Data-driven Green's function retrieval from reflection data

Kees Wapenaar, Evert Slob, Filippo Broggini,
Roel Snieder, Jan Thorbecke and Joost van der Neut

Outline

- Introduction
- Focusing functions
- Green’s function retrieval
- Imaging
- Conclusions

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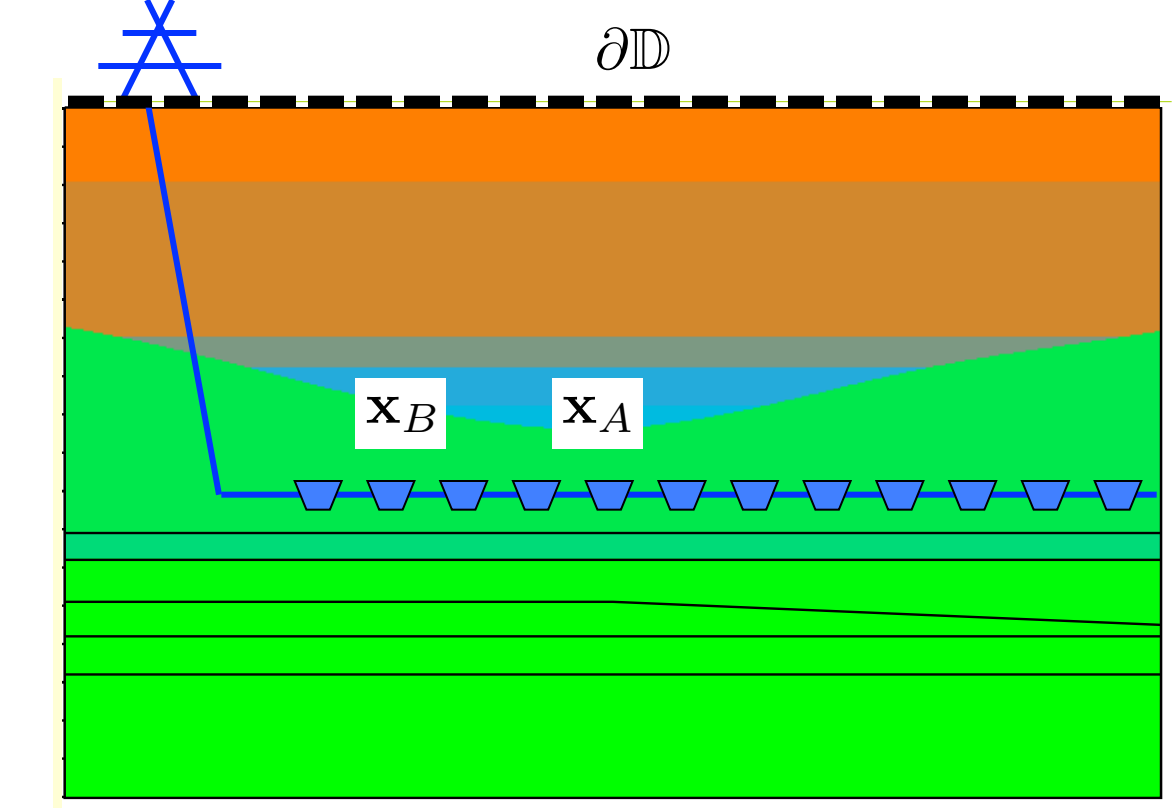
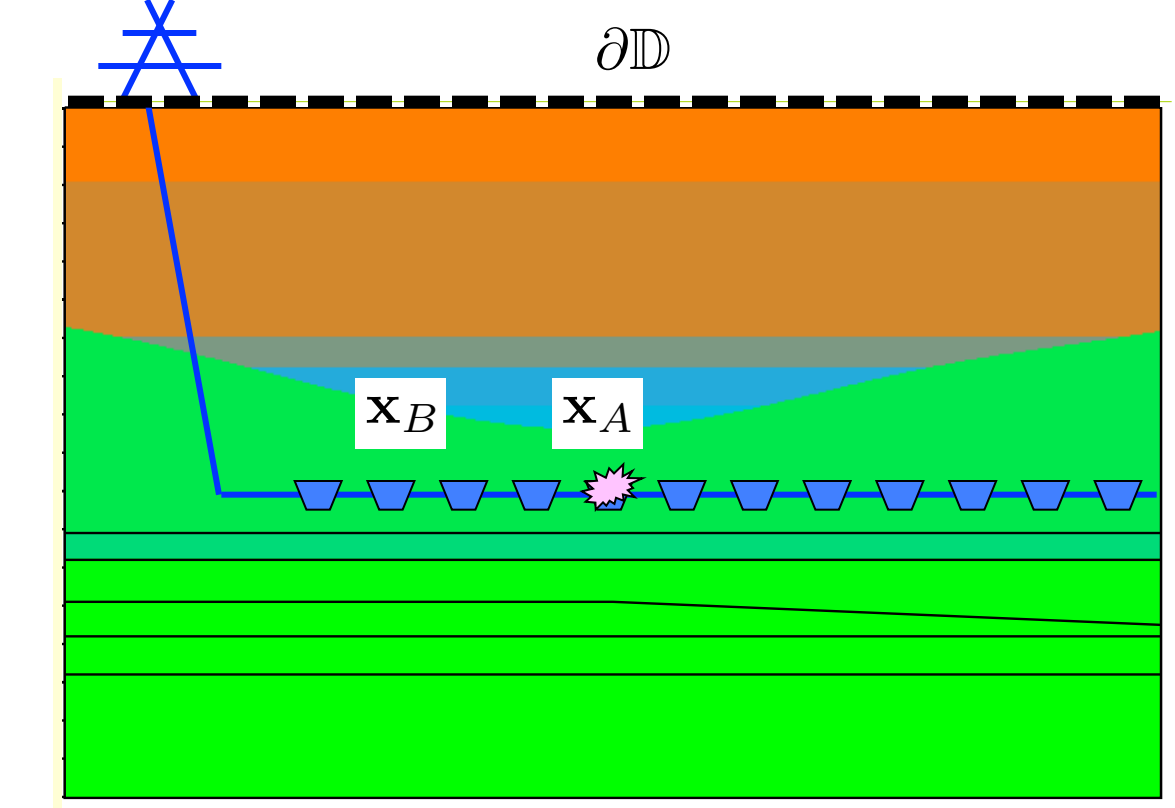


Interferometric Green's function representation:

$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \int_{\partial\mathbb{D}} \mathrm{d}\mathbf{x} \int G(\mathbf{x}_B, \mathbf{x}, t + t') G(\mathbf{x}_A, \mathbf{x}, t') \mathrm{d}t'$$

(Phys. Rev. Lett., 2004)



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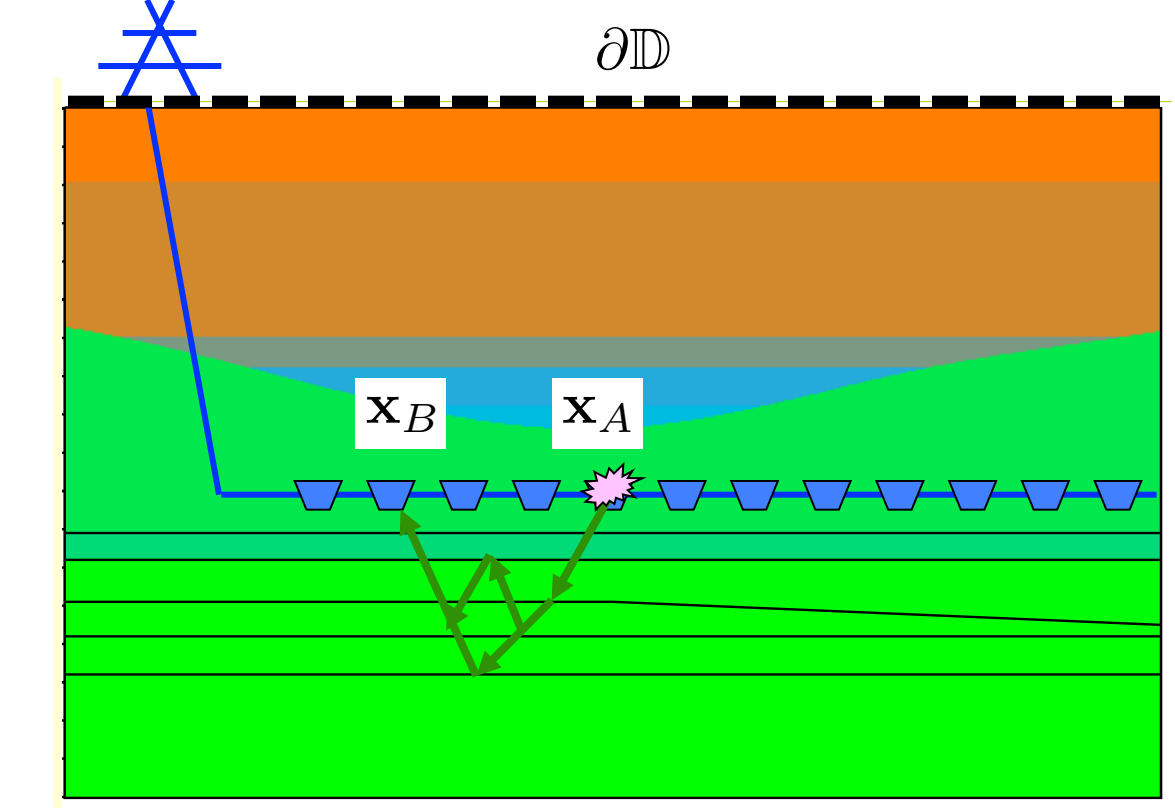

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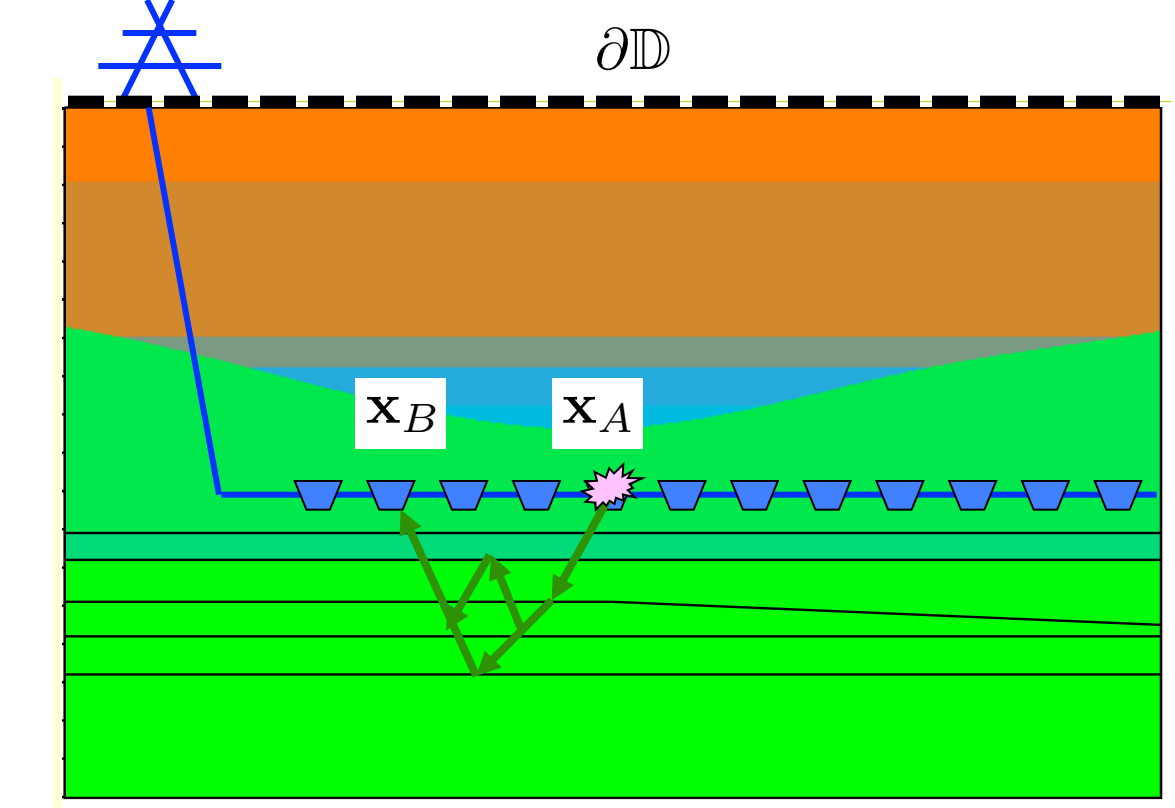

Interferometric Green's function representation:

Receivers *inside* the medium, at VS position

$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \int_{\partial\mathbb{D}} \mathrm{d}\mathbf{x} \int G(\mathbf{x}_B, \mathbf{x}, t + t') G(\mathbf{x}_A, \mathbf{x}, t') \mathrm{d}t'$$

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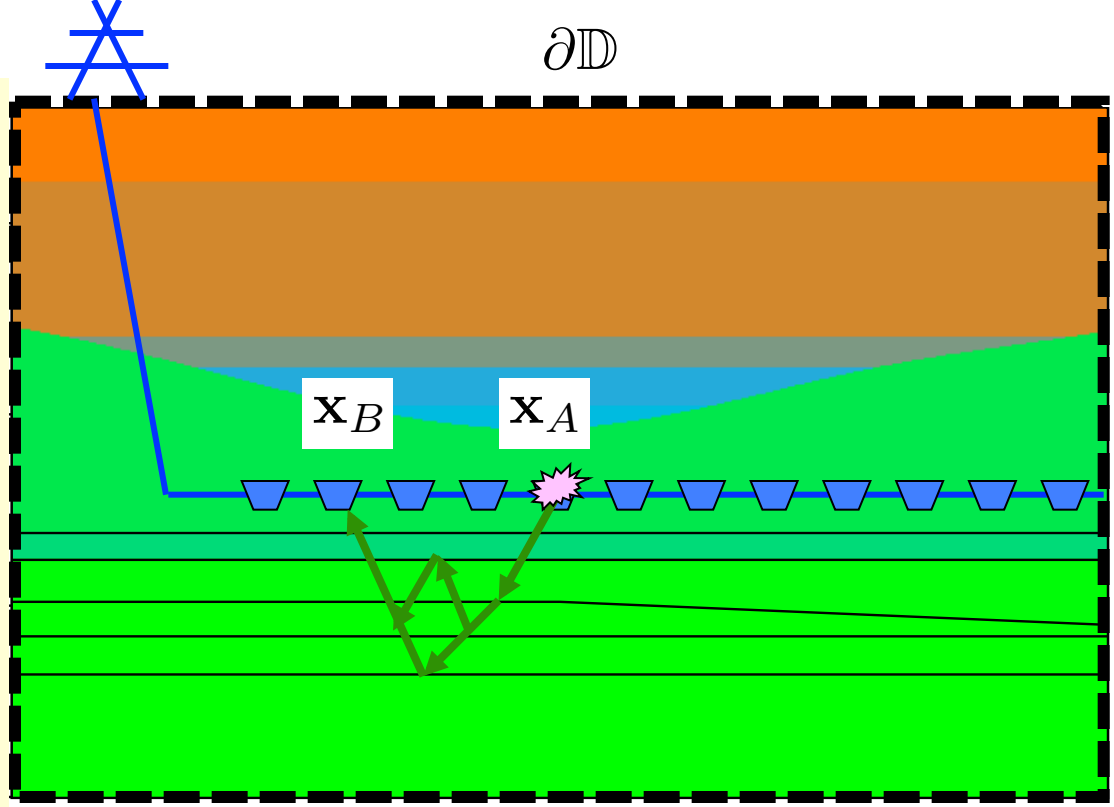

Interferometric Green's function representation:

Receivers *inside* the medium, at VS position

Sources at $\partial\mathbb{D}$, *enclosing* the medium

$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \int_{\partial\mathbb{D}} \mathrm{d}\mathbf{x} \int G(\mathbf{x}_B, \mathbf{x}, t + t') G(\mathbf{x}_A, \mathbf{x}, t') \mathrm{d}t'$$

