Turning noise into signal: a new twist of Green's theorem

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Since the last turn of the century, developments in seismology, ultrasonics and underwater acoustics have caused a significant change in the way we think about diffuse wave fields and noise. Contrary to their definition, diffuse wave fields appeared not to be fully disorganized and without any information. It has been shown theoretically and experimentally that the heterogeneities and boundaries of the medium in which a diffuse wave field exists leave an imprint on the wave field which is characteristic for the medium. What was even more surprising was the fact that the inherited information about the medium can be retrieved in a very simple manner, namely by cross-correlation. To be more specific, if one measures the noise of a diffuse wave field at two arbitrary points in space and determines the cross-correlation between these two noise registrations, one obtains the impulse response of the medium (i.e., the Green's function) that would be measured if there were a source at one of the two points and a receiver at the other. In other words, by passively listening at noise and applying cross-correlation, one obtains the same information as would be obtained in a controlled experiment with manmade sources. It appears that this is true no matter how complex the heterogeneities and boundaries of the medium. This very simple principle has far-reaching consequences. To mention a few examples: in the field of ultrasonics noise of thermal fluctuations in a specimen has been successfully turned into pulse-echo measurements of the specimen, in regional seismology recordings of ambient seismic noise have been used to reconstruct the crustal structure of southern California and in exploration seismology recordings of background noise in a desert area have been turned into seismic reflection measurements of the area.

The theory behind the principle of 'Green's function retrieval by cross-correlation of noise' can be approached via different angles. In the presentation I will start with a specific version of Green's theorem as used in holographic imaging (Porter, JOSA 1970) and show that with some simple modifications this can be shaped into a theory for turning noise into signal.