On the relation between seismic interferometry and the simultaneous-source method

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Summary
In seismic interferometry the response to a virtual source is created from responses to sequential transient or simultaneous noise sources. In the simultaneous-source method, overlapping responses to sources with small time delays are recorded. Seismic interferometry and the simultaneous-source method are related. In this paper we make this relation explicit by discussing deblending as a form of seismic interferometry by multidimensional deconvolution.

Introduction

In seismic interferometry the response to a virtual source is created from responses to sequential transient sources or simultaneous noise sources. Most methods use crosscorrelation (Schuster, 2009), but recently seismic interferometry by multidimensional deconvolution (MDD) has been proposed as well (Wapenaar *et al.* , 2008; van der Neut *et al.* , 2010).

In the simultaneous-source method (also known as blended acquisition), overlapping responses to sources with small time delays are recorded (Beasley *et al.* , 1998). The crosstalk that occurs in imaging of simultaneous-source data can be reduced by using phase-encoded sources (Bagaini, 2006) or simultaneous noise sources (Howe *et al.* , 2008), by randomizing the time interval between the shots (Stefani *et al.* , 2007) possibly followed by a noise filtering process (Moore *et al.* , 2008), by prediction and subtraction (Spitz *et al.* , 2008; Mahdad *et al.* , 2011), or by inverting the blending operator using sparseness constraints (Berkhout, 2008).

Seismic interferometry and the simultaneous-source method are related. In this paper we make this relation explicit by discussing deblending as a form of seismic interferometry by MDD.

Seismic interferometry by multidimensional deconvolution (MDD)

Consider the configuration in Figure 1a, with sources (red stars) at the surface and receivers (black triangles) in a horizontal borehole below a complex overburden. We define the correlation function for sequential transient-source responses as (Bakulin and Calvert, 2006; Mehta *et al.* , 2007)

$$C_{\rm seq}(\mathbf{x}_B,\mathbf{x}_A,t)=\sum_i u^{\rm up}(\mathbf{x}_B,\mathbf{x}_S^{(i)},t)*u^{\rm down}(\mathbf{x}_A,\mathbf{x}_S^{(i)},-t), \tag{1}$$

where $u^{\rm down}(\mathbf{x}_A,\mathbf{x}_S^{(i)},t)$ and $u^{\rm up}(\mathbf{x}_B,\mathbf{x}_S^{(i)},t)$ are the downgoing and upgoing wavefields at two different receivers $\mathbf{x}_A$ and $\mathbf{x}_B$ in the borehole, related to the same source $\mathbf{x}_S^{(i)}$ at the surface. Figure 1b shows the correlation function $C_{\rm seq}(\mathbf{x}_B,\mathbf{x}_A,t)$ for fixed $\mathbf{x}_A$ (the red dot in Figure 1a) and variable $\mathbf{x}_B$ (the black triangles in Figure 1a). This is usually interpreted as the response to a virtual source at $\mathbf{x}_A$, observed by a receiver at $\mathbf{x}_B$, i.e., the Green's function $G(\mathbf{x}_B,\mathbf{x}_A,t)$. A more precise relation between the correlation function and the Green's function is (Wapenaar *et al.* , 2011)

$$C_{\rm seq}(\mathbf{x}_B,\mathbf{x}_A,t)=\int_{\mathbb{S}}\bar{G}_{\rm d}(\mathbf{x}_B,\mathbf{x},t)*\Gamma_{\rm seq}(\mathbf{x},\mathbf{x}_A,t)\,{\rm d}\mathbf{x} \tag{2}$$

($\mathbb{S}$ coincides with the borehole), where $\Gamma_{\rm seq}(\mathbf{x},\mathbf{x}_A,t)$ is the so-called point-spread function, defined as (van der Neut *et al.* , 2010)

$$\Gamma_{\rm seq}(\mathbf{x},\mathbf{x}_A,t)=\sum_i u^{\rm down}(\mathbf{x},\mathbf{x}_S^{(i)},t)*u^{\rm down}(\mathbf{x}_A,\mathbf{x}_S^{(i)},-t). \tag{3}$$