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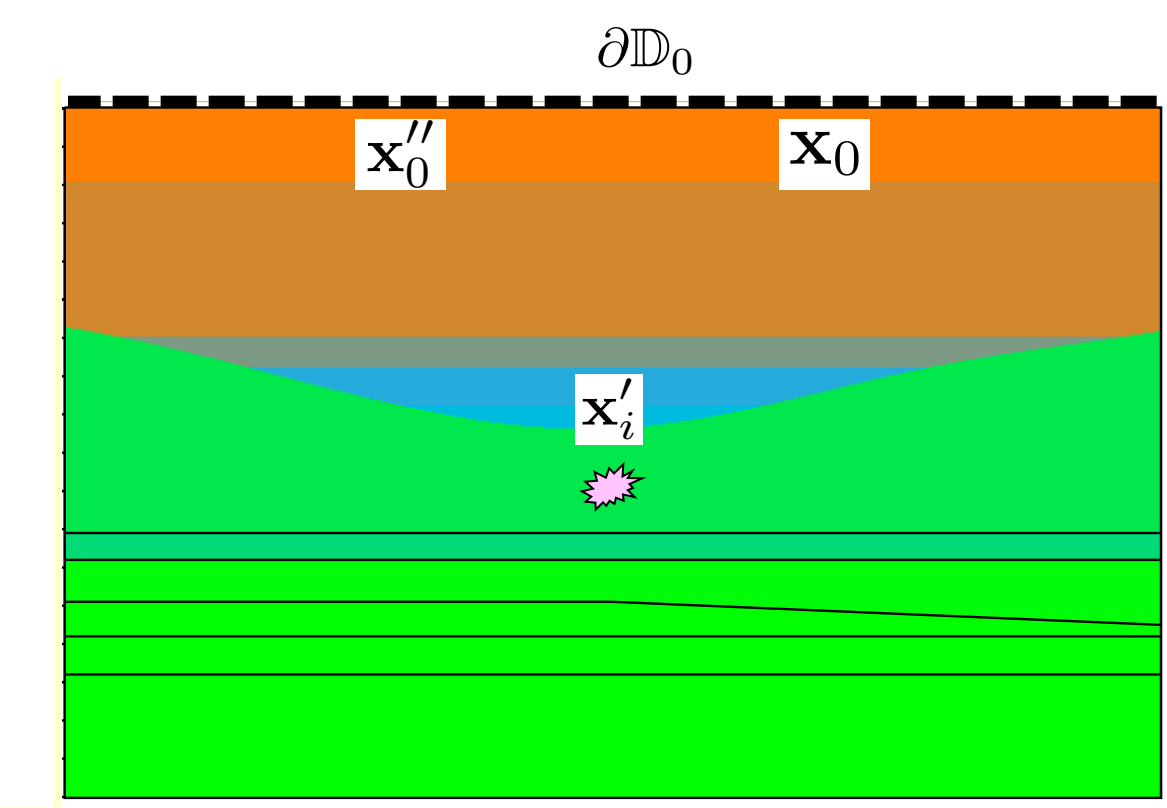
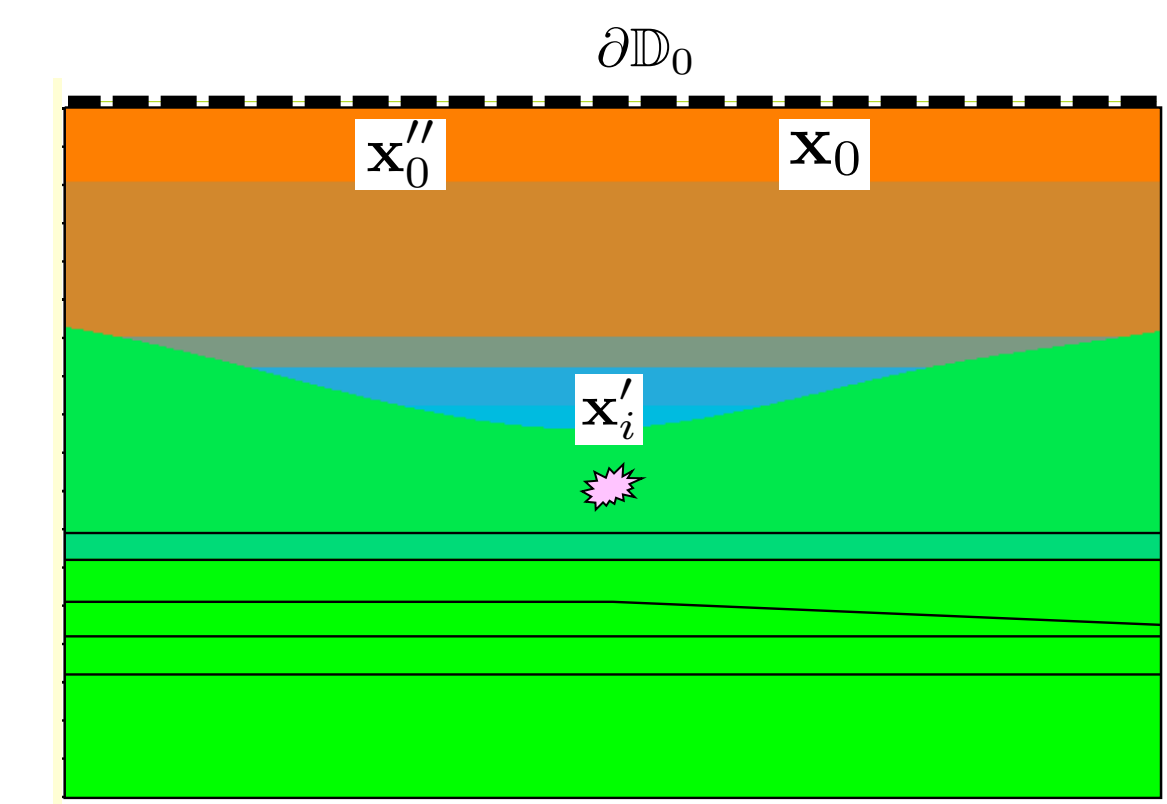


Single-sided Green's function representation:

$$G(\mathbf{x}_0'', \mathbf{x}_i', t) = f(\mathbf{x}_i', \mathbf{x}_0'', -t) + \int_{\partial\mathbb{D}_0} \mathrm{d}\mathbf{x}_0 \int_{-\infty}^{t} R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f(\mathbf{x}_i', \mathbf{x}_0, t') \mathrm{d}t'$$

(Phys. Rev. Lett., 2013)



Single-sided Green's function representation:

No receiver at VS!

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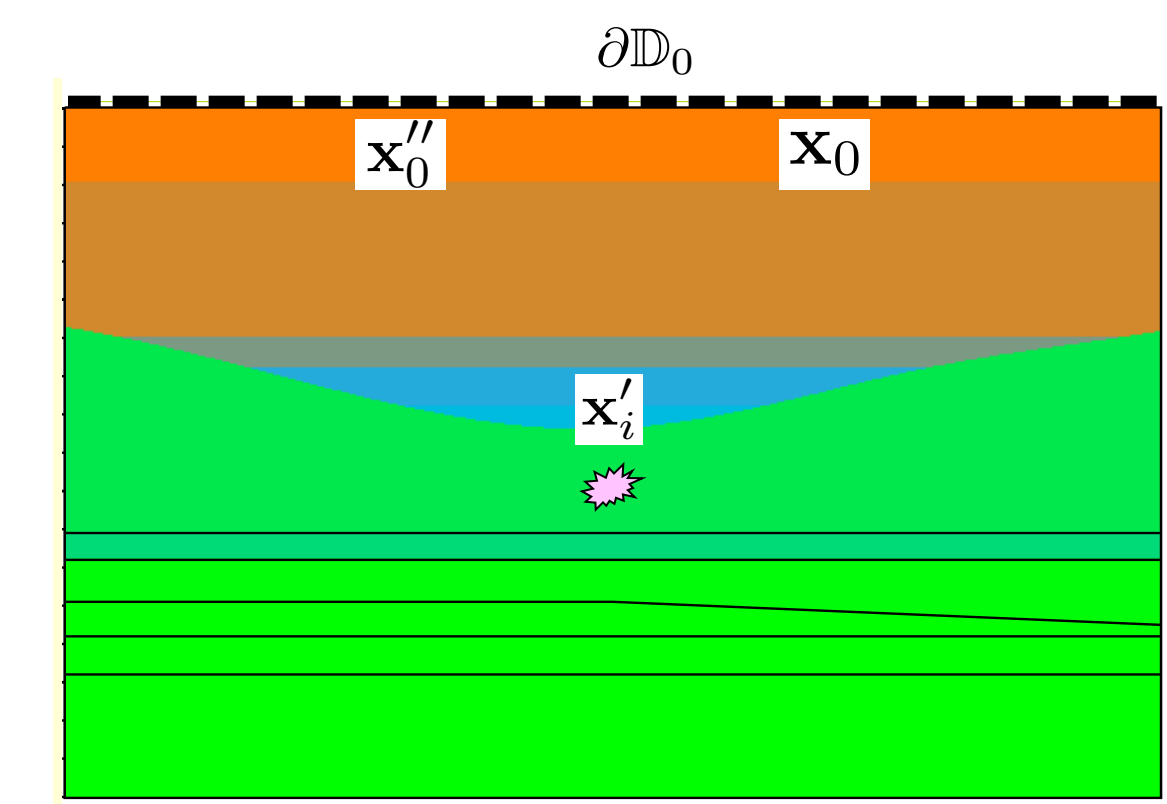

Single-sided Green's function representation:

No receiver at VS!

Sources *and* receivers at $\partial\mathbb{D}_0$, at *one side* of the medium

$$G(\mathbf{x}_0'', \mathbf{x}_i', t) = f(\mathbf{x}_i', \mathbf{x}_0'', -t) + \int_{\partial\mathbb{D}_0} \mathrm{d}\mathbf{x}_0 \int_{-\infty}^{t} R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f(\mathbf{x}_i', \mathbf{x}_0, t') \mathrm{d}t'$$

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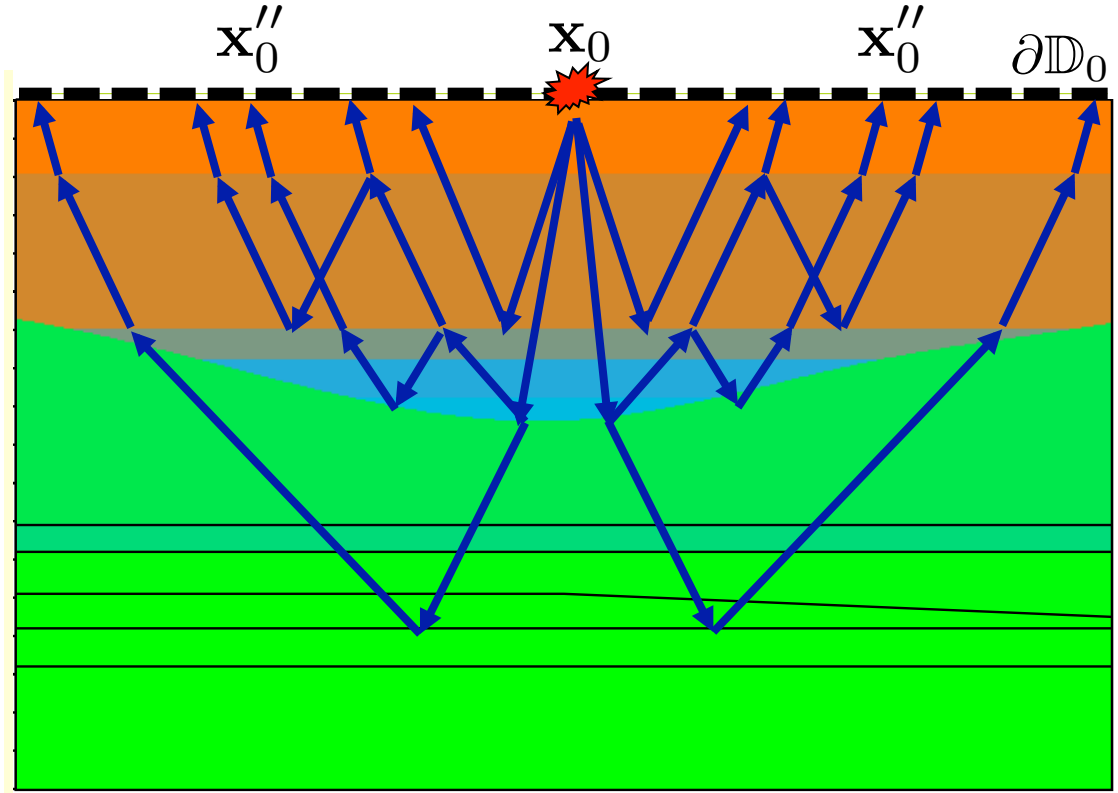

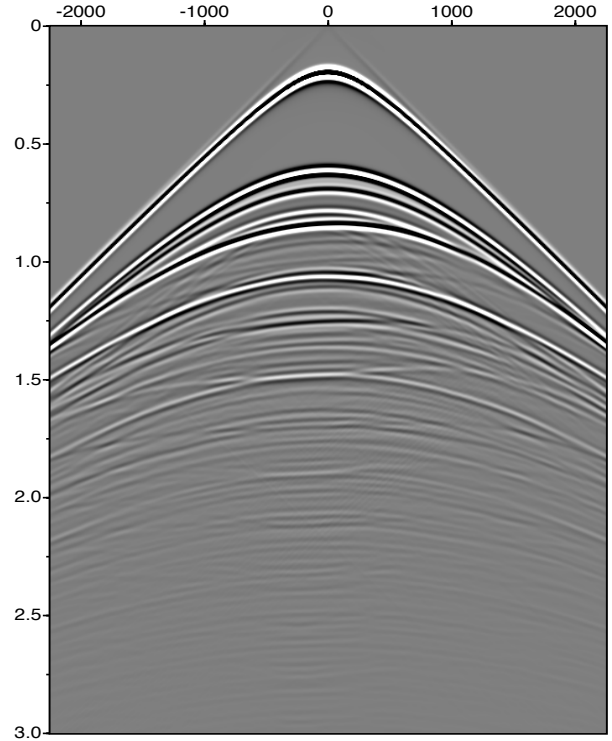

$$G(\mathbf{x}_0'', \mathbf{x}_i', t) = f(\mathbf{x}_i', \mathbf{x}_0'', -t) + \int_{\partial\mathbb{D}_0} \mathrm{d}\mathbf{x}_0 \int_{-\infty}^{t} R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f(\mathbf{x}_i', \mathbf{x}_0, t') \mathrm{d}t'$$

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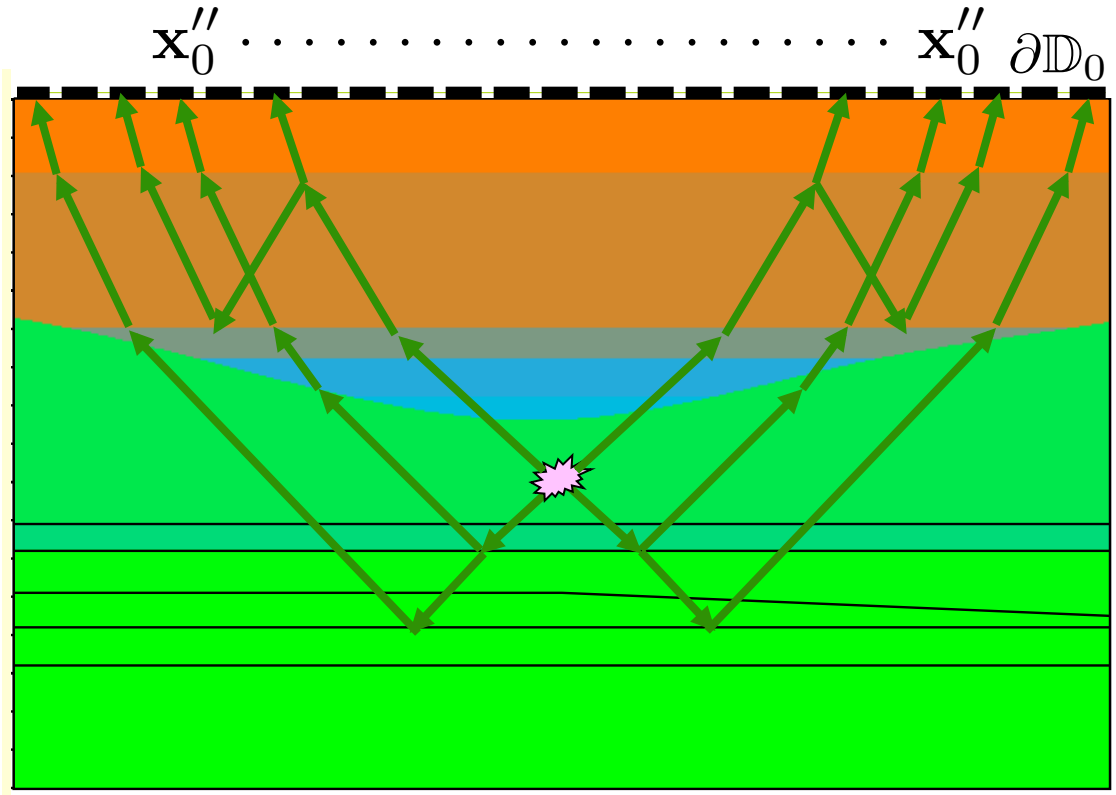

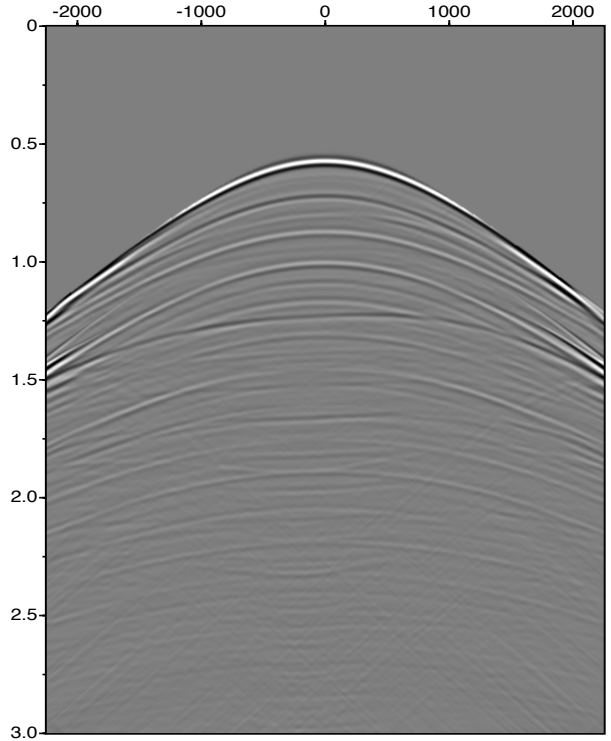

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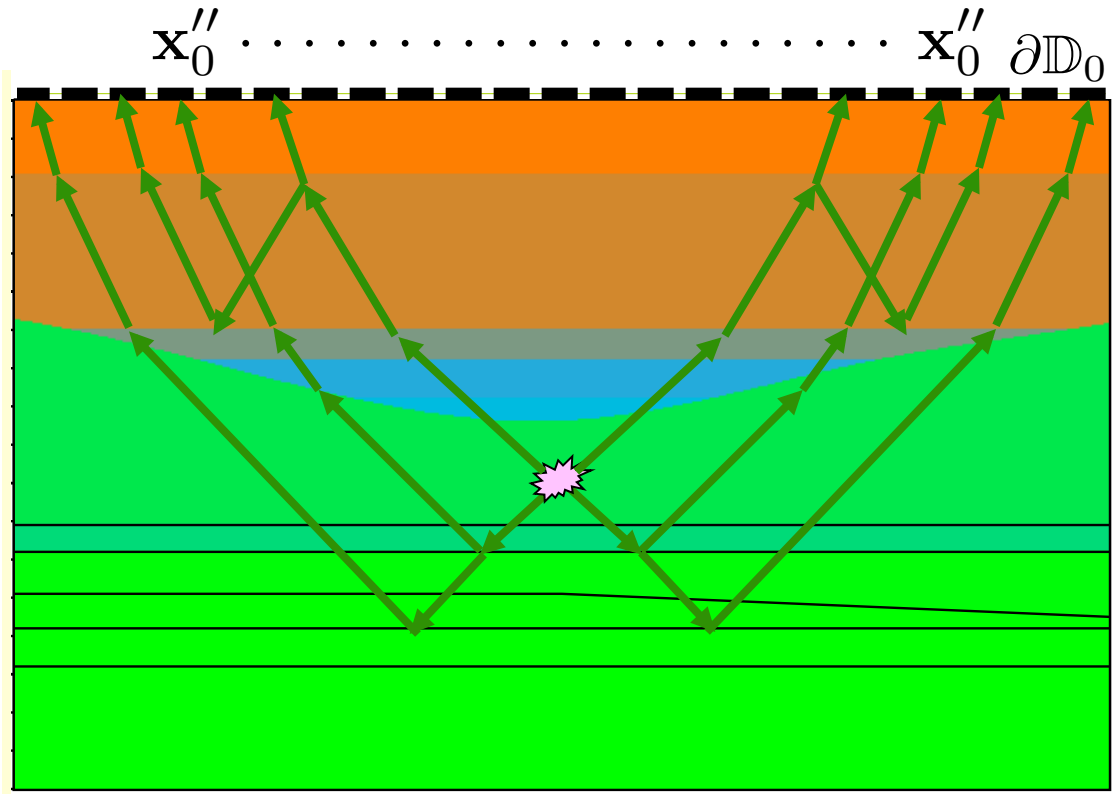

