SCALING BEHAVIOUR OF THE TRANSMISSION RESPONSE OF RESERVOIR ROCK

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Introduction

A couple of years ago, ultrasonic transmission measurements have been carried out on Rotliegend reservoir sandstone samples (Den Boer, Dillen, Duijndam and Fokkema, 1996, EAGE; Swinnen, 1997, M.Sc. thesis, Leuven / Delft). The experiments were carried out for a range of different ambient pressures. Figure 1 shows the transmission responses for pressures ranging from 2 MPa (the latest arrival) to 20 MPa (the first arrival). It appeared that not only the arrival time reduces when the ambient pressure increases, but that the width of the wavelet reduces by approximately the same relative amount. In other words, the time-axis seems to be scaled by a single factor when the ambient pressure is changed from one value to another. Also the amplitude changes with changing pressure. It has been carefully checked that these time and amplitude changes are not source effects, but that it is the propagation through the sandstone that changes with changing pressure. The aim of this paper is to give a possible theoretical explanation for the observed scaling of the time axis.

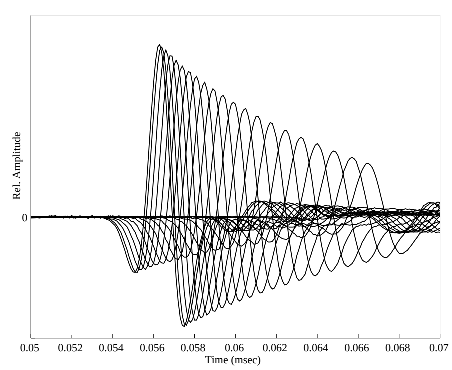

Figure 1: Transmission responses of Rotliegend reservoir sandstone for varying ambient pressure.

The binary layered medium approach

We start by making a strong simplification, that is, we assume that the sandstone is horizontally layered. Although this is not very realistic, it is a suitable starting point for studying the scaling behaviour analytically. The second assumption is that the layered medium consists of only two types of material (hence the name binary layered medium); the layer thicknesses are in some way randomly distributed. The third assumption is that changes in the ambient pressure do not affect the layer thicknesses, but only the material parameters. With these assumptions the depth-dependent normal-incidence plane-wave reflectivity $r(z)$ obeys the following scaling relation

$$r_B(z) = \alpha\, r_A(z), \tag{1}$$

where the subscripts A and B refer to two different ambient pressure states. When the material parameters of both layer types react similarly (in a relative sense) to changes in the ambient pressure then $\alpha = 1$; when they react differently then $\alpha \neq 1$. The average slowness $\bar{s}$ of the material obeys the following relation

$$\bar{s}_B = \beta\, \bar{s}_A. \tag{2}$$

It is beyond the scope of this paper to specify the scaling factors α and β and their mutual relation. For our analysis it is sufficient to assume that relations (1) and (2) hold for some value of α and β. Mercerat (2001, M.Sc. thesis, Delft University) has carried out numerical modeling experiments with binary layered media and observed a scaling behaviour of the transmission response, similar to that observed in Figure 1. In the following section we will evaluate the scaling behaviour of the transmission response of binary layered media analytically.

Scaling behaviour of the transmission response

The normal-incidence plane-wave transmission response of a layered medium can be expressed in the frequency domain in terms of a 'generalized primary' propagator $\mathcal{W}(z_1, z_0, \omega)$, according to

$$\begin{aligned} \mathcal{W}(z_1, z_0, \omega) &= \mathcal{P}(z_1, z_0, \omega)\mathcal{M}(z_1, z_0, \omega) \\ &= \exp\{-j\omega\bar{s}\Delta z\} \exp\{-\mathcal{A}(2\omega\bar{s})\Delta z\}, \quad \Delta z = z_1 - z_0, \end{aligned} \tag{3}$$

where the first exponential describes the (flux-normalized) primary propagation from depth level z_0 to z_1 and the second exponential accounts for the internal multiples generated between those two depth levels. The function $\mathcal{A}$ is the Fourier transform of the 'causal part' of $\mathcal{S}(z)$, according to

$$\mathcal{A}(k) = \int_0^\infty \exp\{-jkz\}\mathcal{S}(z)\mathrm{d}z, \tag{4}$$

where $\mathcal{S}(z)$ is the autocorrelation of the reflection function $r(z)$. Note that equation (3) is the well-known O'Doherty-Anstey relation, except that $\mathcal{A}(k)$ in equation (4) is expressed in terms of a spatial rather than a temporal autocorrelation function. The depth-time conversion takes place in equation (3), where $\mathcal{A}(k)$ is evaluated at $k = 2\omega\bar{s}$. Assuming $r(z)$ obeys equation (1), $\mathcal{A}(k)$ has the following scaling behaviour

$$\mathcal{A}_B(k) = \alpha^2 \mathcal{A}_A(k), \tag{5}$$

where the subscripts A and B refer again to two different ambient pressure states. For these two pressure states the generalized primary propagators read

$$\mathcal{W}_A(z_1, z_0, \omega) = \mathcal{P}_A(z_1, z_0, \omega)\mathcal{M}_A(z_1, z_0, \omega) = \exp\{-j\omega\bar{s}_A\Delta z\} \exp\{-\mathcal{A}_A(2\omega\bar{s}_A)\Delta z\} \tag{6}$$

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or, using equations (2) and (5),

$$\begin{aligned} \mathcal{W}_B(z_1, z_0, \omega) &= \exp\{-j\beta\omega\bar{s}_A\Delta z\} \exp\{-\alpha^2\mathcal{A}_A(2\beta\omega\bar{s}_A)\Delta z\} \\ &= \mathcal{P}_A(z_1, z_0, \beta\omega)[\mathcal{M}_A(z_1, z_0, \beta\omega)]^{\alpha^2}. \end{aligned} \tag{8}$$

Discussion and conclusions

Equation (8) quantifies the scaling behaviour of the transmission response. Note that in both terms at the right-hand side the frequency is scaled with the same factor β. This agrees with our earlier observation that the arrival time and the width of the wavelet scale by approximately the same amount when the ambient pressure is changed. The exponent α^2 in the second term accounts for the amplitude change. Since this exponent is applied to a frequency-dependent term, there is not a simple scaling relation in the time-domain.

Although we have made a number of simplifying assumptions, it is worthwile to use equation (8) as a first approximate model for observations like those in Figure 1. Estimating the parameters α and β from that type of measurements for a range of different ambient pressures gives valuable information about the pressure-dependent behaviour of the reservoir rock.