected material, in order to find local delaminations etc.
For the simulated examples, they both can come with a good solution. It will depend on the actual measurements quality -- e.g. repeatability of the source signal and the total number of layers in the configurations -- which one will be preferred in practice.

References

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