Assumption:
Lossless medium

Approximation:
Evanescent field
not included

$$G(\mathbf{x}_0'', \mathbf{x}_i', t) = f(\mathbf{x}_i', \mathbf{x}_0'', -t) + \int_{\partial\mathbb{D}_0} \mathrm{d}\mathbf{x}_0 \int_{-\infty}^{t} R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f(\mathbf{x}_i', \mathbf{x}_0, t') \mathrm{d}t'$$

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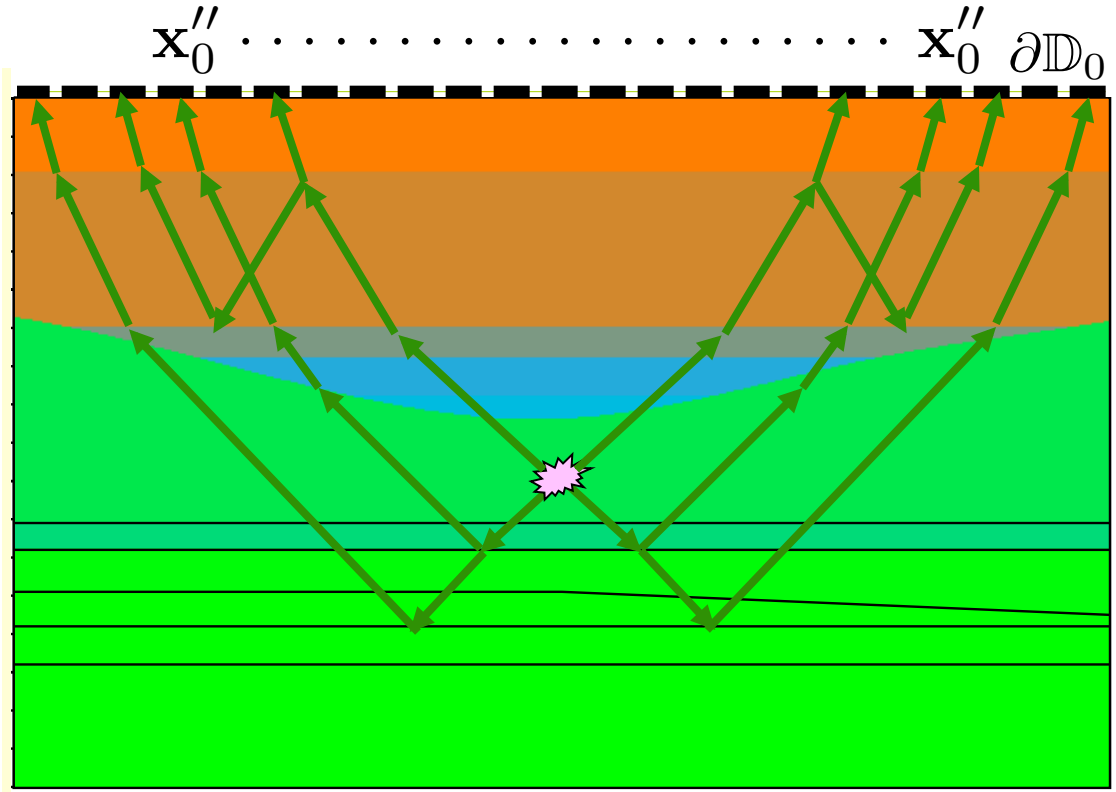

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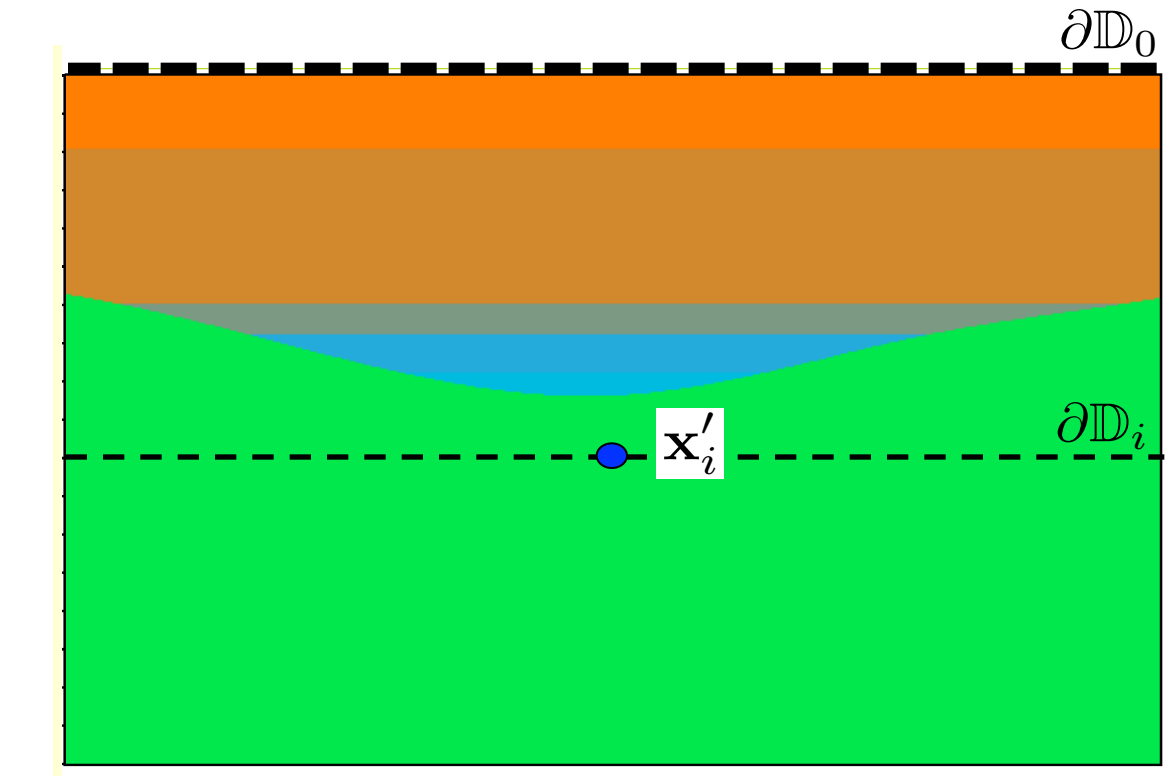
$\partial\mathbb{D}_0$
$\mathbf{x}'_i$
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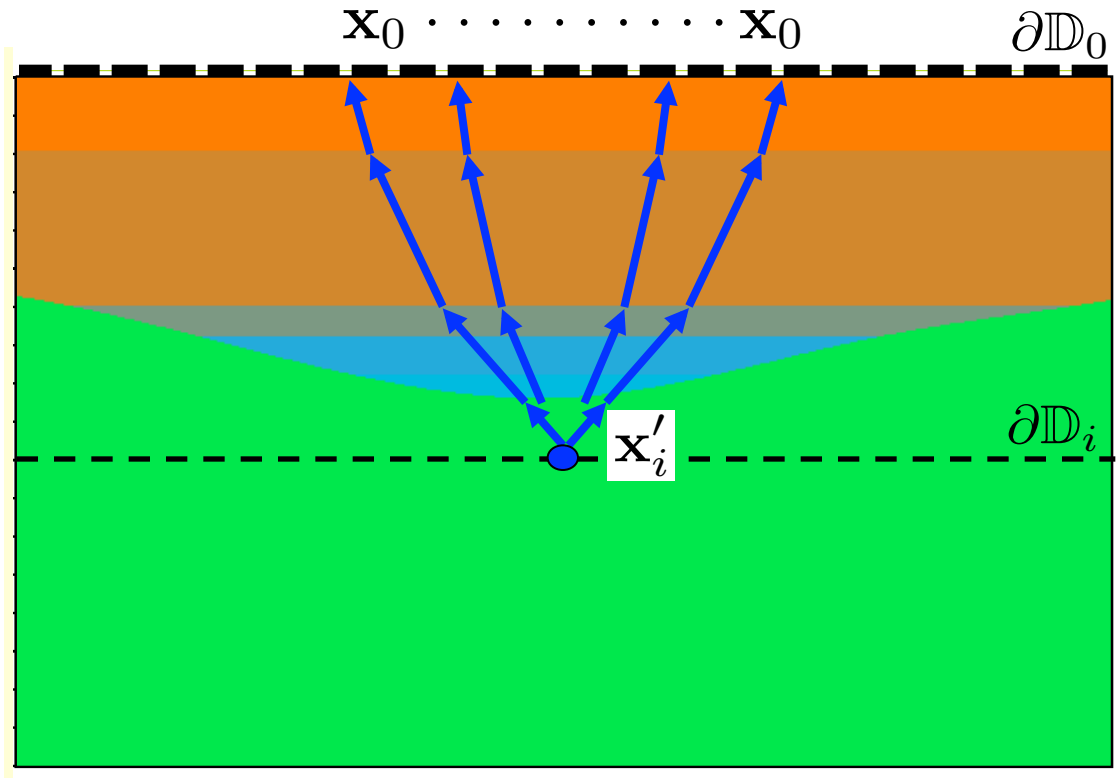
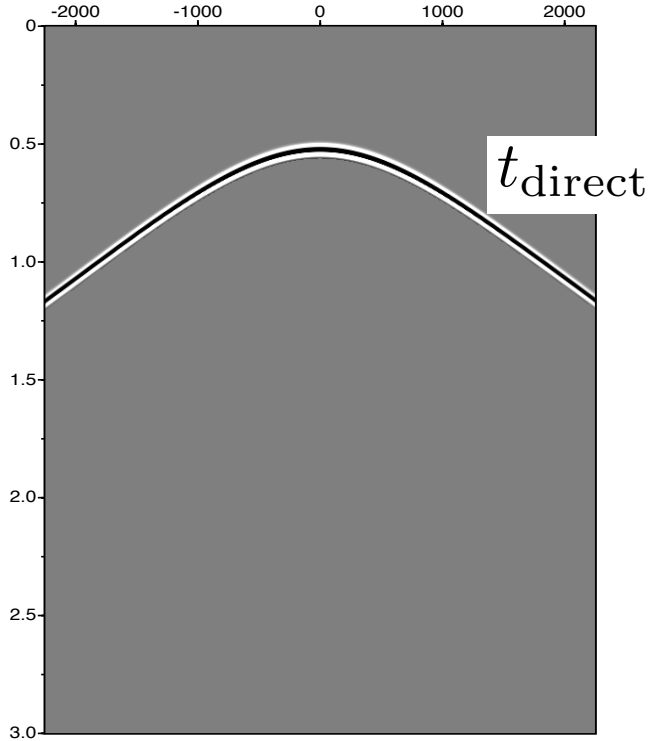
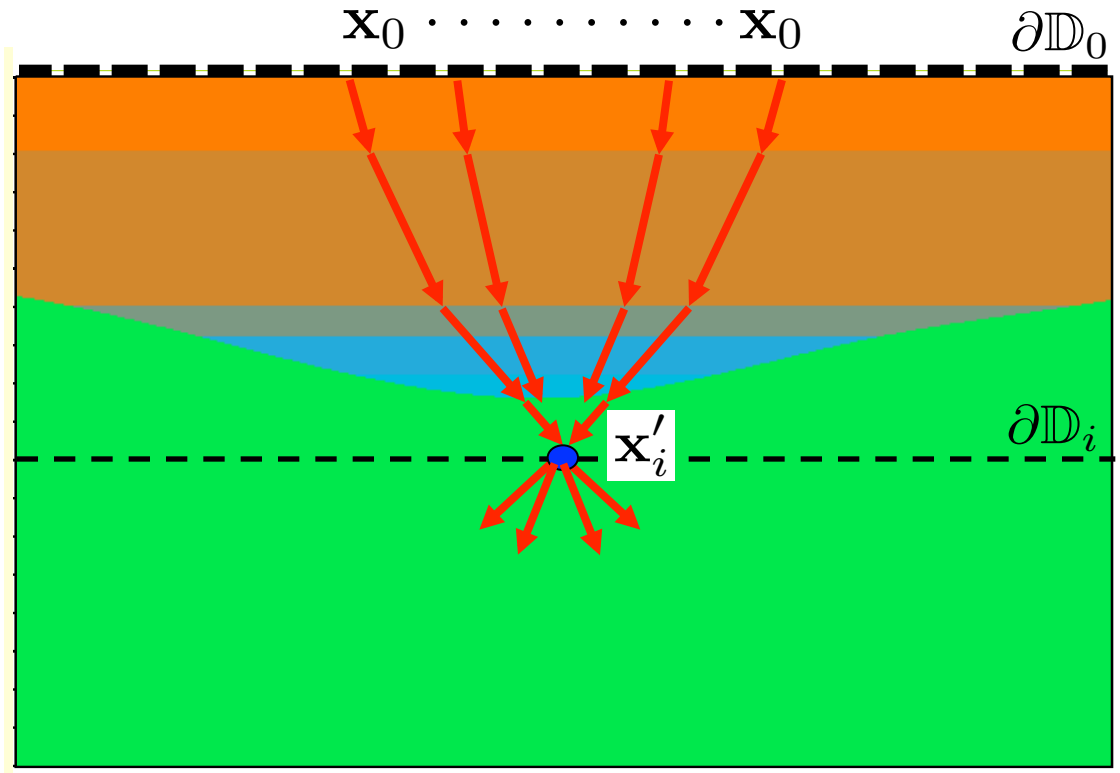
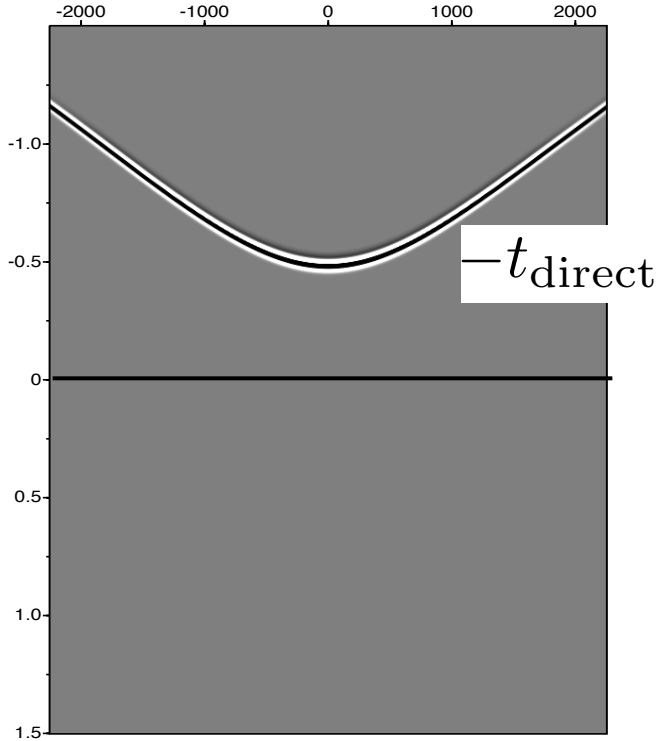




$$G_{\text{d}}(\mathbf{x}_0, \mathbf{x}'_i, t)$$





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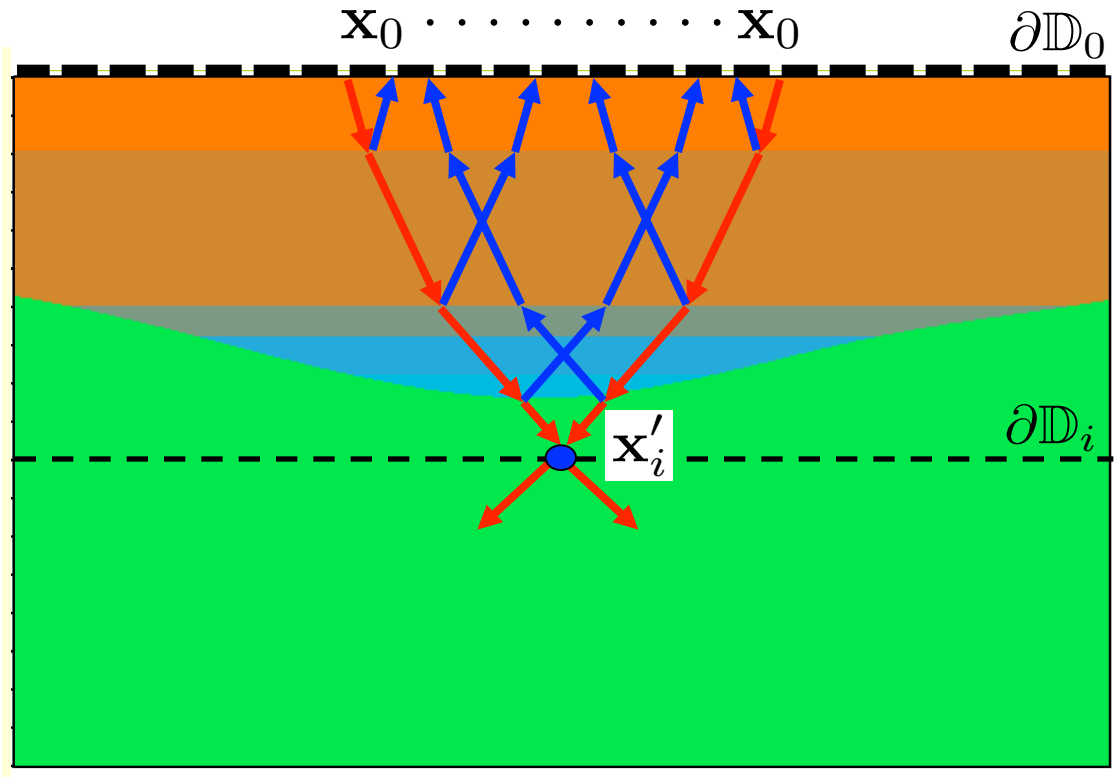

