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ESTIMATION OF EFFECTIVE STRESS FROM SEISMIC REFLECTION COEFFICIENTS AT A NON-WELDED INTERFACE

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SUMMARY

We investigate the possibility to estimate effective stress from reflection coefficients at a non-welded interface. The interface is represented by a planar distribution of penny-shaped cracks, resulting in a linear slip boundary condition with a normal and a tangential compliance. These compliances change as the planar crack density decays exponentially (due to crack closure) with effective stress. The media above and below the interface are VTI and their elastic coefficients are taken from empirical data (dependent on effective stress and pore infill). We compute quasi-PP, -SvP, -SvSv and -PSv reflection coefficients for several realistic scenarios with varying angles of incidence at different stress states with different pore infill (brine or gas). Both effective stress and pore infill affect the model in two ways: a) through the elastic coefficients of the upper and lower media, and b) through the compliances of the interface. Both effects have significant impact on the computed reflection coefficients and their interplay can easily cause misinterpretations. Further, we invert the synthetic reflection coefficient data as generated by the forward model for the effective stress components. Our analysis reveals that the inversion requires a priori, measurements of the vertical wave velocities at each stress state, an estimation of anisotropy and some (theoretical or empirical) relation between fracture compliance and effective stress. We introduce error bars in the estimated stress considering errors in the input wave velocities. Our inversion results demonstrate that we are able to estimate effective stress, assuming that an accurate stress-compliance relation is available.

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INTRODUCTION

Reflection of elastic waves at imperfectly bonded interfaces can be modeled using a linear slip boundary condition, as introduced by Schoenberg (1980). Linear slip means continuity of traction and a jump of displacement, being linearly related to the traction. Schoenberg (1980) defined the ratio of the displacement jump and traction as the interface compliance. It is pointed out by many authors that this compliance decreases with increasing effective stress (= overburden stress minus pore pressure), due to the increase of welded area at the interface. As a result, reflection coefficients at these interfaces hold information on the effective stress in-situ and can possibly be used as a stress indicator. To investigate this, we modeled a non-welded interface as a planar distribution of penny-shaped cracks (Hudson et al., 1996); cracks closing with increasing effective stress (Tod, 2002). Since changes are relatively small, we have to account for stress dependence of the elastic coefficients above and below the interface, which are taken from empirical data by Wang (2002). Both media are Vertical Transverse Isotropic (VTI) and described by their densities and 5 stress dependent elastic coefficients: C_{11}, C_{13}, C_{33}, C_{44}, C_{66}. For each stress state the reflection coefficients are computed with varying angle of incidence. We derived an analytic expression straight from the boundary condition to invert for the interface compliance. Through a stress-compliance relation the stress can then be estimated. This method is successfully tested on the computed reflection coefficients.

THEORY

Consider a non-welded horizontal interface in the (x, z)-plane. Following Schoenberg (1980), the traction at the interface (τ_{iz}) is continuous and the jump of particle velocity (v_i) is proportional to the traction:

$$\begin{pmatrix} \Delta v_x \\ \Delta v_z \end{pmatrix} = i\omega \underline{\underline{Z}} \begin{pmatrix} \tau_{xz} \\ \tau_{zz} \end{pmatrix}, \qquad (1)$$

$$\underline{\underline{Z}} = \begin{pmatrix} Z_T & 0 \\ 0 & Z_N \end{pmatrix}, \qquad (2)$$

Z_T and Z_N being the tangential and normal (interface) compliance, respectively, ω the angular frequency and i the imaginary unit.

Interface compliances

Hudson et al. (1996) showed how the tangential compliance of a planar distribution of cracks can be computed. Tod (2002) derived that for an exponential distribution of aspect ratios the crack number density decays exponentially with effective stress (σ_{eff}) due to crack closures. We can thus formulate a stress dependent expression for the tangential compliance. The normal compliance depends on the infill (gas or fluid) of the cracks. Note that the media above and below the interface contain horizontal fractures or inclusions, causing their transverse isotropic behavior. Schoenberg and Douma (1988) showed how a stack of non-welded fractures constructs an effective medium, and how the average compliance ratio of the individual fractures can be estimated from the elastic coefficients of that effective medium (C_{ij}) and the Lamé parameters of the unfractured background rock. This technique is used to estimate the compliance ratio of upper and lower media. Assuming this ratio holds at the interface too, we can compute the normal compliance from the tangential compliance.

