seismic migration using the wave equation

BOUNDARY CONDITION 1
UNMIGRATED TIME SECTION

X

t

T

HERE THE RE-
CURSION STARTS

BOUNDARY CONDITION 2
NO REFLECTIONS
BELOW T_{max}

BOUNDARY
CONDITION 3
LATERAL
CONTINUITY

SOLUTION PLANE
$t = T$
MIGRATED SECTION
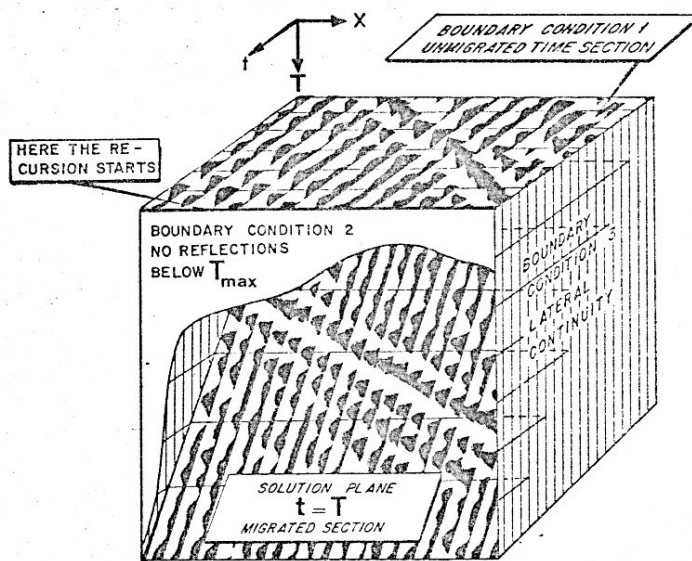
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Cover page of the 'kubus-diktaat' (1978)

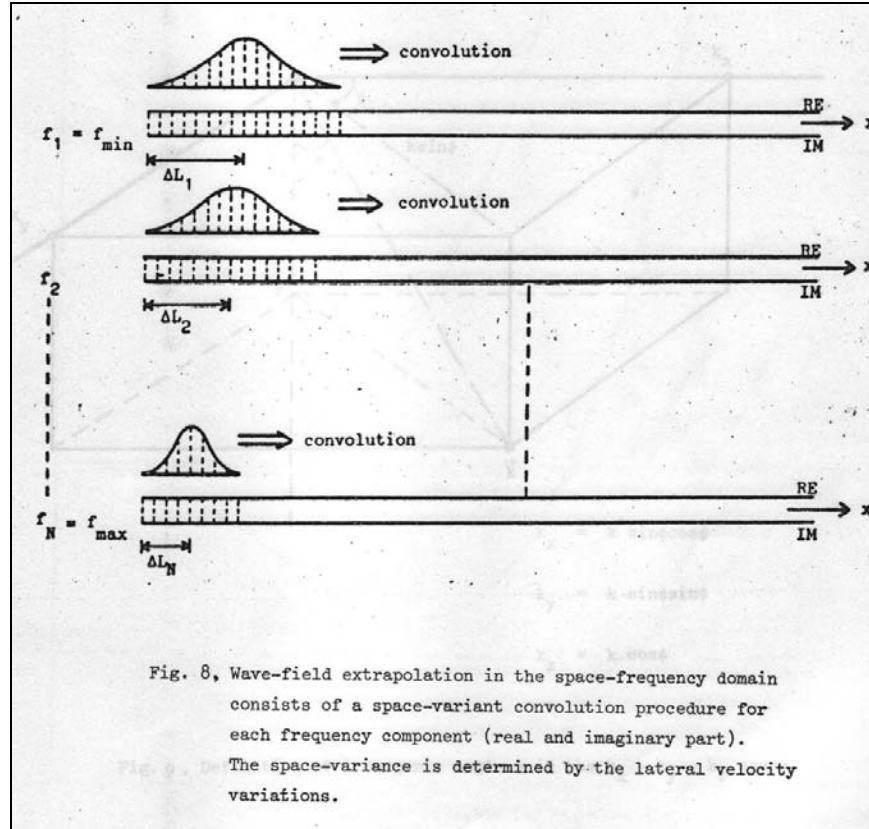# The green line of seismic wave theory and processing