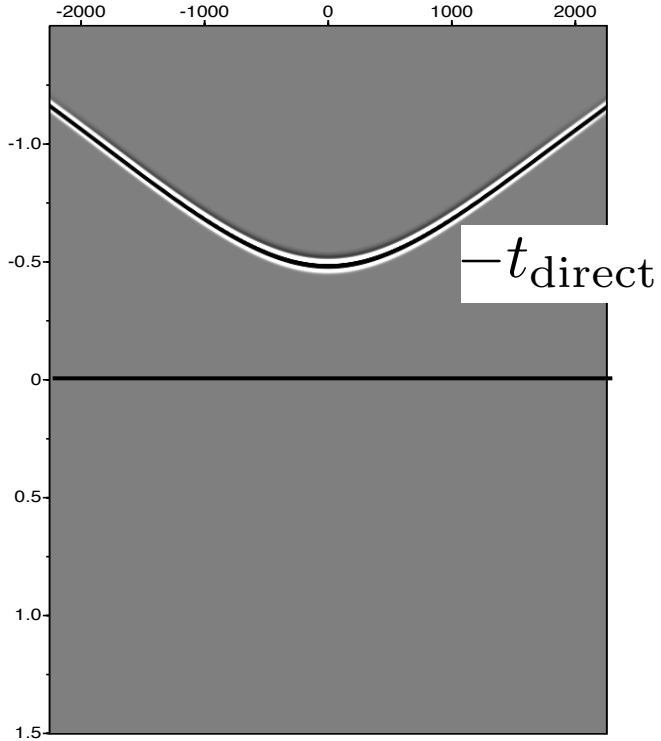

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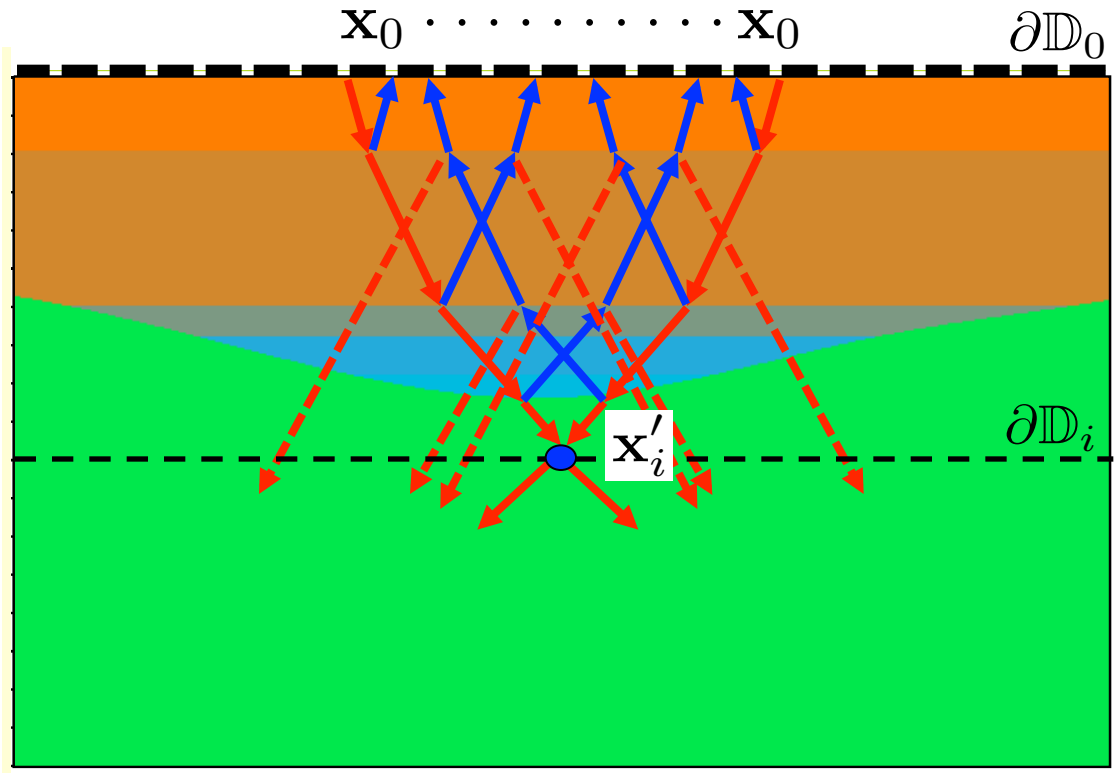

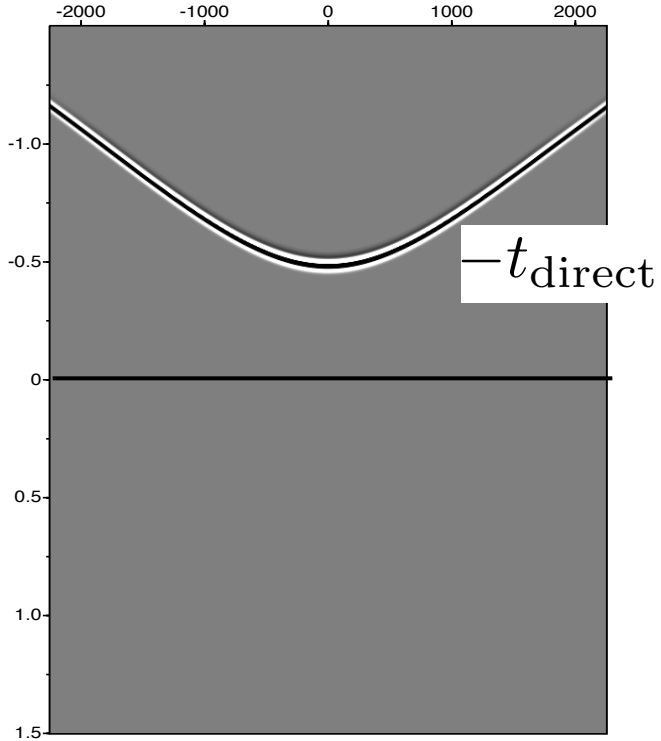

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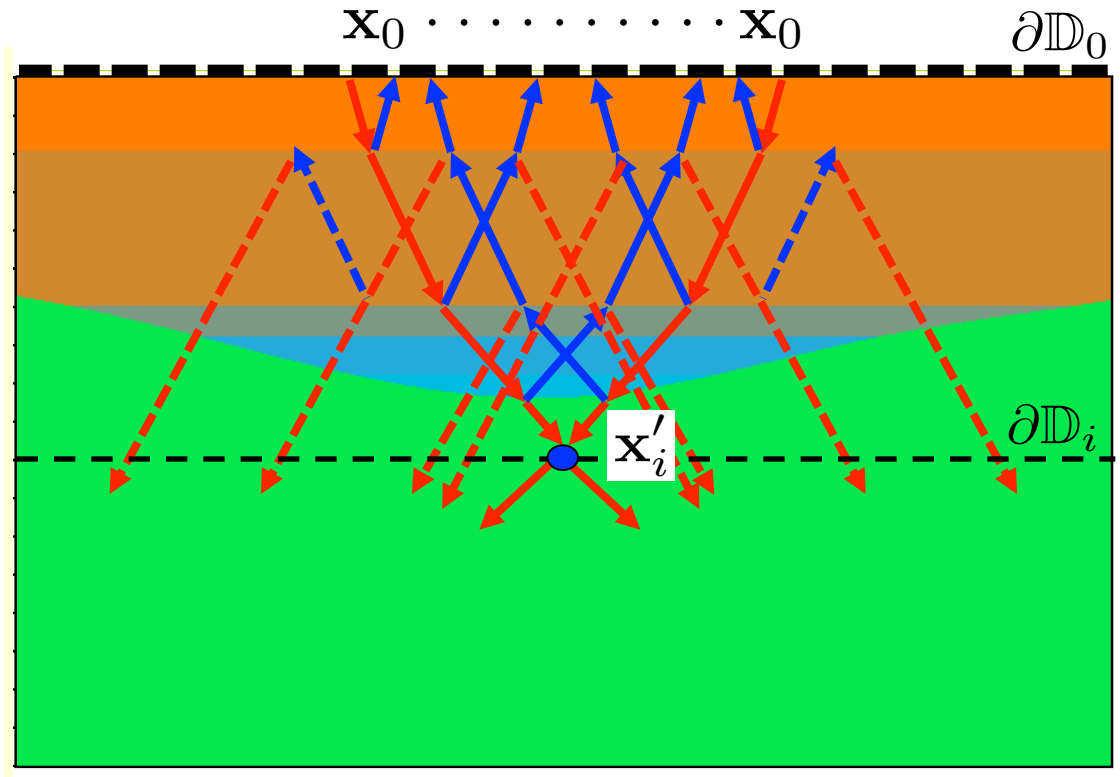

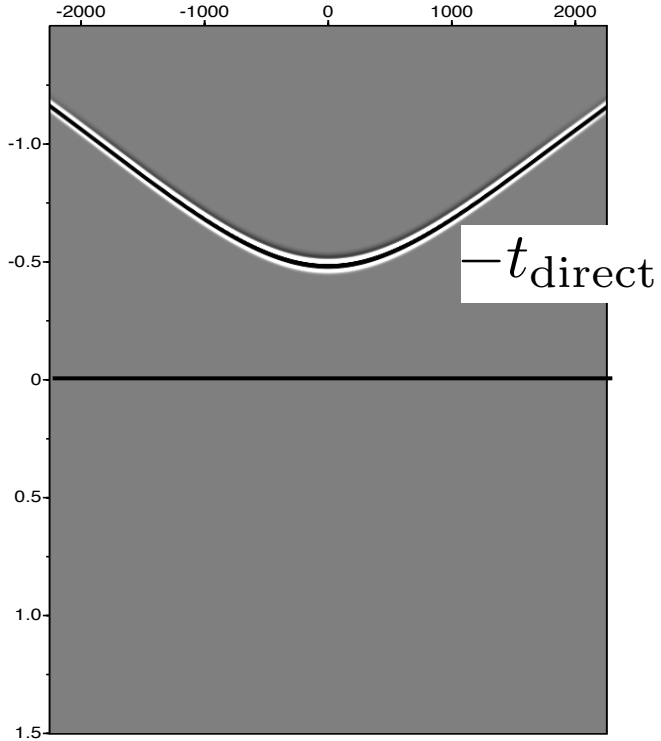

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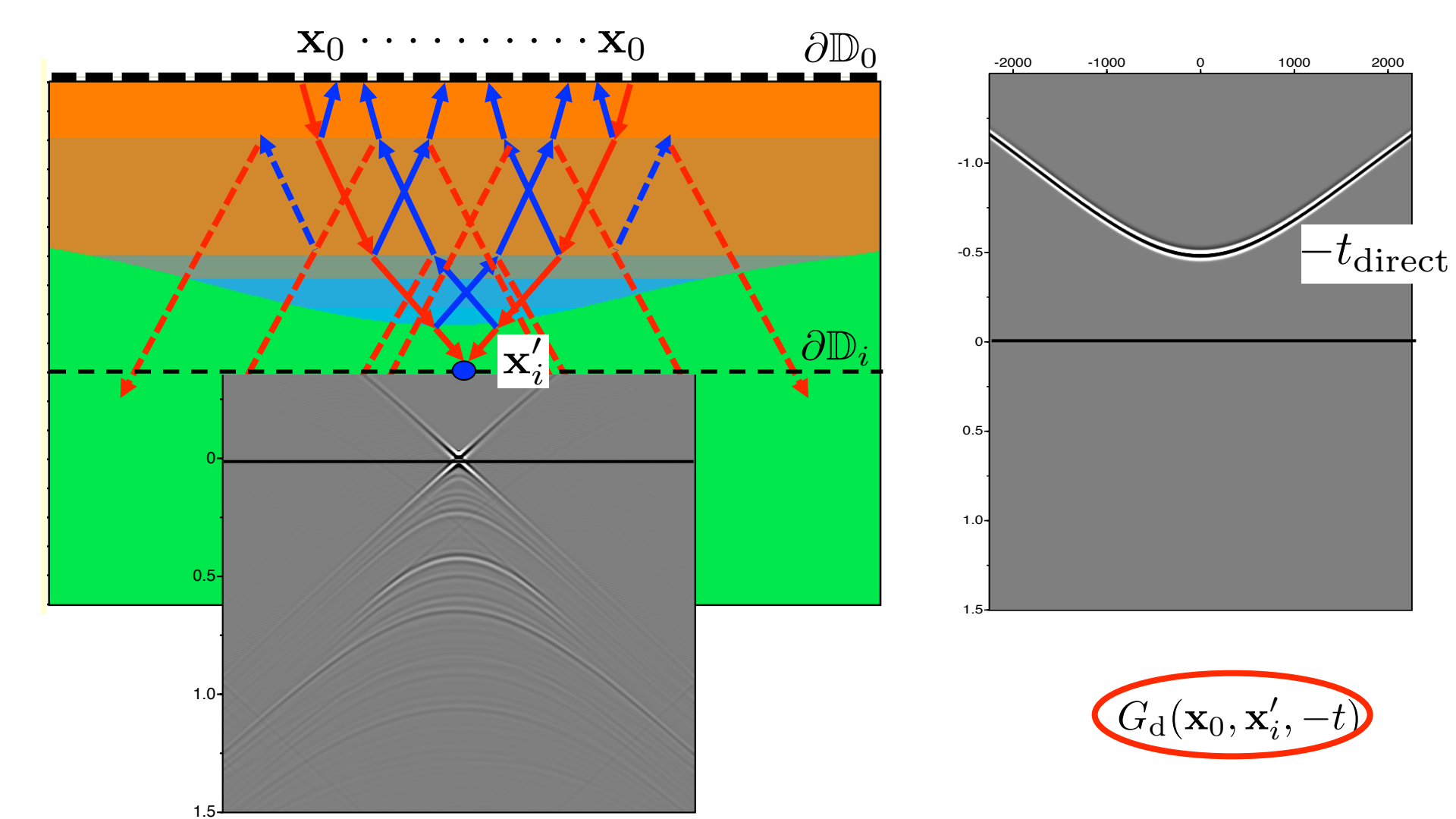
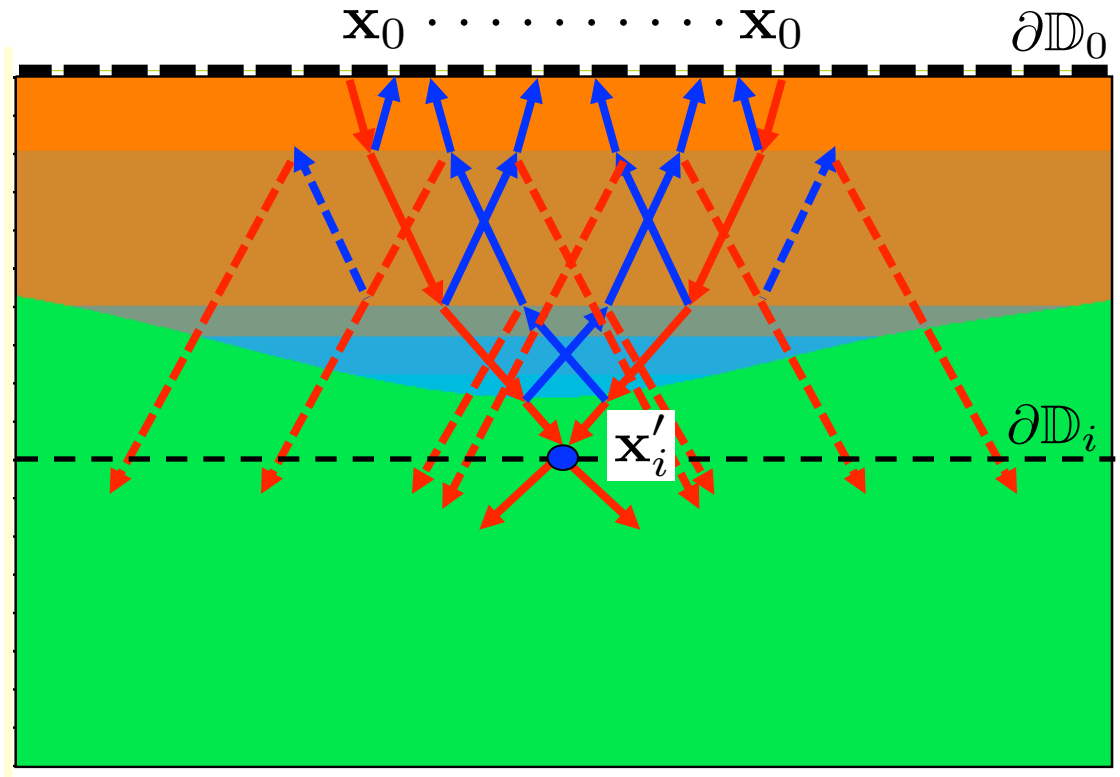
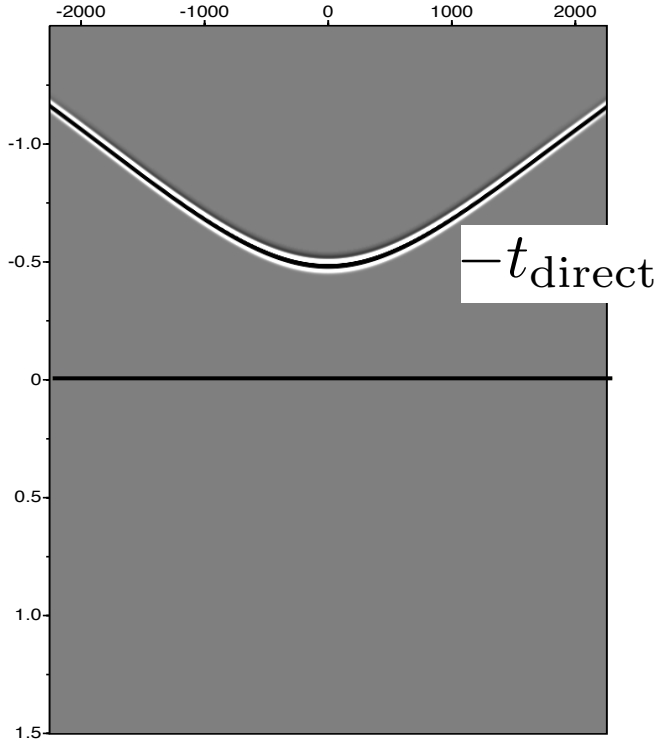