Reflection model

A differential equation for wave propagation in VTI media is formulated in the frequency-wavenumber domain (Wapenaar and Berkhout, 1989):

$$\partial_z \begin{pmatrix} v_x & v_z & \tau_{xz} & \tau_{zz} \end{pmatrix}^T = \underline{\underline{A}} \begin{pmatrix} v_x & v_z & \tau_{xz} & \tau_{zz} \end{pmatrix}^T. \qquad (3)$$

T denotes transposition; matrix $\underline{\underline{A}}$ consists of the elastic coefficients, density and horizontal wavenumber k_x. An eigenvalue decomposition is performed on matrix $\underline{\underline{A}}$:

$$\underline{\underline{A}} = \begin{pmatrix} \underline{\underline{L}}_1^+ & \underline{\underline{L}}_1^- \\ \underline{\underline{L}}_2^+ & \underline{\underline{L}}_2^- \end{pmatrix} \begin{pmatrix} \underline{\underline{\Lambda}} & \underline{\underline{0}} \\ \underline{\underline{0}} & -\underline{\underline{\Lambda}} \end{pmatrix} \begin{pmatrix} \underline{\underline{L}}_1^+ & \underline{\underline{L}}_1^- \\ \underline{\underline{L}}_2^+ & \underline{\underline{L}}_2^- \end{pmatrix}^{-1}. \qquad (4)$$

Scaling of the eigenvectors is performed such that the wave field is decomposed into vectors that represent the particle velocities of the P- and Sv-mode. We can define reflection (R) and transmission (T) coefficient matrices (in terms of particle velocities). The non-welded boundary condition can now be formulated, yielding:

$$\begin{pmatrix} \underline{\underline{I}} & i\omega\underline{\underline{Z}} \\ \underline{\underline{0}} & \underline{\underline{I}} \end{pmatrix} \begin{pmatrix} \underline{\underline{L}}_{1,I}^+ & \underline{\underline{L}}_{1,I}^- \\ \underline{\underline{L}}_{2,I}^+ & \underline{\underline{L}}_{2,I}^- \end{pmatrix} \begin{pmatrix} \underline{\underline{I}} \\ \underline{\underline{R}} \end{pmatrix} = \begin{pmatrix} \underline{\underline{L}}_{1,II}^+ & \underline{\underline{L}}_{1,II}^- \\ \underline{\underline{L}}_{2,II}^+ & \underline{\underline{L}}_{2,II}^- \end{pmatrix} \begin{pmatrix} \underline{\underline{T}} \\ \underline{\underline{0}} \end{pmatrix}. \qquad (5)$$

A solution can be found analytically:

$$\underline{\underline{R}} = \left[\underline{\underline{\Xi}}^- - i\omega\underline{\underline{Z}}\,\underline{\underline{L}}_{2,I}^-\right]^{-1} \cdot \left[i\omega\underline{\underline{Z}}\,\underline{\underline{L}}_{2,I}^+ - \underline{\underline{\Xi}}^+\right], \qquad (6)$$

$$\underline{\underline{\Xi}}^\pm = \underline{\underline{L}}_{1,II}^+ \left(\underline{\underline{L}}_{2,II}^+\right)^{-1} \underline{\underline{L}}_{2,I}^\pm - \underline{\underline{L}}_{1,I}^\pm. \qquad (7)$$