Kees Wapenaar

It is autumn 1978. A couple of weeks ago I entered the group of Acoustics as a fourth-year student in Applied Physics. To familiarize myself with the fundamentals of acoustic imaging, I am studying the so-called 'kubus-diktaat', a weighty syllabus written by professor Berkhout, and the predecessor of his successful book 'Seismic migration'. I am reading about Green's theorem and get fascinated by its beautiful application in wave theory: given an acoustic wave field on a closed surface and the parameters of the medium enclosed by that surface, the wave field anywhere in the enclosed medium is represented by the Kirchhoff integral, derived from Green's theorem. A beautiful and precise mathematical formulation of the well-known Huygens principle, which I first encountered a couple of years earlier at high school.

Conclusions
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1) If we know on a closed surface S the sound pressure p and the normal component of the particle velocity v_n, then the sound pressure can be computed in every point inside S with the aid of Kirchhoff-integral eqn (III-4b).
   It is interesting to see that from the particle velocity, only should the normal component be known on S.

2) Using the results of chapter IIF, we may conclude that the Kirchhoff integral states that each pressure field may be synthesized by means of a monopole and dipole distribution on a closed surface S. The strength of each monopole is given by the normal component of the velocity on S, the strength of each dipole by the pressure on S.
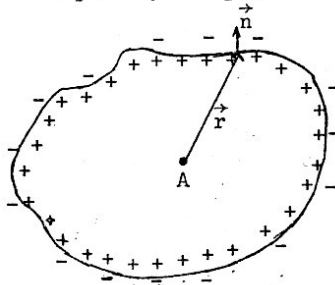
It gets even more interesting a number of pages later, when professor Berkhout transforms this mathematical concept into an algorithm for wave field extrapolation, which can be summarized as follows: decompose the time-domain registrations at each receiver into monochromatic components, re-order the results and apply a spatial convolution along the receiver array for each frequency component. The convolution operator (or 'spatial wavelet') in this algorithm is the Green's function, which played a central role in Green's theorem a few pages back.

Fig. 8, Wave-field extrapolation in the space-frequency domain consists of a space-variant convolution procedure for each frequency component (real and imaginary part). The space-variance is determined by the lateral velocity variations.

The crown on this line of thought comes a couple of chapters later, when professor Berkhout argues that, since the forward problem can be formulated as a spatial convolution process, the inverse problem (seismic migration) is nothing but a spatial *de*convolution. In 1979 he published this, together with the late professor van Wulfften Palthe, in their classic paper 'Migration in terms of spatial deconvolution' in Geophysical Prospecting. This systematic approach set out in the 'kubus-diktaat' is exemplary for Berkhout's research during his entire career.

The combination of the mathematical aspects of wave theory with concepts from signal processing made a deep impression on me as a 22 year old student. Even my fascination today for the applications of Green's theorem in seismic interferometry originally stems from reading the 'kubus-diktaat' in the autumn of 1978. But I'm running ahead now, so let's go back for a moment to the 1980's and 1990's.