$\mathbf{x}_0 \cdots\cdots\cdots \mathbf{x}_0$
$\partial\mathbb{D}_0$
$\partial\mathbb{D}_i$
$\mathbf{x}'_i$
0
0.5
1.0
1.5
-2000
-1000
0
1000
2000
-1.0
-0.5
0
0.5
1.0
1.5
$-t_{\mathrm{direct}}$
$G_{\mathrm{d}}(\mathbf{x}_0, \mathbf{x}'_i, -t)$





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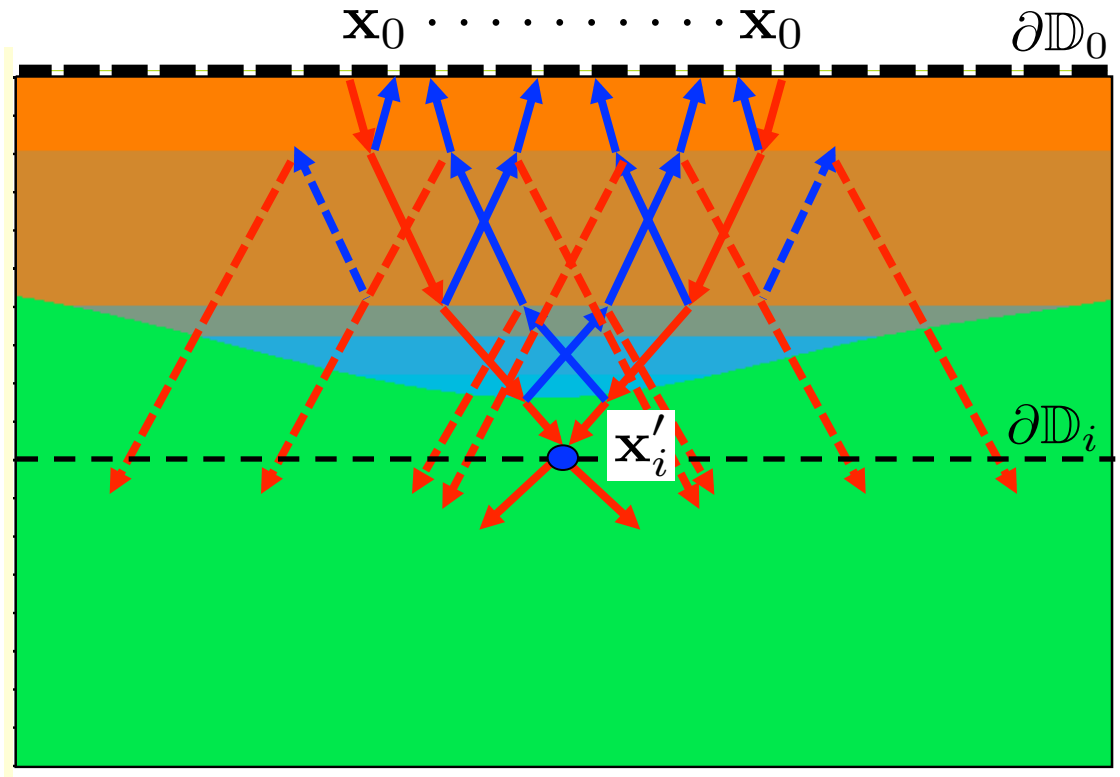
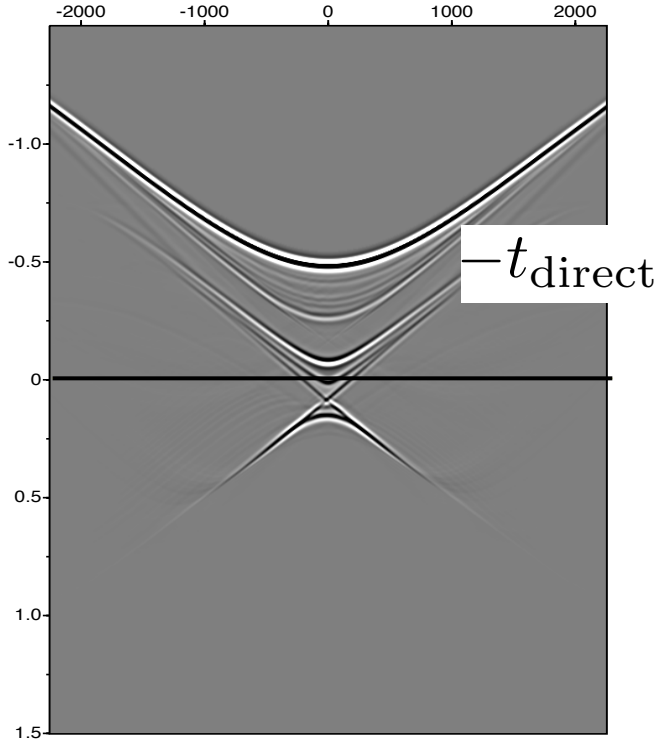
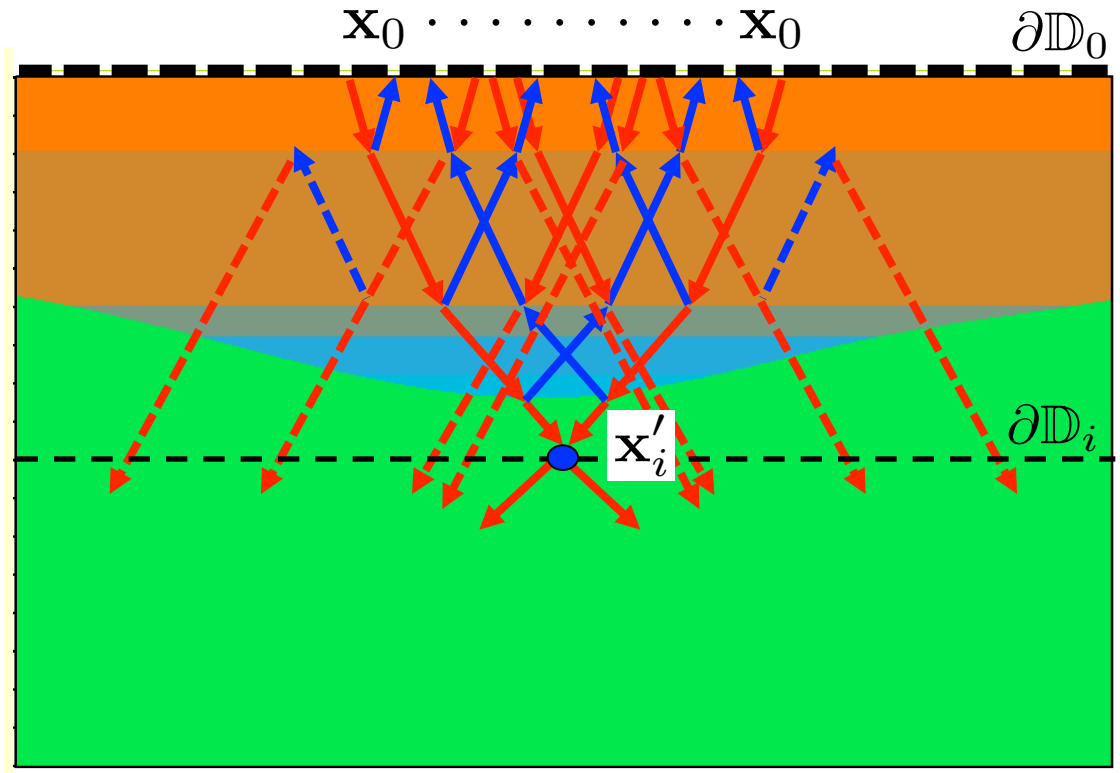
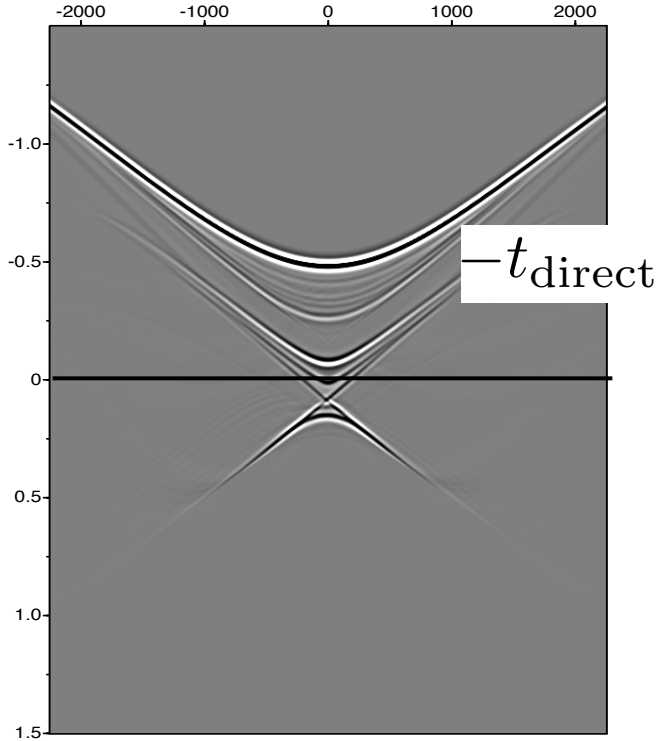




$$f_1^+(\mathbf{x}_0, \mathbf{x}'_i, t)$$





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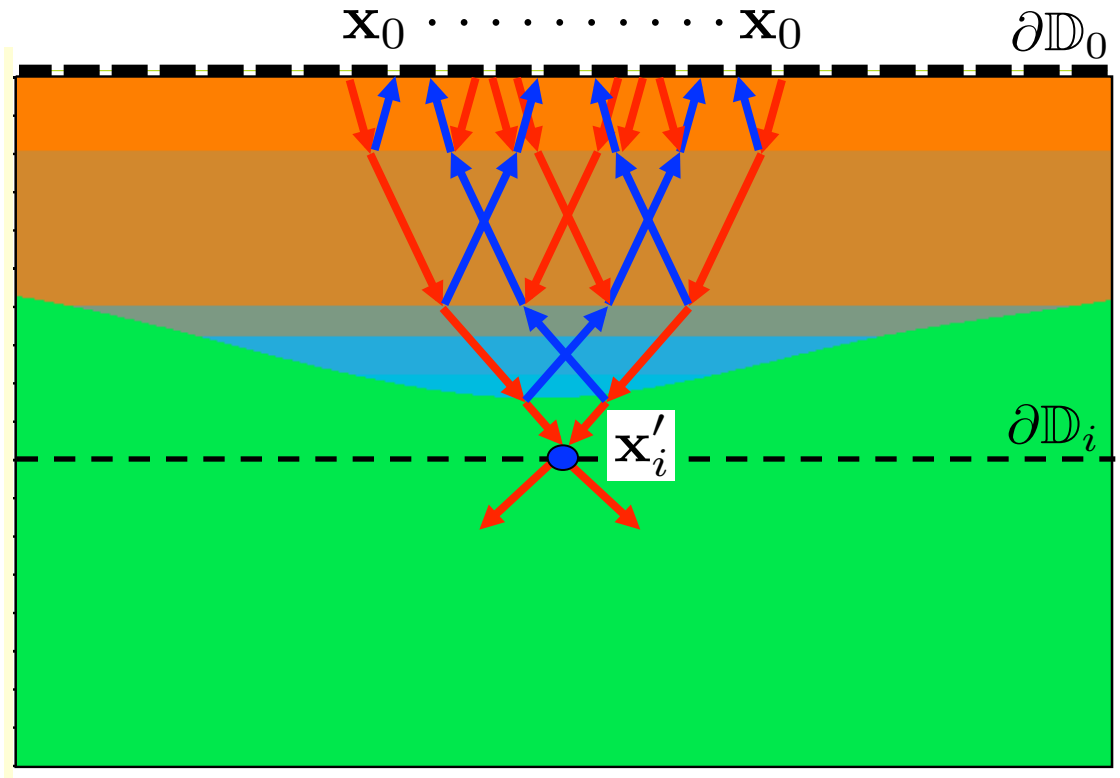