Estimation of effective stress

Estimation of stress can be done either through the tangential or through the normal compliance. Formulations for Z_T and Z_N can be derived from the boundary conditions; in case of an incident P-wave :

$$Z_T = \frac{1}{\omega}\mathrm{Im}\left(\frac{\Xi_{11}^- R_{PP} + \Xi_{12}^- R_{SvP} + \Xi_{11}^+}{L_{2,11,I}^+ + L_{2,11,I}^- R_{PP} + L_{2,12,I}^- R_{SvP}}\right), \qquad (8)$$

$$Z_N = \frac{1}{\omega}\mathrm{Im}\left(\frac{\Xi_{21}^- R_{PP} + \Xi_{22}^- R_{SvP} + \Xi_{21}^+}{L_{2,21,I}^+ + L_{2,21,I}^- R_{PP} + L_{2,22,I}^- R_{SvP}}\right). \qquad (9)$$

Im being the imaginary part (double subscripts being the 2x2 matrix element numbers). The effective stress is estimated with an empirical exponential stress-compliance relation.

RESULTS

Here we discuss some results for a reservoir case study. Stress dependent elastic coefficients are taken from Wang (2002): lower interface is African sandstone E2; upper interface is African shale E1. We consider gas saturation at an initial effective stress of 7.86 MPa rising to 30.34 MPa. Figure 1 shows the magnitudes of R_{PP} and R_{SvP} at both stress states. Dashed lines show the result if a) only the change in interface compliance $\underline{\underline{Z}}$ is accounted for, and b) only the change in elastic coefficients C_{ij} of upper and lower media is accounted for. Note that both the effects have significant impact and neglecting either of them can easily cause misinterpretations. Note that the reflection coefficients are complex valued (due to the interface slip) and their phases have stress dependence too. We invert the data through equations (8) and (9), while introducing small error bars in the required vertical P- and S-wave velocities. Stress is computed using a (stress, compliance)-curve. Results are shown in figure 2. Note that inversion through normal compliance (eq. 9) works better than through tangential compliance (eq. 8). Especially inversion at normal angle of incidence seems fruitful, since R_{SvP} drops in equation (9), making it independent of the vertical S-wave velocity and solely expressible in terms of R_{PP}.

CONCLUSION

Our forward model results reveal that effective stress alters phase and magnitude of the reflection coefficients at a non-welded interface in two ways: a) through the slip at the interface, b) through the changing contrast of the media above and below the interface. Assuming our model is a good representation of reality; stress information can be obtained from the complex valued reflection coefficients in seismic data if a stress-compliance relation, an accurate velocity model (updated for each stress state) and an estimate of anisotropy are available. Normal compliance is a potential stress indicator upon use of PP and P-to-S reflection coefficients. The modeling studies show that the inversion is most accurate at normal incidence as it can be performed solely with PP.

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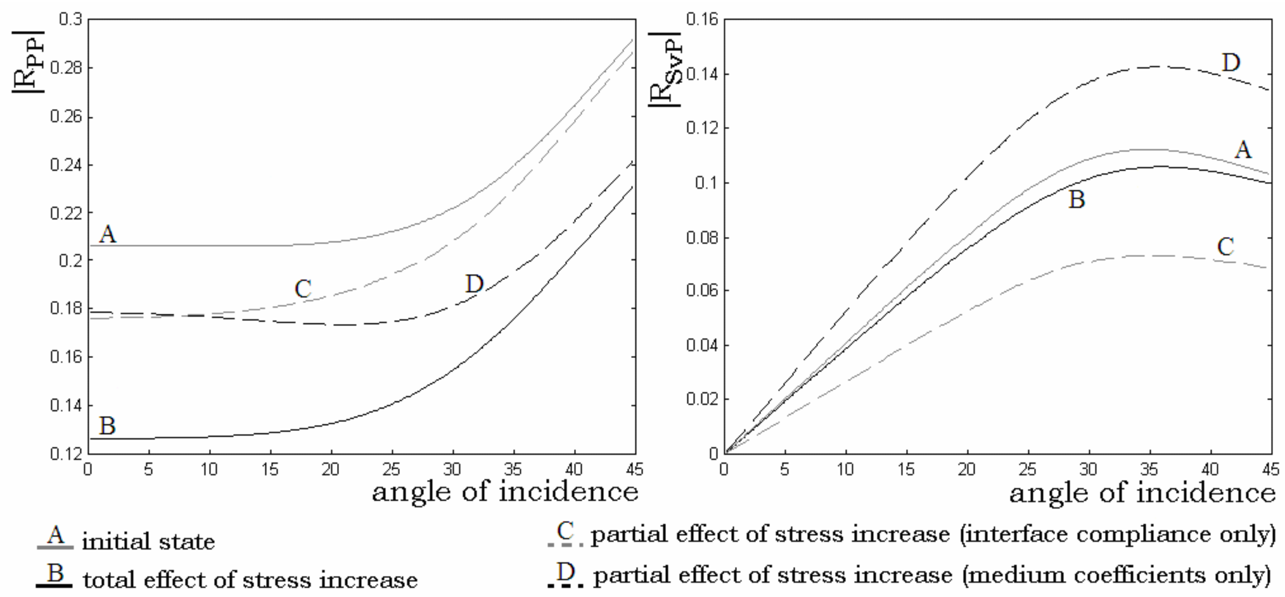