During my PhD research I did fundamental research on acoustic and elastodynamic wave field extrapolation and published a series of papers on this together with prof Berkhout. In addition I implemented a three-dimensional target-oriented prestack migration scheme on a workstation, less powerful than the PC on which I'm writing this paper. The flow chart below for target-oriented migration is reminiscent of the 'kubus-diktaat'.
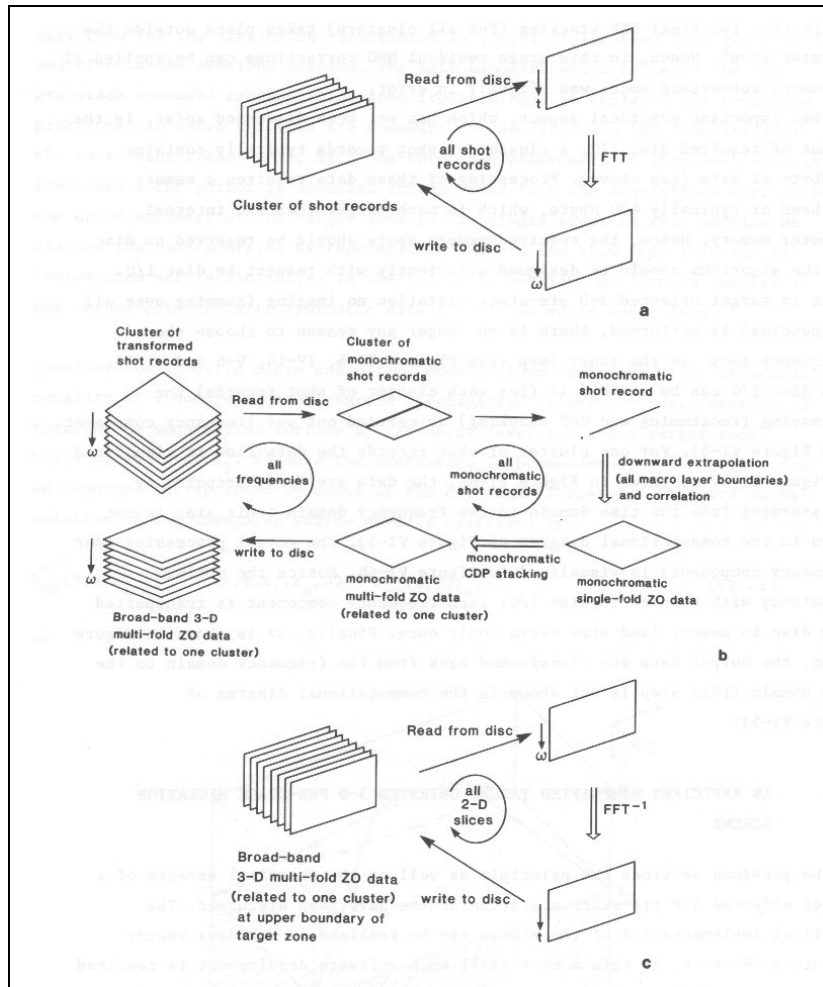
Figure VI-6 of my PhD thesis (1986).

Between my PhD graduation in 1986 and my appointment as professor in 1999 my main effort was the co-projectleadership of the Delphi consortium, established by Guus in 1986 (note that 'Berkhout' became 'Guus' after my PhD). The Delphi consortium project was and still is the perfect example of the system-oriented research approach advocated by Guus. It rightfully receives worldwide recognition from industry and academia.

