Surface wave retrieval from ambient noise using multi-dimensional deconvolution

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Seismic interferometry is an effective tool to retrieve surface waves between two receiver stations by cross-correlating ambient background noise over sufficiently long recording times. This method assumes an azimuthally uniform distribution of noise sources. Unfortunately this assumption is not always fulfilled in practice. If noise sources are located on one side of a receiver array only, surface waves can also be retrieved by multi-dimensional deconvolution of passive records. We show how this method can effectively correct for azimuthal variations in the noise source distribution. We do not take backscattering of the surface waves into account, but this can be overcome if wavefield decomposition is incorporated.

Introduction

Shapiro et al. (2005) and various others have shown that surface waves can be retrieved by cross-correlation of ambient seismic noise records. One of the major assumptions underlying this concept is that the passive sources have a uniform azimuthal distribution. We propose a method that corrects for a non-uniform distribution of passive sources by replacing cross-correlation with multi-dimensional deconvolution. We require an array of receivers to obtain directional information of the illuminating wavefields. Further we assume that all passive sources are located on one side of this array and that backscattering at the other side of the array can be neglected or that wavefield decomposition can be incorporated.

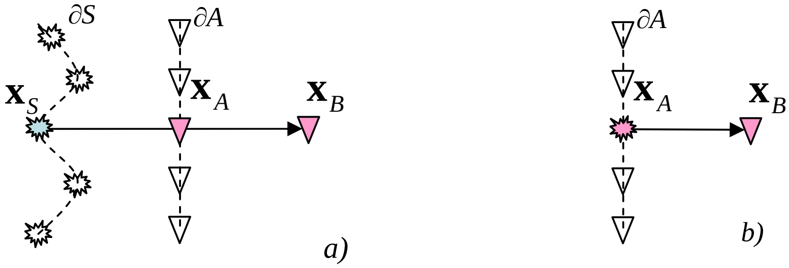

Figure 1a) Plane view of a passive experiment; noise sources are located at ∂S ,left of receiver array ∂A; b) Retrieved surface wave with a virtual source at receiver location $\mathbf{x}_A$ (at ∂A) and a receiver at $\mathbf{x}_B$ (right of ∂A).

Surface wave retrieval by multi-dimensional deconvolution with transient sources

In Figure 1a we have depicted a passive experiment, where sources are located at the left side of receiver array ∂A. Aim is to transform the real acquisition of Figure 1a into a virtual acquisition as if there was a source at location $\mathbf{x}_A$ (at ∂A) and a receiver at $\mathbf{x}_B$ (right of ∂A) - see Figure 1b. First, let us assume that the sources are transients with distinct excitation times. If we assume that all waves are rightgoing at ∂A (this means that backscattering is neglected or decomposition is applied), the arriving wavefield at the array, $\hat{P}^{+,}(\mathbf{x}_A,\mathbf{x}_S,\omega)$ (superscript $+,$ denotes "rightgoing at the receiver"), can be extrapolated to location $\mathbf{x}_B$ (Wapenaar and Berkhout, 1989):

$$\hat{P}(\mathbf{x}_B,\mathbf{x}_S,\omega)=\int_{\partial A}\hat{D}^{,+}(\mathbf{x}_B,\mathbf{x}_A,\omega)\hat{P}^{+,}(\mathbf{x}_A,\mathbf{x}_S,\omega)d\mathbf{x}_A . \tag{1}$$

Here $\hat{P}(\mathbf{x}_B,\mathbf{x}_S,\omega)$ is the full wavefield at receiver $\mathbf{x}_B$ and $\hat{D}^{,+}(\mathbf{x}_B,\mathbf{x}_A,\omega)$ is an extrapolator (superscript $,+$ denotes "rightgoing at the source"). The integral is over receiver locations $\mathbf{x}_A$. It should be noted that $\hat{D}^{,+}$ is a scaled dipole impulse response that we can write as $\hat{D}^{,+}(\mathbf{x}_B,\mathbf{x}_A,\omega)=-(2/\rho)\partial_1\left\{\hat{G}^{,+}(\mathbf{x}_B,\mathbf{x}_A,\omega)\right\}$, where $\hat{G}^{,+}$ is a monopole Green's function, ρ is the medium density and ∂_1 represents a spatial derivative normal to the receiver array ∂A. The extrapolator $\hat{D}^{,+}$ can be interpreted as the unknown Green's function that we aim to retrieve with seismic interferometry. This is done by inverting equation 1, a procedure that is also known as multi-dimensional deconvolution (Wapenaar et al., 2008). We can show that solving equation 1 for $\hat{D}^{,+}$ in a least-squares sense is equivalent to solving the following normal equation (Menke, 1989):