Figure 1 - Magnitudes of reflection coefficients PP and SvP - stress rise, initial state = 7.86 MPa, new state = 30.34 MPa (frequency = 80 Hz).

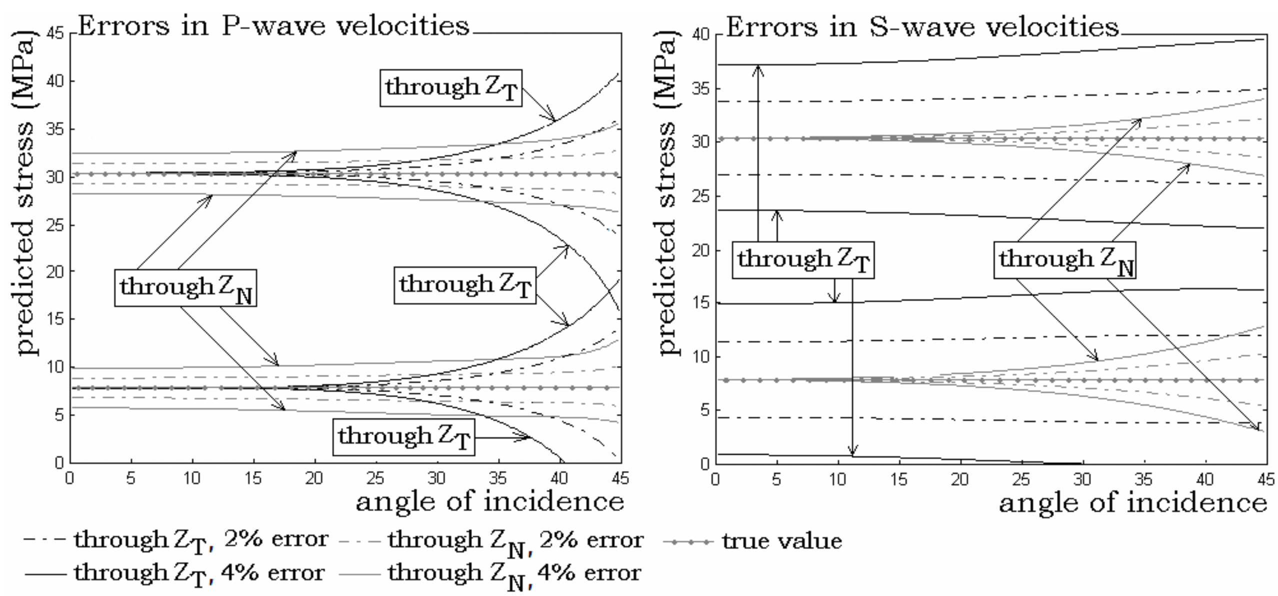

Figure 2 - Estimation of the effective stress states (7.86 and 30.34 MPa).