During those years Guus and I published a lot together, essentially on the many aspects of 'the application of Green's theorem in seismic processing'. It was a productive cooperation: all together we co-authored one book, twenty journal papers, numerous conference proceedings and, not in the last place, the yearly Delphi consortium books, which all exceeded the 'kubus-diktaat' in weight. We didn't use LaTeX in those days, so the manuscript editing involved a lot of handwork:
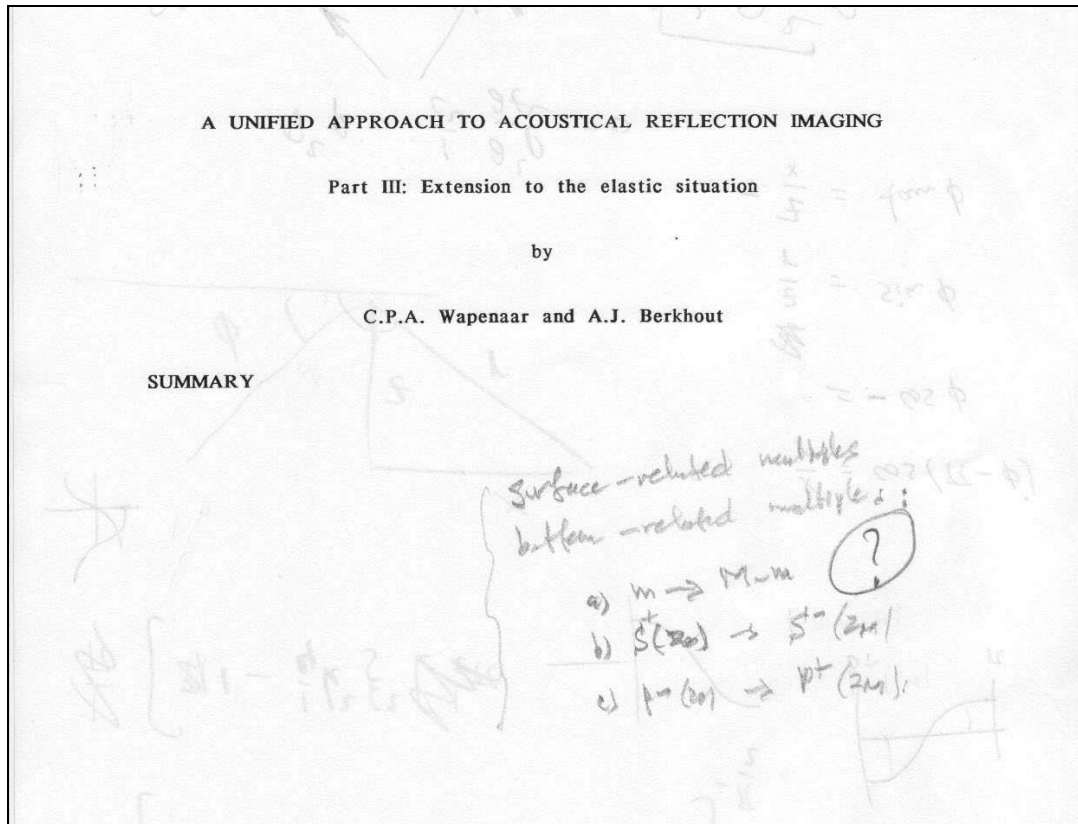
Title page of the manuscript of a paper which appeared in JASA in 1993.
Through the years I've learned to decipher Guus' scribblings flawlessly.

Approximately simultaneously with Guus' move to the university board in 1998, I moved to the Earth Science Department (which today is called the Department of Geotechnology). In the following I want to highlight one particular line of current research.

In 2001 our section was visited by Professor Harvey Butcher, director of Astron, who gave a very interesting presentation about the Lofar project (Low Frequency Array), which involves the construction of a major multi-element interferometric radio telescope with stations spread over the northern part of the Netherlands and a part of Germany. The construction of such a telescope implies the development of an infrastructure for the measuring network, including facilities for fiber-optic data transport, massive data storage and parallel number crunching. Professor Butcher offered us the possibility to join the project and share the facilities for geophysical measurements. This was the start of the Persimmon project (Permanent Seismic Monitoring Network), co-sponsored by ISES, and carried out in cooperation with TNO and the KNMI. The network will be fully operational in 2009 and will be used for the imaging of structures and the observation of processes in the subsurface of the Netherlands. However, the main reason for mentioning this project is that it challenged me to think about ways of processing the passive data gathered with this network. I remembered having seen a paper by Claerbout in Geophysics from 1968 in which he showed that the autocorrelation of the transmission response of a horizontally layered medium is identical to the reflection response of the