$$\hat{C}^{,+}(\mathbf{x}_B,\mathbf{x}'_A,\omega)=\int_{\partial A}\hat{D}^{,+}(\mathbf{x}_B,\mathbf{x}_A,\omega)\hat{\Gamma}^{+,+}(\mathbf{x}_A,\mathbf{x}'_A,\omega)d\mathbf{x}_A . \tag{2}$$

We will refer to $\hat{C}^{,+}$ as the cross-correlation function. It represents a source integral of cross-correlations of the rightgoing wavefield at location $\mathbf{x}'_A$ (at ∂A) and the total wavefield at location $\mathbf{x}_B$ (right of ∂A):

$$\hat{C}^{,+}\left(\mathbf{x}_B,\mathbf{x}'_A,\omega\right)=\int_{\partial S}\hat{P}\left(\mathbf{x}_B,\mathbf{x}_S,\omega\right)\left\{\hat{P}^{+,}\left(\mathbf{x}'_A,\mathbf{x}_S,\omega\right)\right\}^* d\mathbf{x}_S\,. \tag{3}$$

$\hat{\Gamma}^{+,+}$ is referred to as the interferometric resolution function. It represents a source integral over cross-correlations of rightgoing wavefields at locations $\mathbf{x}'_A$ and $\mathbf{x}_A$ (both at ∂A):

$$\hat{\Gamma}^{+,+}\left(\mathbf{x}_A,\mathbf{x}'_A,\omega\right)=\int_{\partial S}\hat{P}^{+,}\left(\mathbf{x}_A,\mathbf{x}_S,\omega\right)\left\{\hat{P}^{+,}\left(\mathbf{x}'_A,\mathbf{x}_S,\omega\right)\right\}^* d\mathbf{x}_S\,. \tag{4}$$

If the sources are azimuthally uniformly distributed, $\hat{\Gamma}^{+,+}$ collapses into a bandlimited delta function: $\hat{\Gamma}^{+,+}\left(\mathbf{x}'_A,\mathbf{x}_A,\omega\right)\approx\left|\hat{S}\left(\omega\right)\right|^2\delta\left(\mathbf{x}'_A-\mathbf{x}_A\right)$, where $\hat{S}\left(\omega\right)$ is the emitted source wavelet. As a consequence, $\hat{C}^{,+}\approx\left|\hat{S}\left(\omega\right)\right|^2\hat{D}^{,+}$ and thus the cross-correlation function provides a fair representation of the desired virtual source data, imprinted by the squared source wavelet. This representation confirms the concept of interferometry by cross-correlation (Halliday et al., 2007), where it is claimed that Green's functions can be properly retrieved if the source distribution is uniform. If the source distribution is not uniform, multi-dimensional deconvolution (meaning inversion of equation 2) can in some cases improve the retrieved response.

Surface wave retrieval by multi-dimensional deconvolution with noise sources

In passive records we assume noise signals to be recorded simultaneously, such that the source integrals can not be implemented through equations 3 and 4. However, if the noise sources are white and uncorrelated, the source integrals can be replaced by spatial ensemble averages (Wapenaar, 2004). Say $\hat{P}\left(\mathbf{x}_B,\omega\right)$ is the signal at receiver $\mathbf{x}_B$ and $\hat{P}^{+,}\left(\mathbf{x}_A,\omega\right)$ is the (rightgoing) signal at receiver $\mathbf{x}_A$, due to uncorrelated noise sources left of receiver array ∂A. Both fields are related through the following extrapolation equation:

$$\hat{P}\left(\mathbf{x}_B,\omega\right)=\int_{\partial A}\hat{D}^{,+}\left(\mathbf{x}_B,\mathbf{x}_A,\omega\right)\hat{P}^{+,}\left(\mathbf{x}_A,\omega\right)d\mathbf{x}_A\,. \tag{5}$$