Subscript d in $\bar{G}_{\rm d}(\mathbf{x}_B,\mathbf{x},t)$ denotes that the source of this Green's function is a dipole (at $\mathbf{x}$ on $\mathbb{S}$); the bar denotes a reference situation (i.e., a homogeneous medium above $\mathbb{S}$). Equation (2) states that the correlation function $C_{\rm seq}(\mathbf{x}_B,\mathbf{x}_A,t)$, defined by equation (1), is proportional to the Green's function $\bar{G}_{\rm d}(\mathbf{x}_B,\mathbf{x},t)$, with its source smeared in space and time by the point-spread function $\Gamma_{\rm seq}(\mathbf{x},\mathbf{x}_A,t)$.

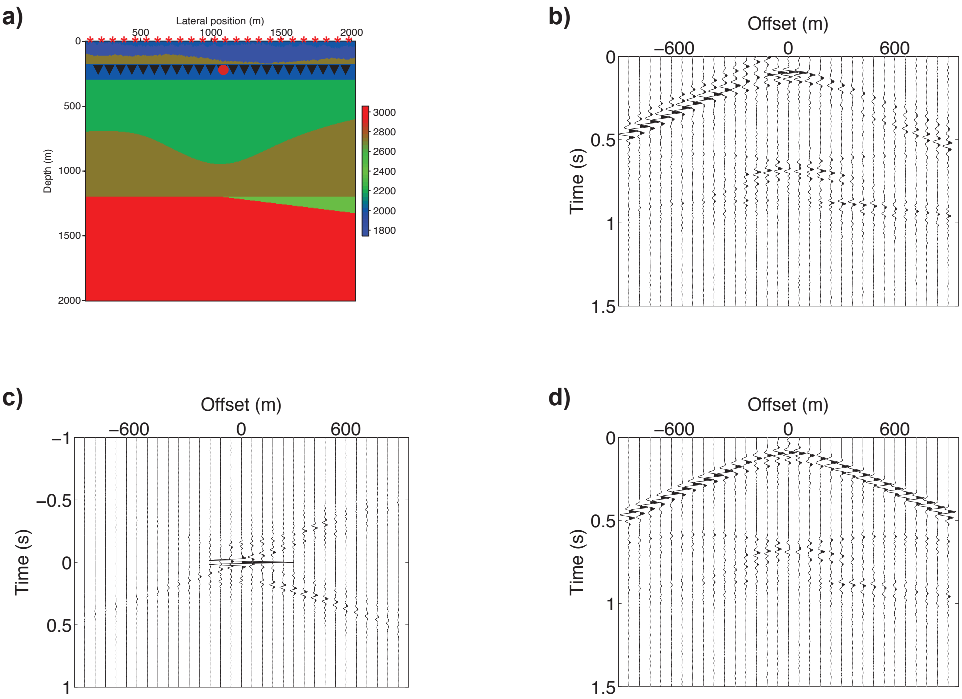

Figure 1: *Numerical example of seismic interferometry by MDD. (a) Inhomogeneous medium configuration. The sources are situated at the surface (red stars) and the receivers in a horizontal borehole (the black triangles at a depth of 200 m) below a complex overburden. (b) Correlation function. (c) Point-spread function. (d) Result of seismic interferometry by MDD. (Figures (b) – (d) show every fourth trace.)*