same medium. If it would be possible to generalize this concept to 3-D inhomogeneous media, we would have a means to use the Persimmon network in a passive mode and to obtain the subsurface reflection response just by cross-correlating the passive noise registrations at the different stations. The solution appeared to lie in …. a combination of Green's theorem with some concepts of signal processing. Let's see how this works.
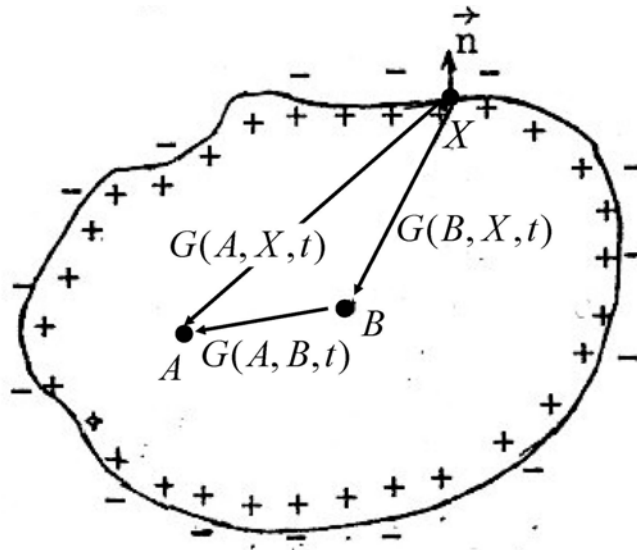
The response at *A* of a source at *B* can be synthesized from the cross-correlation of
responses of a monopole and dipole distribution on a closed surface.

Consider recordings *G(A,X,t)* and *G(B,X,t)* at two receivers *A* and *B* due to impulsive monopole and dipole sources at *X* on an arbitrary surface enclosing these receivers. Take the cross-correlation of these recordings for each source [i.e., *G(A,X,-t)*G(B,X,t)*] and integrate the results along the sources at *X* on the closed surface. According to a modified form of Green's theorem this gives the Green's function *G(A,B,t)*, which is the response at *A* as if there were an impulsive source at *B*, plus its time-reversed version *G(A,B,-t)*. Hence, new sources can be synthesized at positions where only receivers are available! Moreover, one doesn't need to know anything about the medium: all the required information is already implicitly present in the measurements *G(A,X,t)* and *G(B,X,t)* and is unraveled by the cross-correlation and integration. This theorem holds for any inhomogeneous medium and it easily extends to the full elastic situation. If the receivers are situated on the earth's free surface, the closed surface containing the sources can be replaced by an arbitrary open surface in the subsurface. This was the wave theoretical part of the derivation.

Next, consider a distribution of uncorrelated noise sources instead of impulsive point sources. Using the modified form of Green's theorem and some simple concepts of signal processing it easily follows that a direct cross-correlation of the acoustic noise responses *N(A,t)* and *N(B,t)* at *A* and *B* gives the Green's function *G(A,B,t)* (plus its time-reversed version), convolved with the autocorrelation *S(t)* of the noise sources. Hence, the integration along the sources is not necessary anymore (since the measured signals *N(A,t)* and *N(B,t)* each consist of a superposition of the noise responses of all sources).
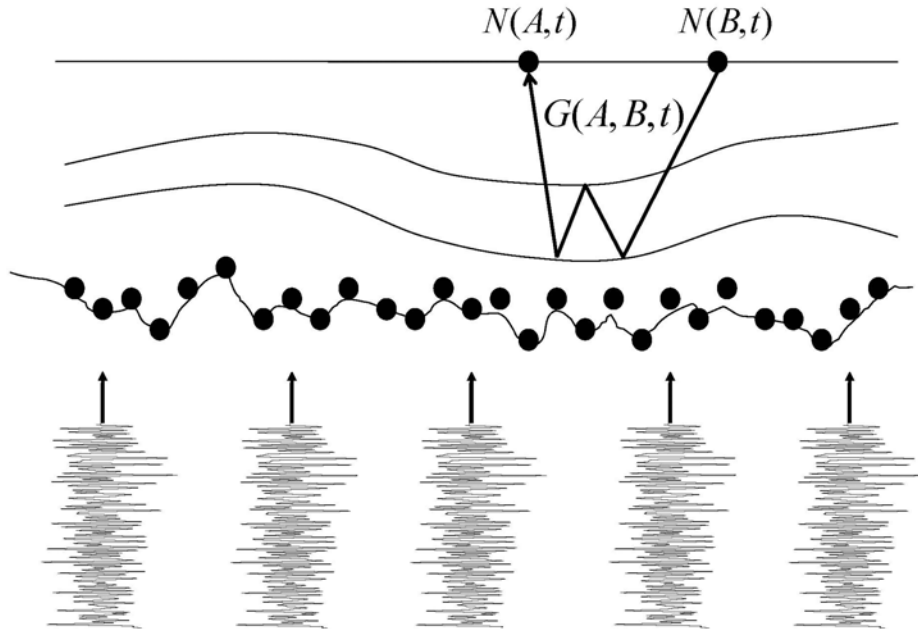
Turning noise into signal: the cross-correlation of the noise responses at *A* and *B*
yields the impulse response at *A* as if there were a source at *B*.