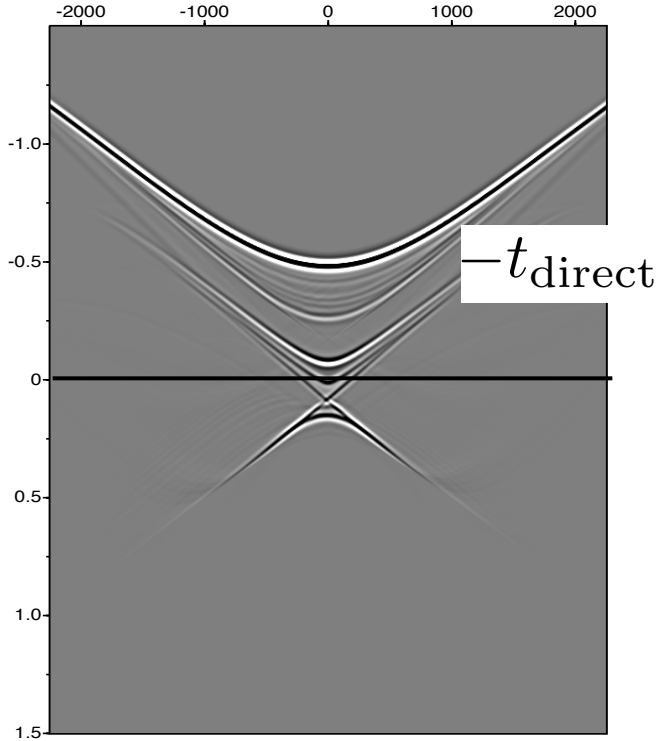

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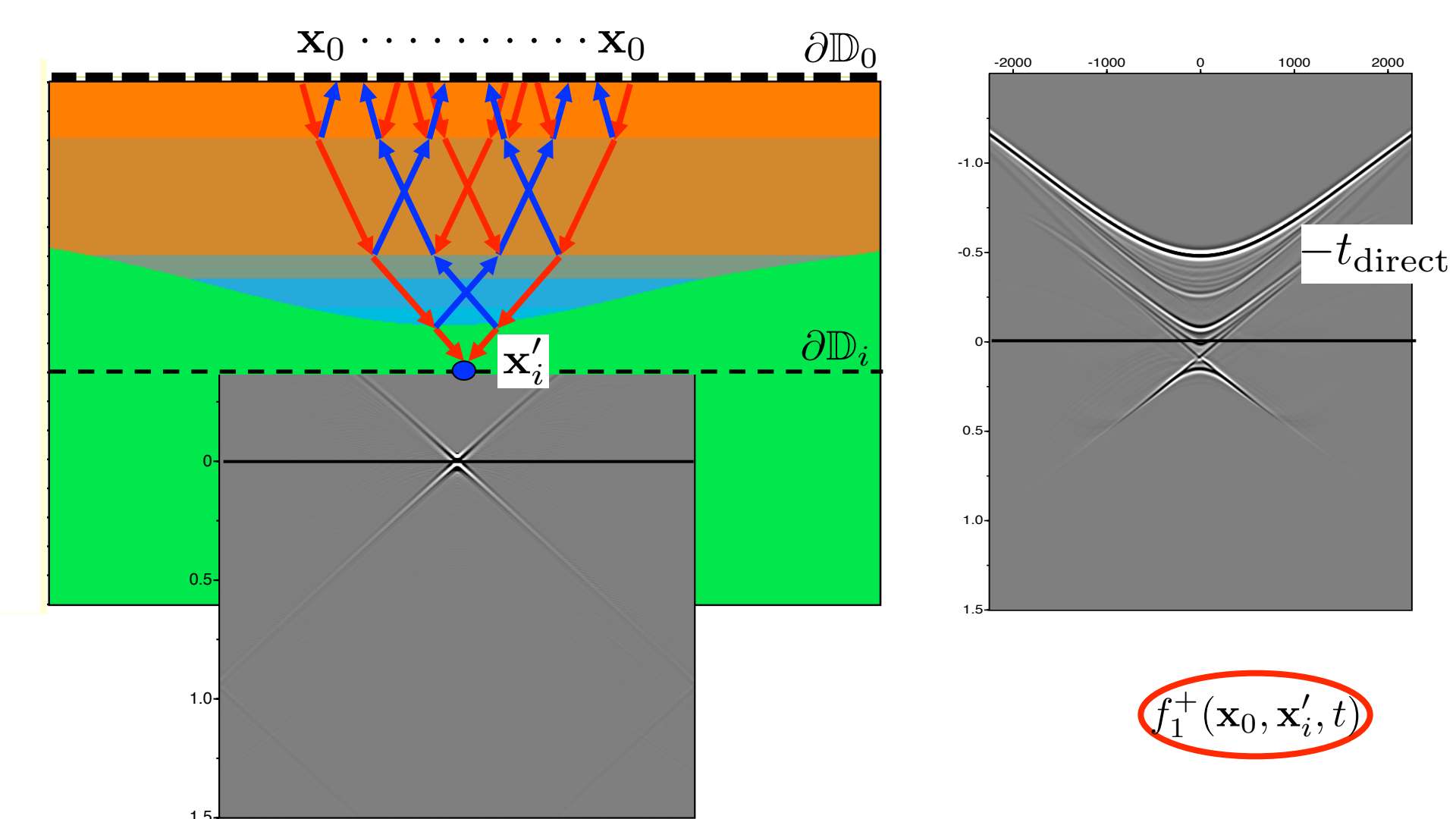
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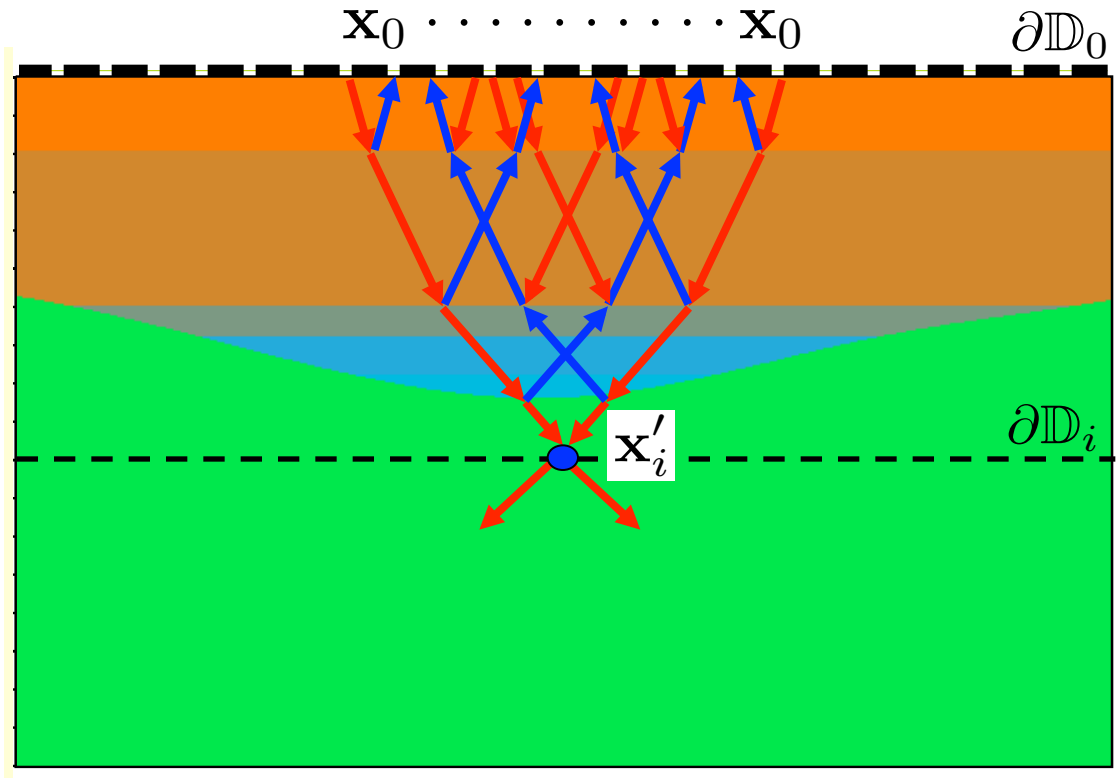

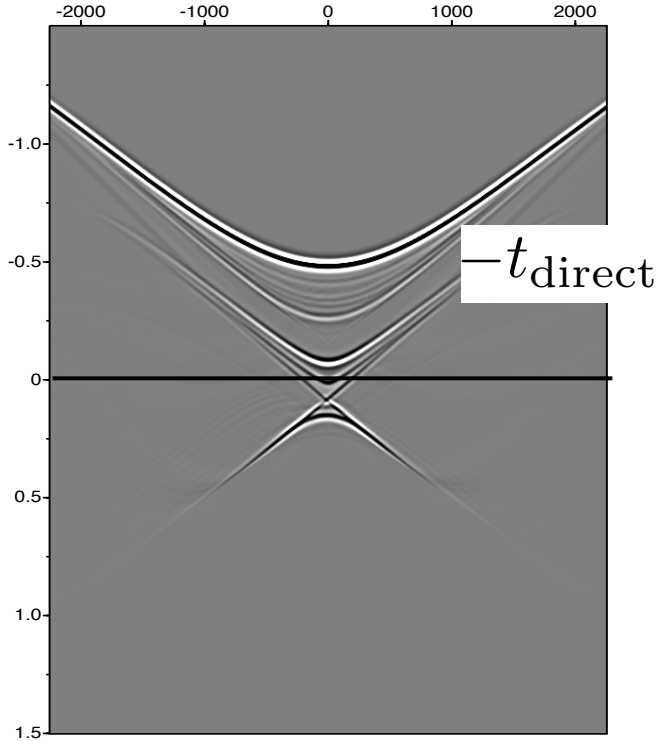

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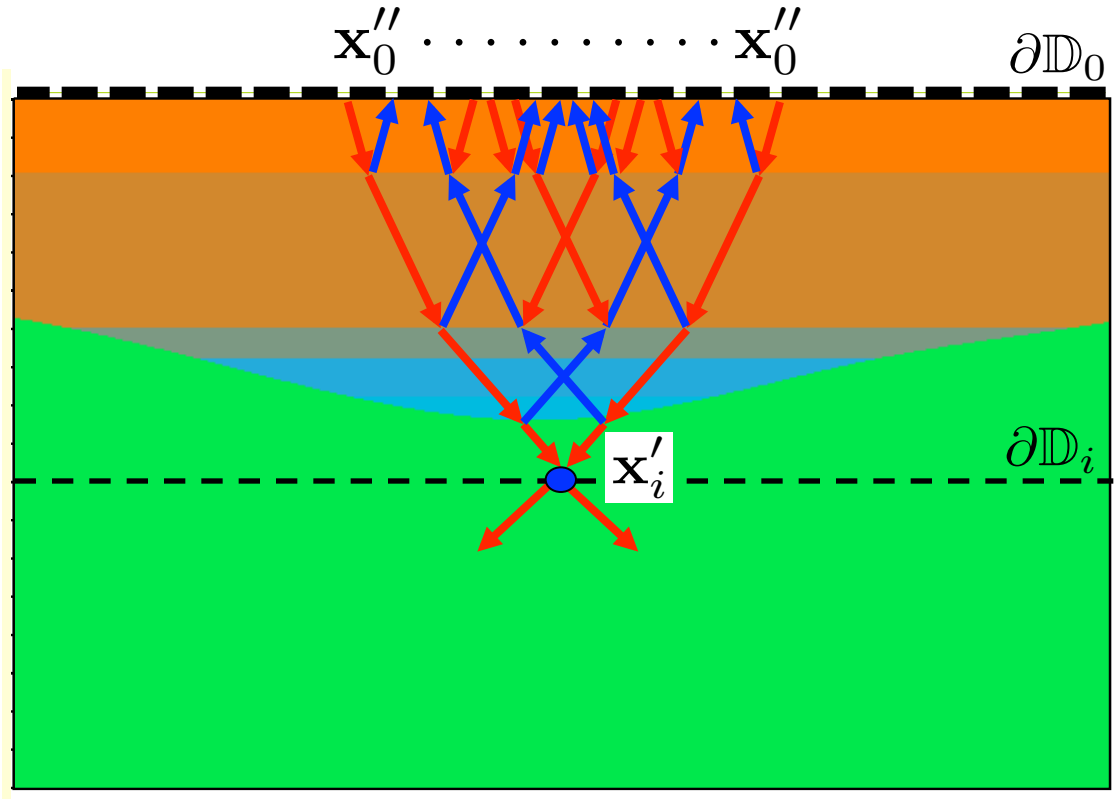
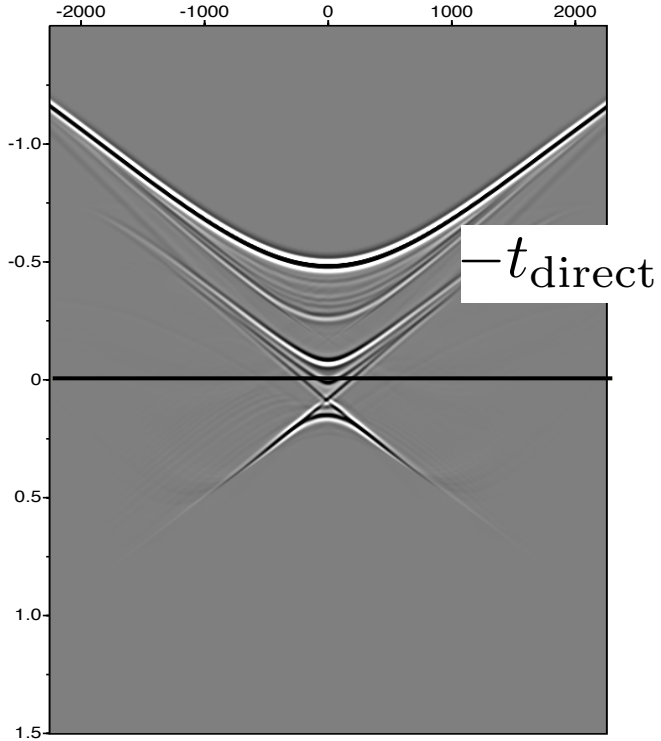
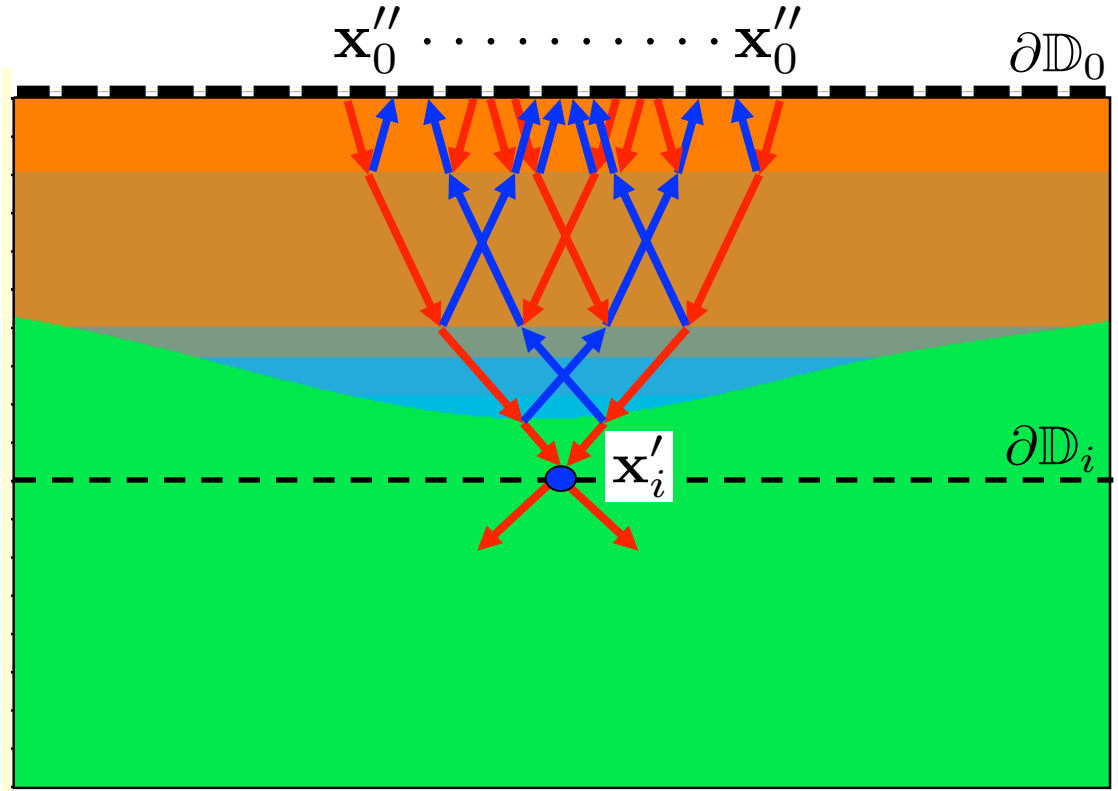
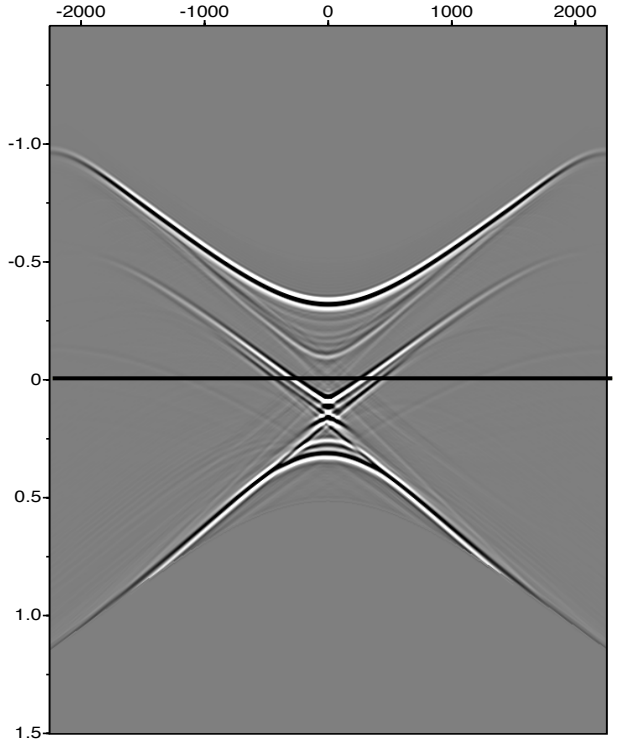




$$f_1^-(\mathbf{x}_0'', \mathbf{x}_i', t) = \int_{\partial\mathbb{D}_0} \mathrm{d}\mathbf{x}_0 \int_{-\infty}^{t} R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}_i', t')\, \mathrm{d}t'$$