This point-spread function is defined, according to equation (3), as the crosscorrelation of the downward propagating fields at $\mathbf{x}_A$ and $\mathbf{x}$, summed over the source positions $\mathbf{x}_S^{(i)}$. Under ideal circumstances the point-spread function approaches a temporally and spatially band-limited delta function (Wapenaar *et al.*, 2011, Appendix). Hence, under ideal circumstances the correlation function $C_{\rm seq}(\mathbf{x}_B, \mathbf{x}_A, t)$ is a temporally and spatially band-limited version of the Green's function $\bar{G}_{\rm d}(\mathbf{x}_B, \mathbf{x}, t)$. In more realistic situations the point-spread function can deviate significantly from a band-limited delta function, as is shown in Figure 1c for fixed $\mathbf{x}_A$ (the red dot in Figure 1a) and variable $\mathbf{x}$ (the black triangles in Figure 1a). In order to retrieve the Green's function from the correlation function, the effect of the point-spread function needs to be removed by inverting equation (2). Because equation (2) represents a multidimensional convolution process along the temporal and spatial axes, inversion of this equation is equivalent to multi-dimensional deconvolution (MDD). The result is shown in Figure 1d. This response accurately resembles the directly modeled Green's function $\bar{G}_{\rm d}(\mathbf{x}_B, \mathbf{x}, t)$ (not shown here).

Deblending by multidimensional deconvolution

Figure 2a shows a similar configuration as Figure 1a, but this time we consider simultaneous-source acquisition. We form 32 source groups $\boldsymbol{\sigma}^{(m)}$, each containing four adjacent sources which emit transient wavelets. The ignition times within one source group are chosen randomly from a uniform distribution between 0 and 1 s. We define the correlation function for the simultaneous-source responses as

$$C_{\rm sim}(\mathbf{x}_B, \mathbf{x}_A, t) \quad = \quad \sum_m u^{\rm up}(\mathbf{x}_B, \boldsymbol{\sigma}^{(m)}, t) * u^{\rm down}(\mathbf{x}_A, \boldsymbol{\sigma}^{(m)}, -t), \tag{4}$$

where $u^{\rm down}(\mathbf{x}_A, \boldsymbol{\sigma}^{(m)}, t)$ and $u^{\rm up}(\mathbf{x}_B, \boldsymbol{\sigma}^{(m)}, t)$ are the downgoing and upgoing wavefields in the borehole, related to the same source group $\boldsymbol{\sigma}^{(m)}$ at the surface. Figure 2b shows the correlation function $C_{\rm sim}(\mathbf{x}_B, \mathbf{x}_A, t)$ for fixed $\mathbf{x}_A$ (the red dot in Figure 2a) and variable $\mathbf{x}_B$ (the black triangles in Figure 2a). The noise is a result of the crosstalk between the responses to different sources within the source groups.