Solving this equation is equal to solving normal equation 2, where the cross-correlation and resolution functions are replaced by these spatial ensemble averages over sufficiently long recording times:

$$\hat{C}^{,+}\left(\mathbf{x}_B,\mathbf{x}'_A,\omega\right)=\left\langle\hat{P}\left(\mathbf{x}_B,\omega\right)\left\{\hat{P}^{+,}\left(\mathbf{x}'_A,\omega\right)\right\}^*\right\rangle, \tag{6}$$

$$\hat{\Gamma}^{+,+}\left(\mathbf{x}_A,\mathbf{x}'_A,\omega\right)=\left\langle\hat{P}^{+,}\left(\mathbf{x}_A,\omega\right)\left\{\hat{P}^{+,}\left(\mathbf{x}'_A,\omega\right)\right\}^*\right\rangle. \tag{7}$$

If the noise sources are distributed uniformly, the resolution function collapses to a band-limited delta-function and the cross-correlation function gives a fair representation of the desired surface wave. However, if the distribution is not uniform, $\hat{C}^{,+}$ can be deconvolved by the estimated resolution function $\hat{\Gamma}^{+,+}$, which has captured the imprint of the non-uniform illumination at array ∂A.

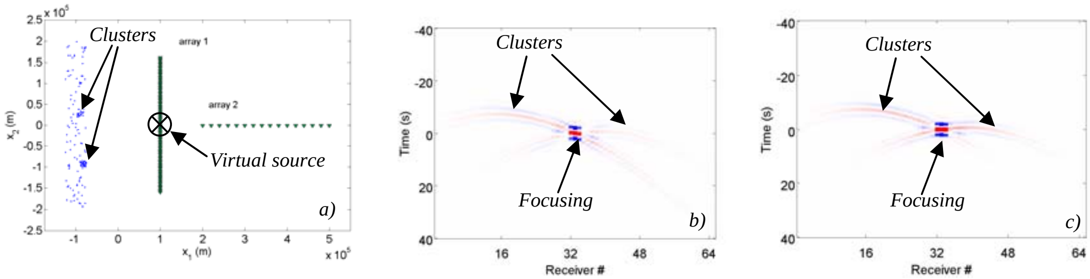

Figure 2a) Configuration of synthetic passive seismic experiment 1; blue dots are noise sources, green triangles are receivers; a virtual source is generated at receiver #33 of array 1; b) resolution function, using noise sources; c) resolution function, using transient sources.

Examples

We compute a dispersion curve for the upper 300 km of the PREM model (Dziewonski and Anderson, 1981) using an approach as described in Wathelet et al. (2004) and use this to model fundamental Rayleigh waves. The configuration is shown in Figure 2a. We use 150 simultaneously acting white-noise sources, represented by the blue stars, with irregular distribution. In particular, we use two clusters of noise sources –one is concentrated around x2=30000 m and consists of 20 sources, while the other is concentrated around x2=-100000 m and consists of 30 sources. We record the emitted noise fields at two mutually perpendicular receiver arrays, which in the figure are represented by the green triangles. Array 1 is parallel to the source-distribution geometry and contains 65 receivers spaced at 5000 m. Arrays 2 contains 16 receivers spaced at 20000 m. The frequency spectrum of the noise sources peaks at 0.6 Hz, which is the double-frequency microseismic peak. The two arrays record the total noise for nearly 42 hours. We compute the resolution function at receiver #33 of array 1 with equation 7 – see Figure 2b. Note that we can clearly see the imprint of the two noise clusters. In Figure 2c we show the result of a similar experiment with transient sources using equation 4. Note that the same imprint can be observed. The resolution seems well able to capture the illumination imprint of the source distribution, either for noise or transient sources. In Figure 3a we show the retrieved response by cross-correlation of the noise source responses at receiver #33 of array 1 with the other receivers of array 2 (in red). The response is overlaid (in black) by a dipole response that we computed by placing an active source at the virtual source location (receiver #33 of array 1). Note that the dispersion characteristics are not retrieved exactly. Since we know the imprint of the noise sources that hampers the results (Figure 2b), we can invert equation 2 for improvement. The result of this process, known is multi-dimensional deconvolution, is shown in Figure 3b. Note that we have improved the response significantly. Next we repeat the experiment with a slightly different acquisition having two parallel arrays of 65 and 12 receivers – see Figure 4a. We create a virtual source at location #33 by cross-correlation and multi-dimensional deconvolution and compare the retrieved surface wave with a reference response. Note that the cross-correlation based response (Figure 4b) has suffered significantly from the noise source distribution imprint, whereas this effect is almost completely corrected for by multi-dimensional deconvolution (Figure 4c).