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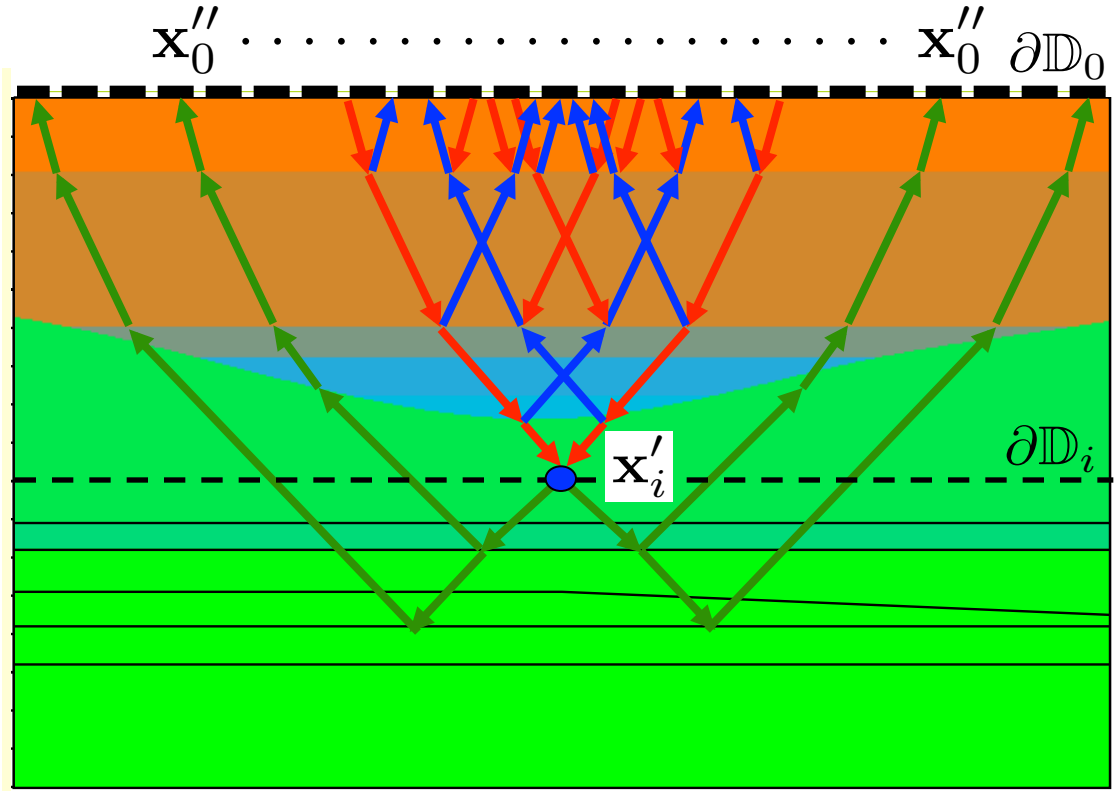
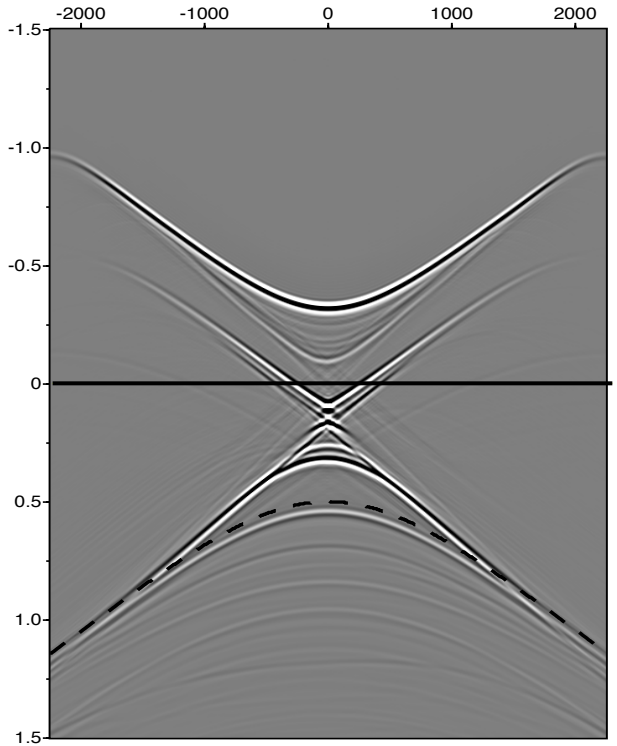
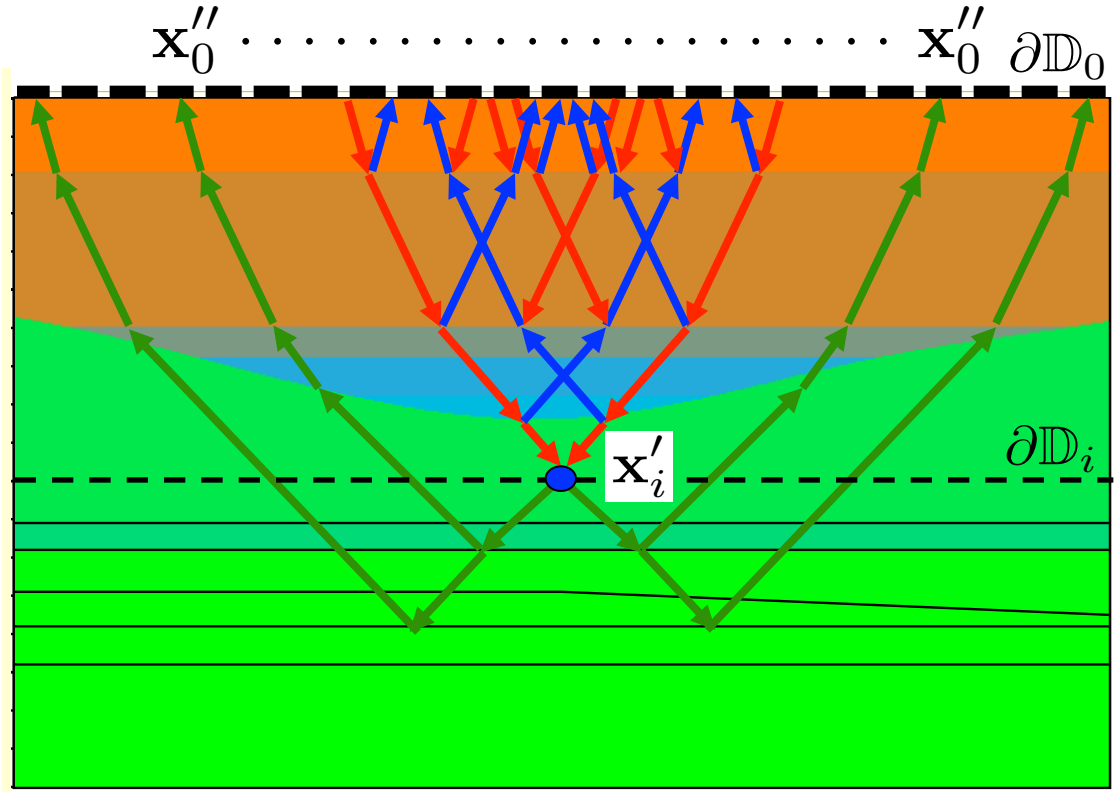
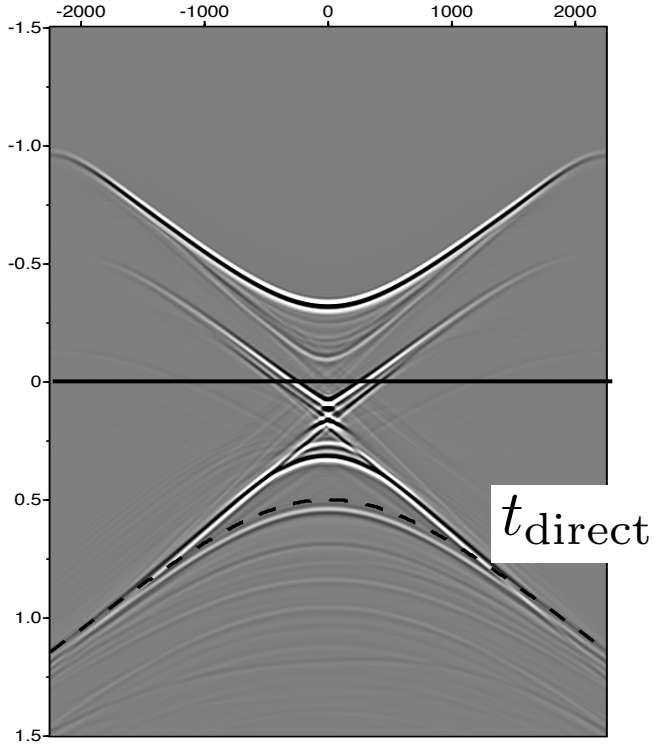


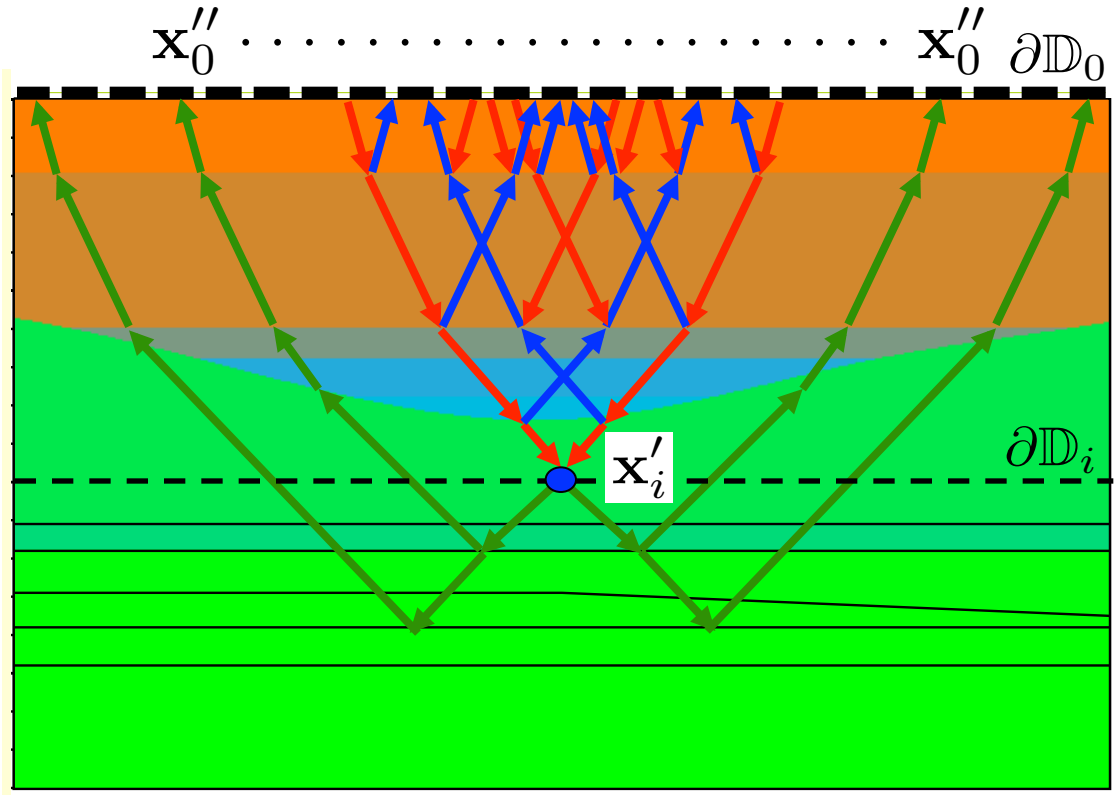


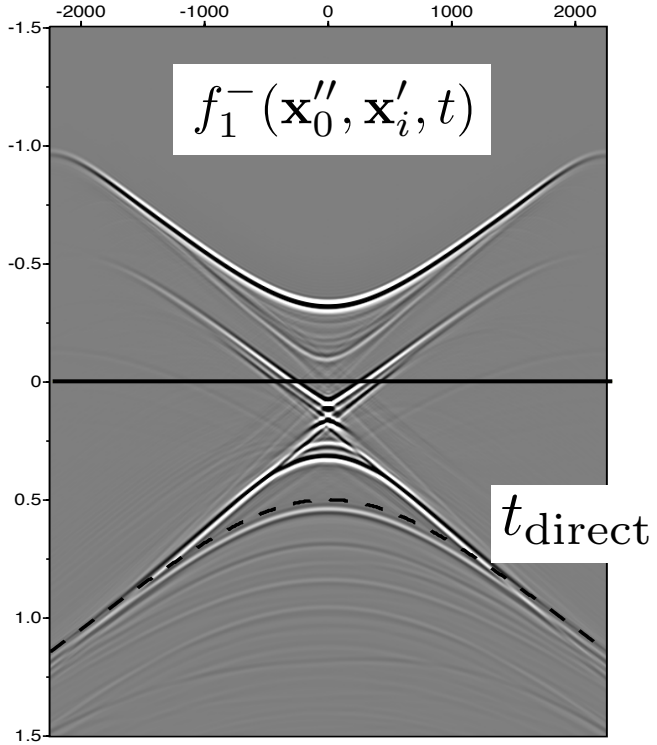
$$G^{-,+}(\mathbf{x}_0'', \mathbf{x}_i', t) + f_1^-(\mathbf{x}_0'', \mathbf{x}_i', t) = \int_{\partial\mathbb{D}_0} \mathrm{d}\mathbf{x}_0 \int_{-\infty}^{t} R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}_i', t') \mathrm{d}t'$$



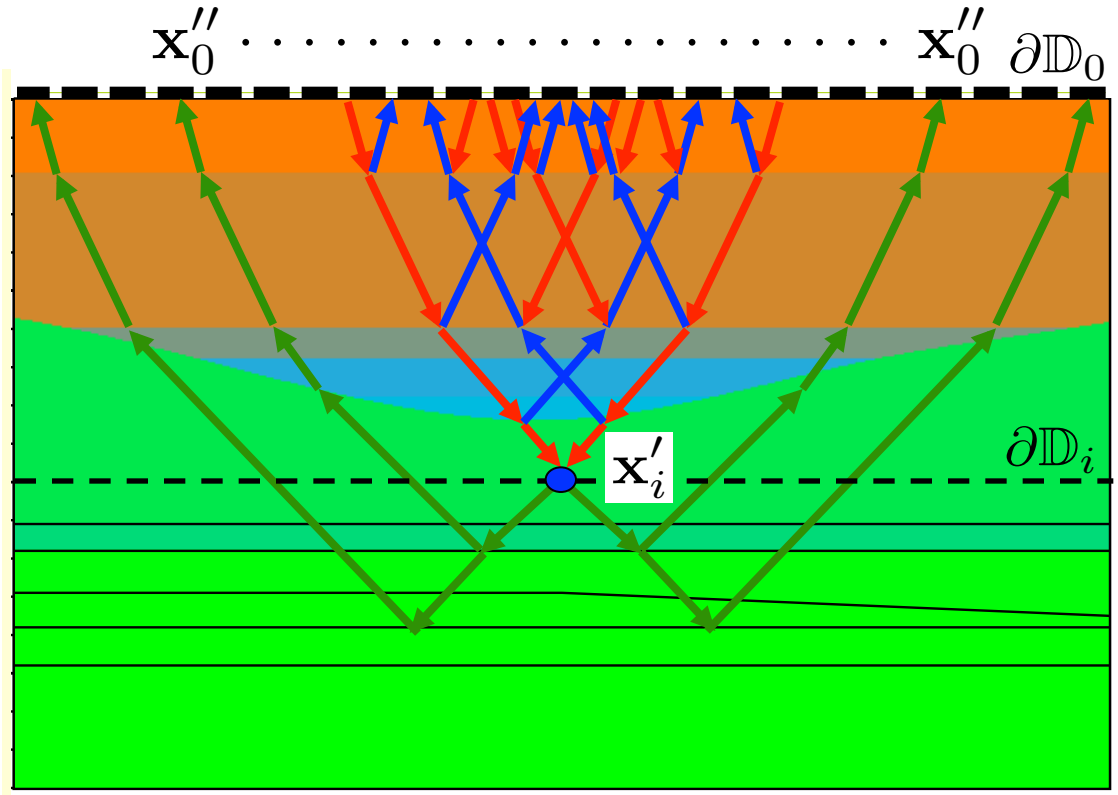


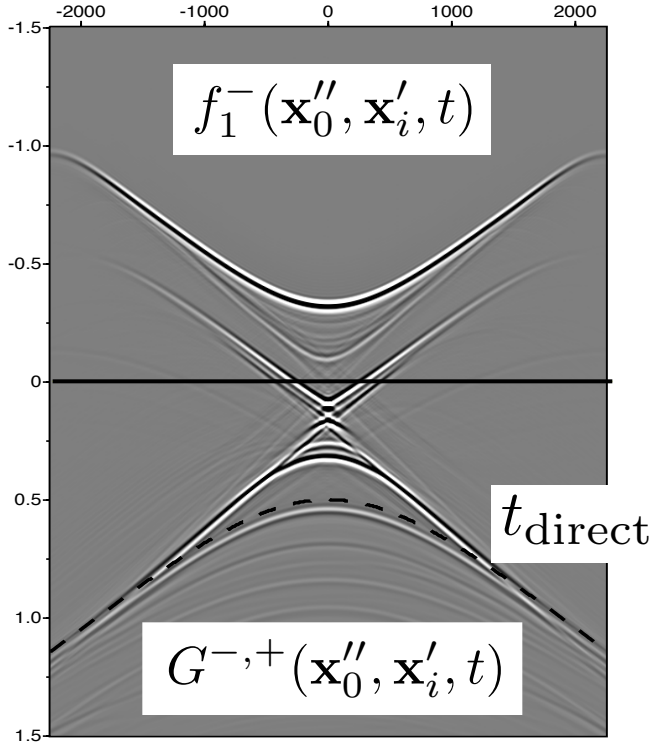
$$G^{-,+}(\mathbf{x}_0'', \mathbf{x}_i', t) + f_1^-(\mathbf{x}_0'', \mathbf{x}_i', t) = \int_{\partial\mathbb{D}_0} \mathrm{d}\mathbf{x}_0 \int_{-\infty}^{t} R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}_i', t') \mathrm{d}t'$$



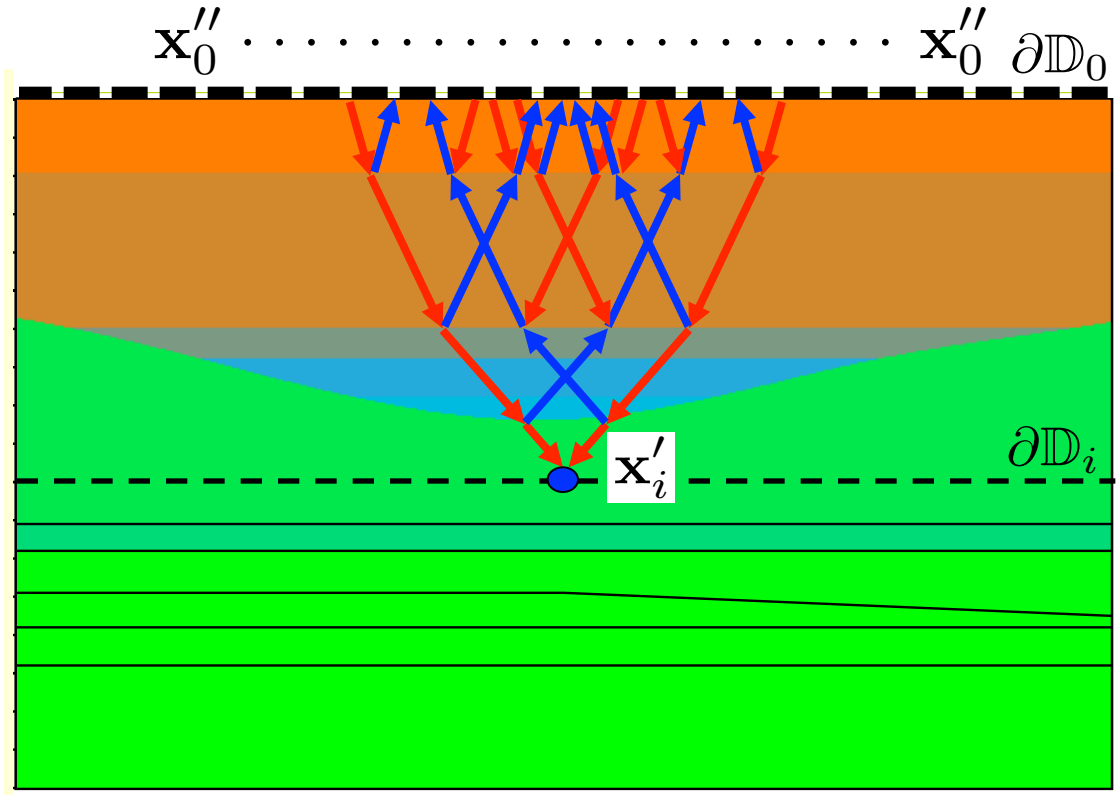


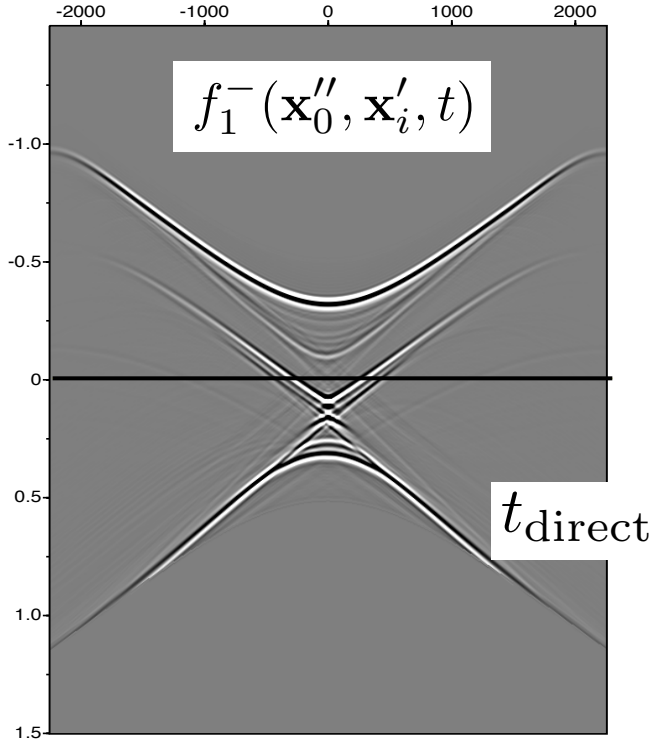
$$G^{-,+}(\mathbf{x}_0'', \mathbf{x}_i', t) + f_1^-(\mathbf{x}_0'', \mathbf{x}_i', t) = \int_{\partial\mathbb{D}_0} \mathrm{d}\mathbf{x}_0 \int_{-\infty}^{t} R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}_i', t') \mathrm{d}t'$$