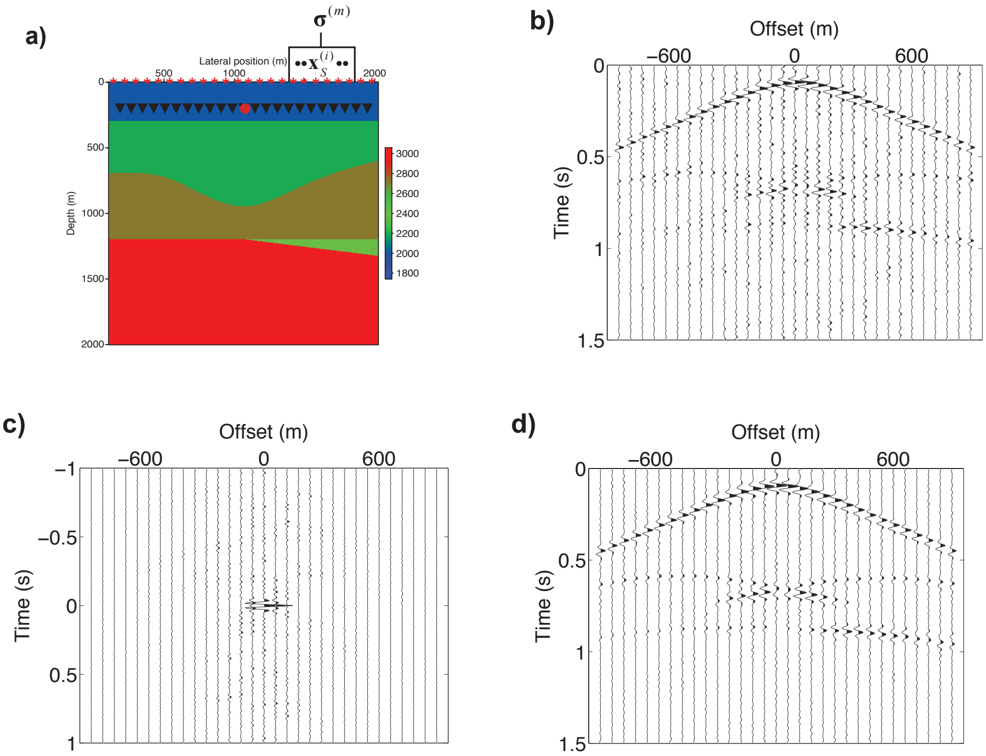

Figure 2: *Numerical example of deblending by MDD. (a) Inhomogeneous medium configuration. The source groups* $\boldsymbol{\sigma}^{(m)}$ *are situated at the surface and the receivers in a horizontal borehole (the black triangles at a depth of 200 m). (b) Correlation function. (c) Point-spread function. (d) Result of deblending by MDD. (Figures (b) − (d) show every fourth trace.)*

Analogous to equation (2), the relation between the correlation function and the Green's function is formulated as

$$C_{\rm sim}(\mathbf{x}_B, \mathbf{x}_A, t) = \int_{\mathbb{S}} \bar{G}_{\rm d}(\mathbf{x}_B, \mathbf{x}, t) * \Gamma_{\rm sim}(\mathbf{x}, \mathbf{x}_A, t)\, {\rm d}\mathbf{x}, \tag{5}$$

where $\Gamma_{\rm sim}(\mathbf{x}, \mathbf{x}_A, t)$ is the point-spread function for simultaneous-source acquistion, defined as

$$\Gamma_{\rm sim}(\mathbf{x}, \mathbf{x}_A, t) \quad = \quad \sum_m u^{\rm down}(\mathbf{x}, \boldsymbol{\sigma}^{(m)}, t) * u^{\rm down}(\mathbf{x}_A, \boldsymbol{\sigma}^{(m)}, -t). \tag{6}$$

This point-spread function is shown in Figure 2c for fixed $\mathbf{x}_A$ (the red dot in Figure 2a) and variable $\mathbf{x}$ (the black triangles in Figure 2a). Similar as in interferometry by MDD, the Green's function is retrieved from the correlation function by removing the effect of the point-spread by inverting equation (5). The result is shown in Figure 2d. This response accurately resembles the directly modeled Green's function $\bar{G}_{\rm d}(\mathbf{x}_B, \mathbf{x}, t)$ (not shown here).

Conclusions

We have shown that deblending can be seen as a form of seismic interferometry by multidimensional deconvolution (MDD). Because seismic interferometry is a form of data-driven redatuming, we have discussed deblending also as a form of data-driven redatuming. However, the wave fields involved in the deblending process, i.e., $u^{\rm down}(\mathbf{x}_A, \boldsymbol{\sigma}^{(m)}, t)$ and $u^{\rm up}(\mathbf{x}_B, \boldsymbol{\sigma}^{(m)}, t)$, are not necessarily measured wave fields in a borehole, but can also be obtained by forward and inverse extrapolation of the source groups and their responses, respectively, from the surface to a datum in the subsurface (assuming of course that a model of the overburden is available). In the presentation we will show that the scheme discussed here can be transformed into a deblending scheme that acts directly on the data at the surface. We will also discuss how it is possible that the seemingly underdetermined deblending problem can be solved by a direct inversion process rather than by an iterative procedure.

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