Conclusion

We have shown that multi-dimensional deconvolution can be a fruitful alternative to cross-correlation in interferometric surface wave retrieval from ambient seismic noise if a densely sampled array of sufficient receivers is available. Noise sources should be located on one side of the receiver array only and backscattering is not accounted for. A different approach would be to separate right- and leftgoing wavefields prior to cross-correlation. An additional advantage of the deconvolution based strategy is that the noise source wavelet is deconvolved automatically, which can be highly benefitial if the noise has a complicated signature.

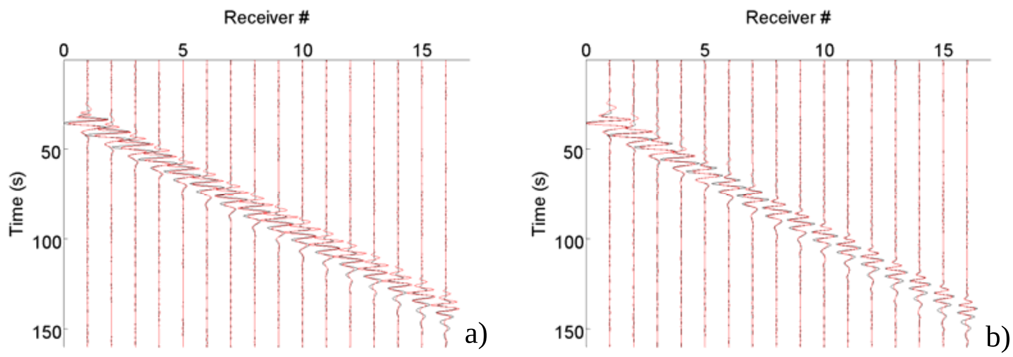

Figure 3a) Retrieved response by cross-correlation using noise sources (red) overlaid by the reference response (black); b) Retrieved response by multi-dimensional deconvolution using noise sources (red) overlaid by the reference response (black).

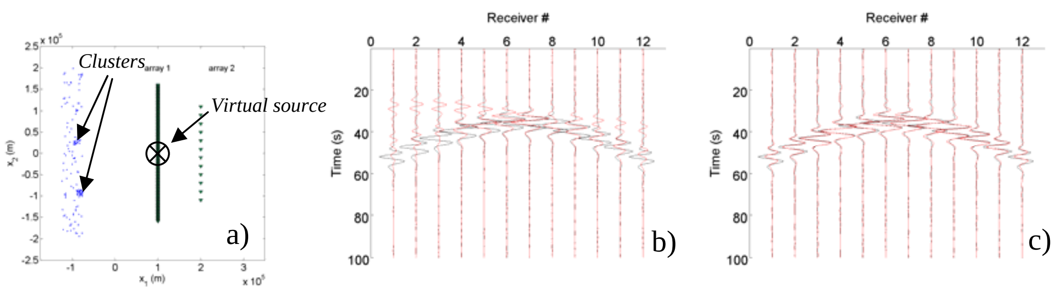

Figure 4a) Configuration of synthetic passive seismic experiment 2; blue dots are noise sources, green triangles are receivers; a virtual source is generated at receiver #33 in the middle of array 1; b) Retrieved response by cross-correlation using noise sources (red) overlaid by the reference response (black); c) Retrieved response by multi-dimensional deconvolution using noise sources (red) overlaid by the reference response (black).

Acknowledgements

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