Hence,

$$\{G(A,B,t) + G(A,B,-t)\} * S(t) = \langle N(A,-t) * N(B,t) \rangle, \qquad (1)$$

where $<\ >$ denotes a spatial ensemble average, which in practice is approximated by averaging long enough over time. Since $G(A,B,t)$ is a causal function of time, it does not overlap with $G(A,B,-t)$, so it is easily obtained from the left-hand side of equation (1).

Once again, the combination of wave theory (Green's theorem) and signal processing (cross-correlation of noise signals) led to a new seismic processing scheme. The amazingly simple equation (1) shows how noise is turned into signal by cross-correlation. It can be generalized to a large class of applications, including waves and diffusion in flowing media, electromagnetic wave and diffusion phenomena, bending waves in mechanical structures, electroseismic waves in poroelastic media, propagators in quantum mechanics, etc. Recently my PhD student Deyan Draganov applied equation (1) successfully to real data, in cooperation with Shell. In a desert area in the middle-east, passive data were recorded by an array of 17 geophones for about ten hours. Draganov cross-correlated these data for all combinations of geophones and obtained seismic shot records as if there were active sources at all the geophone positions. The results compare nicely with reflection data acquired along the same line.

These developments occurred more or less in parallel with similar developments in other branches of science at several places around the world. In ultrasonics it was shown that the noise of thermal fluctuations in a specimen can be turned into pulse-echo

measurements. In regional geophysics, recordings of ambient seismic noise have been transformed into surface wave responses to reconstruct the crustal structure of southern California and other regions. In exploration seismics the group of prof Jerry Schuster pioneered interferometric imaging of controlled source data. A special issue of Geophysics (July-August, 2006) is dedicated to an overview of all these applications. Despite the large differences in the underlying theories and the practical implementations, all these methods have in common that cross-correlations are used to generate new data. Today the common denominator for these methods (at least in geophysics) is 'seismic interferometry'.

One of the most surprising aspects of seismic interferometry is its robustness. In the age of chaos theory, one would expect a strong dependency on initial conditions. Nevertheless, seismic interferometry works, without knowledge of the positions of the sources (the initial conditions), the properties of the noise and the parameters of the medium. All that is needed is a good distribution of noise sources to get the impulse response of the earth. Randomness is no longer at odds with determinism, but has instead become a new tool providing insights into the deterministic response of the physical world (Nature, 2007, Chaos tamed, in press).

I hope I have conveyed my enthousiasm for doing seismic research. The seeds for this enthousiasm were planted almost 30 years ago when I read about Green's theorem and its application in seismic migration in Guus' 'kubus-diktaat'. When I read it again last week I relived the sense of magic I experienced 30 years ago. I treasure those days and the good years of productive cooperation.