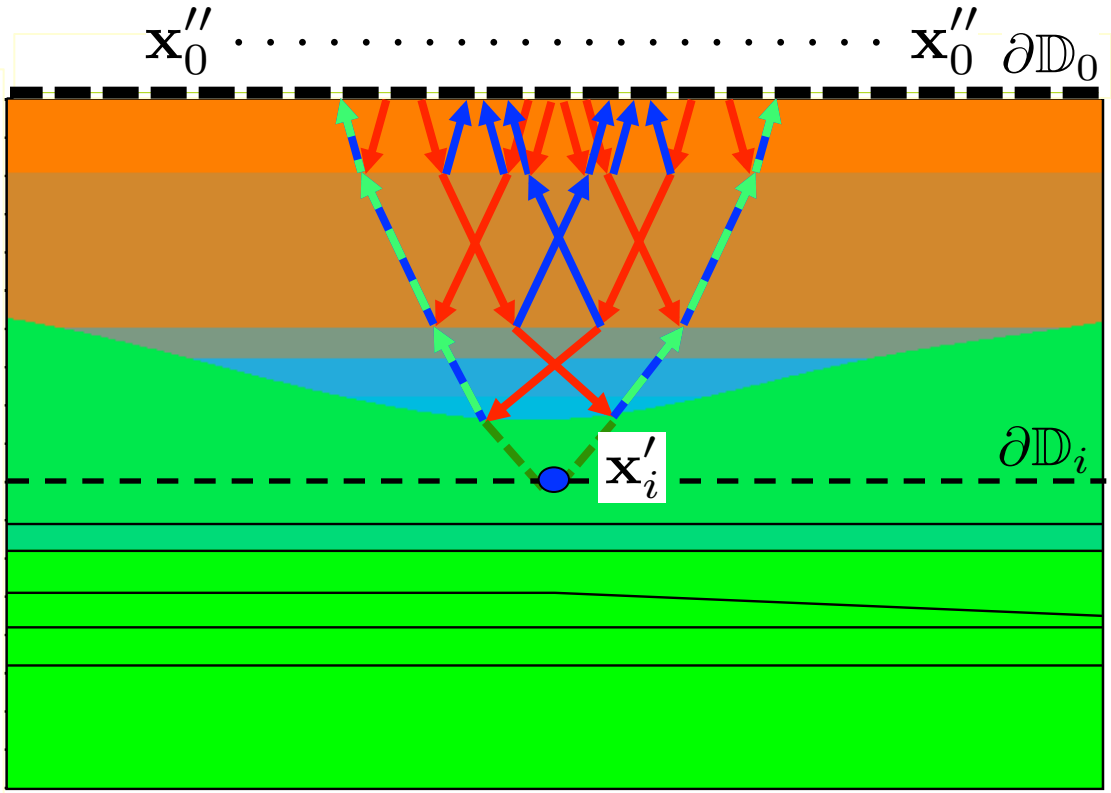


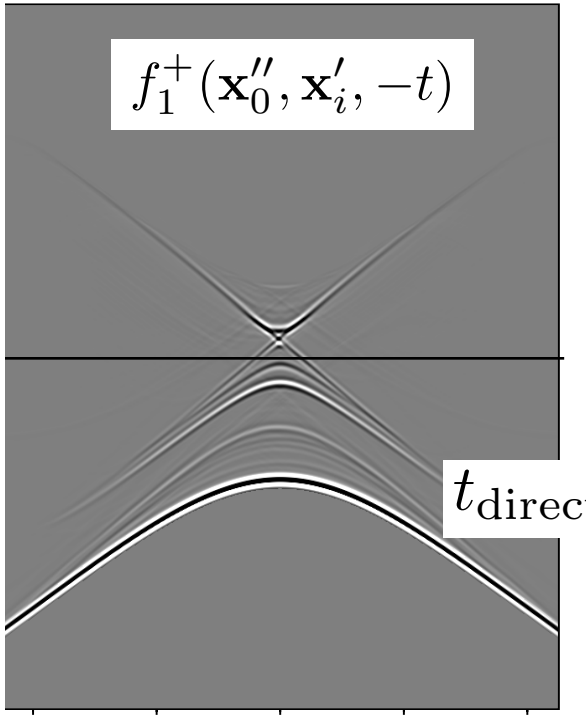
$$G^{-,+}(\mathbf{x}_0'', \mathbf{x}_i', t) + f_1^-(\mathbf{x}_0'', \mathbf{x}_i', t) = \int_{\partial\mathbb{D}_0} \mathrm{d}\mathbf{x}_0 \int_{-\infty}^{t} R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}_i', t') \mathrm{d}t'$$





$$t < t_{\text{direct}}: \quad f_1^-(\mathbf{x}_0'', \mathbf{x}_i', t) = \int_{\partial\mathbb{D}_0} \mathrm{d}\mathbf{x}_0 \int_{-\infty}^{t} R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}_i', t') \mathrm{d}t'$$





$$t < t_{\text{direct}}: \quad f_1^+(\mathbf{x}_0'', \mathbf{x}_i', -t) = \int_{\partial\mathbb{D}_0} \mathrm{d}\mathbf{x}_0 \int_{-\infty}^{t} R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^-(\mathbf{x}_0, \mathbf{x}_i', -t') \mathrm{d}t'$$

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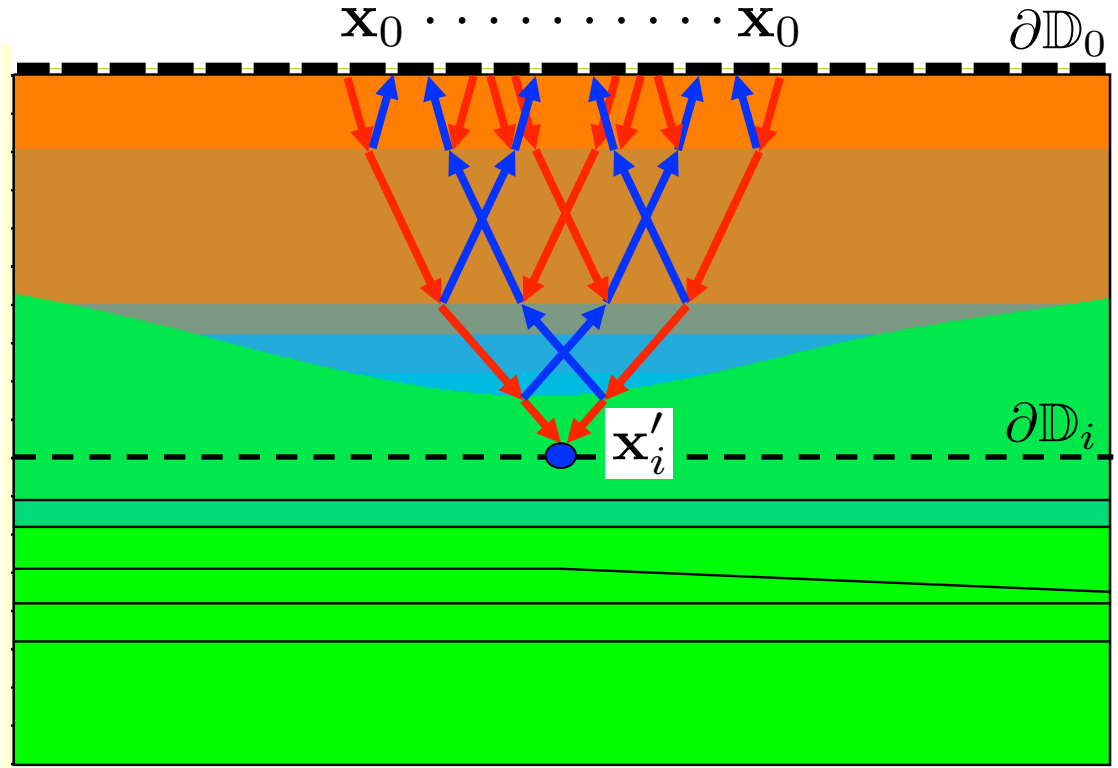
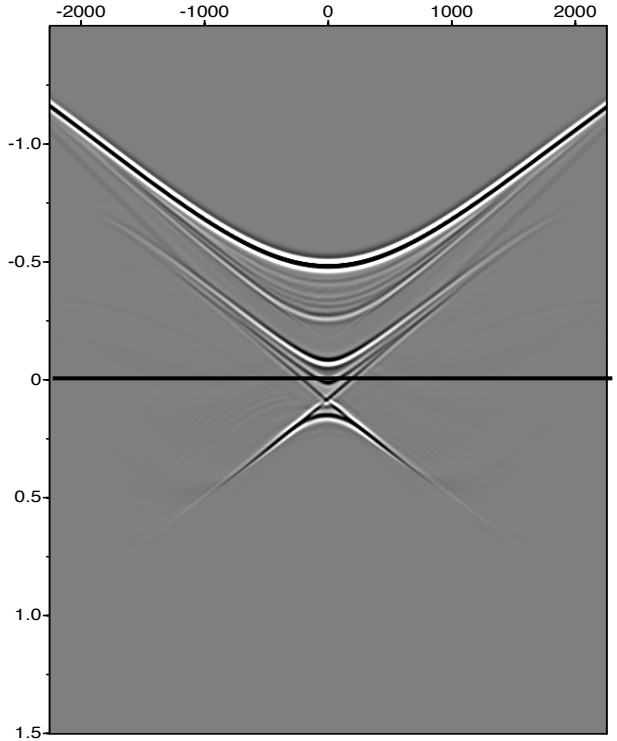
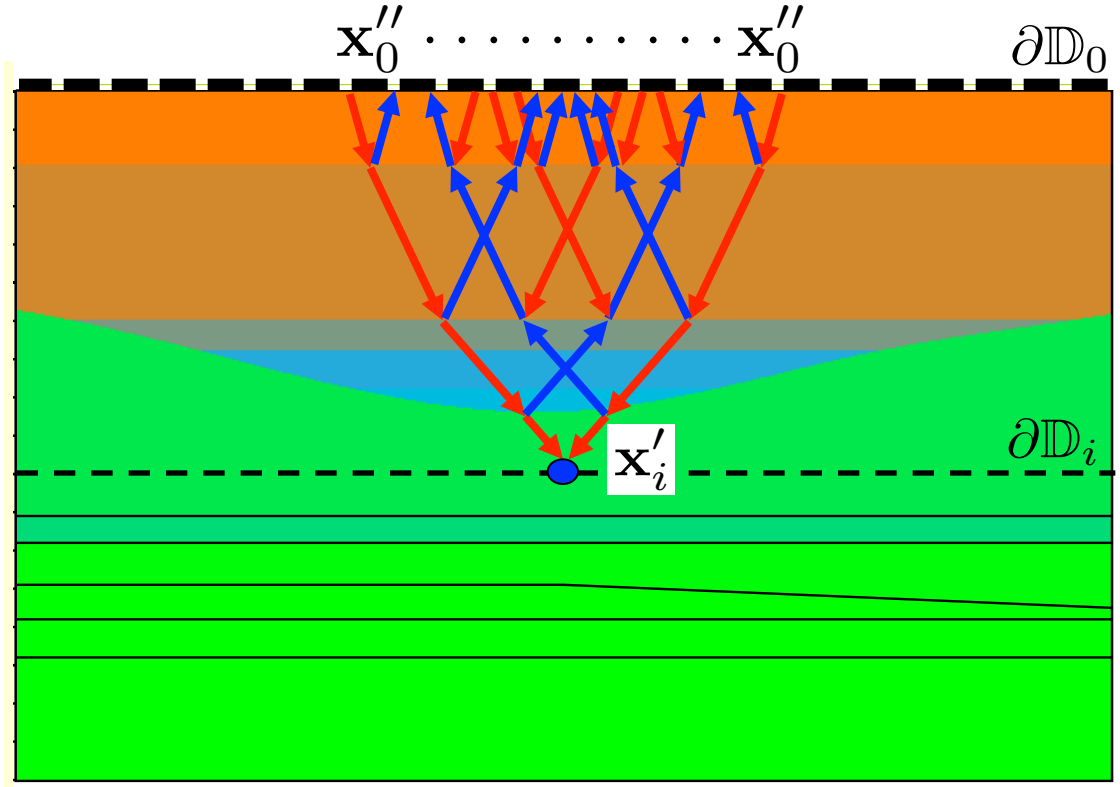
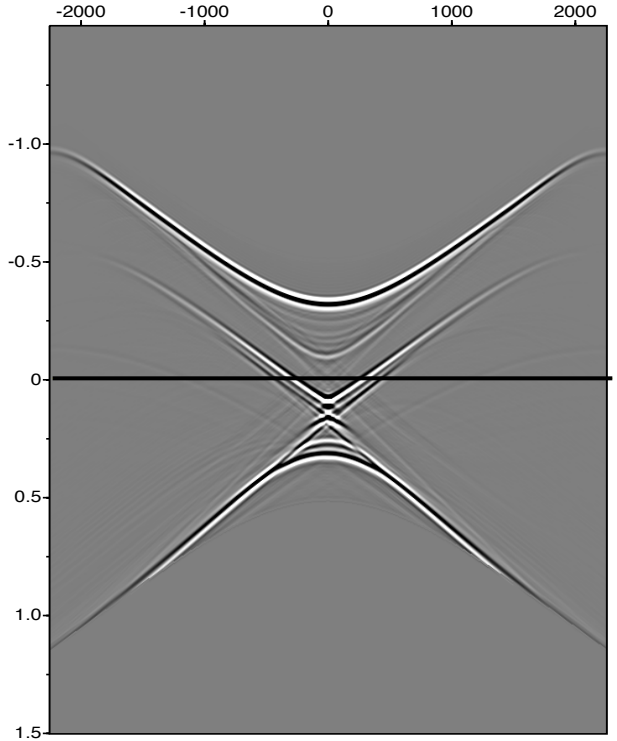


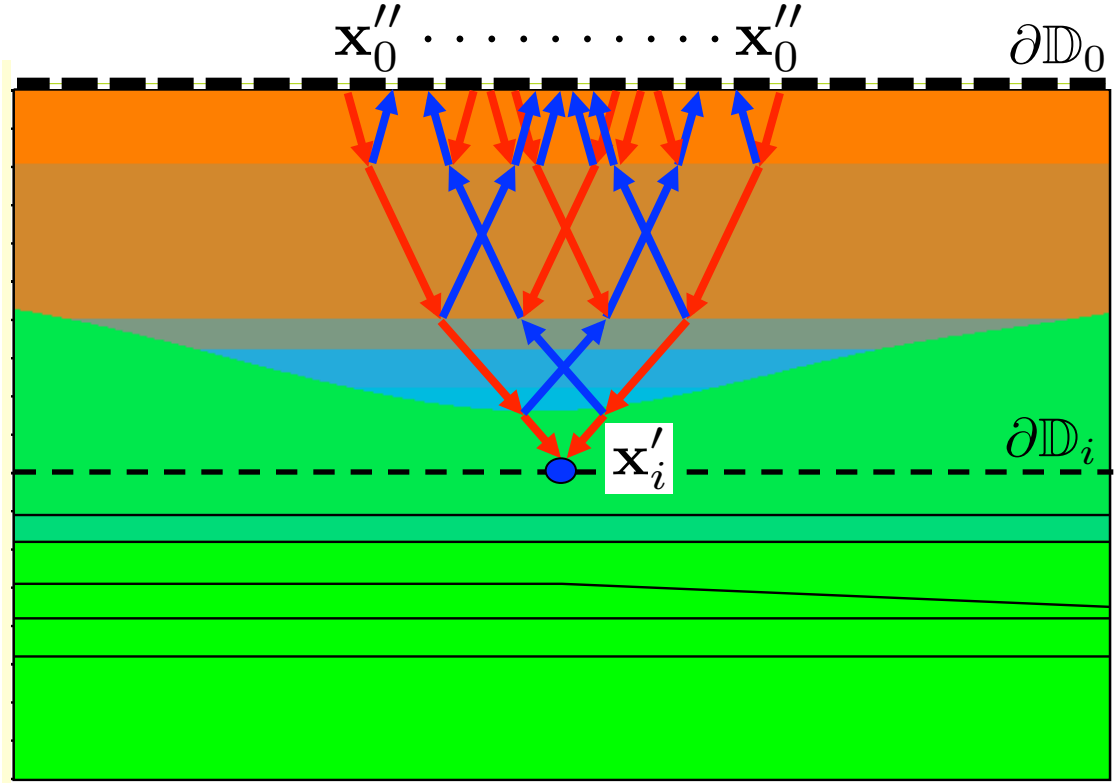


$$f_1^+(\mathbf{x}_0, \mathbf{x}'_i, t)$$

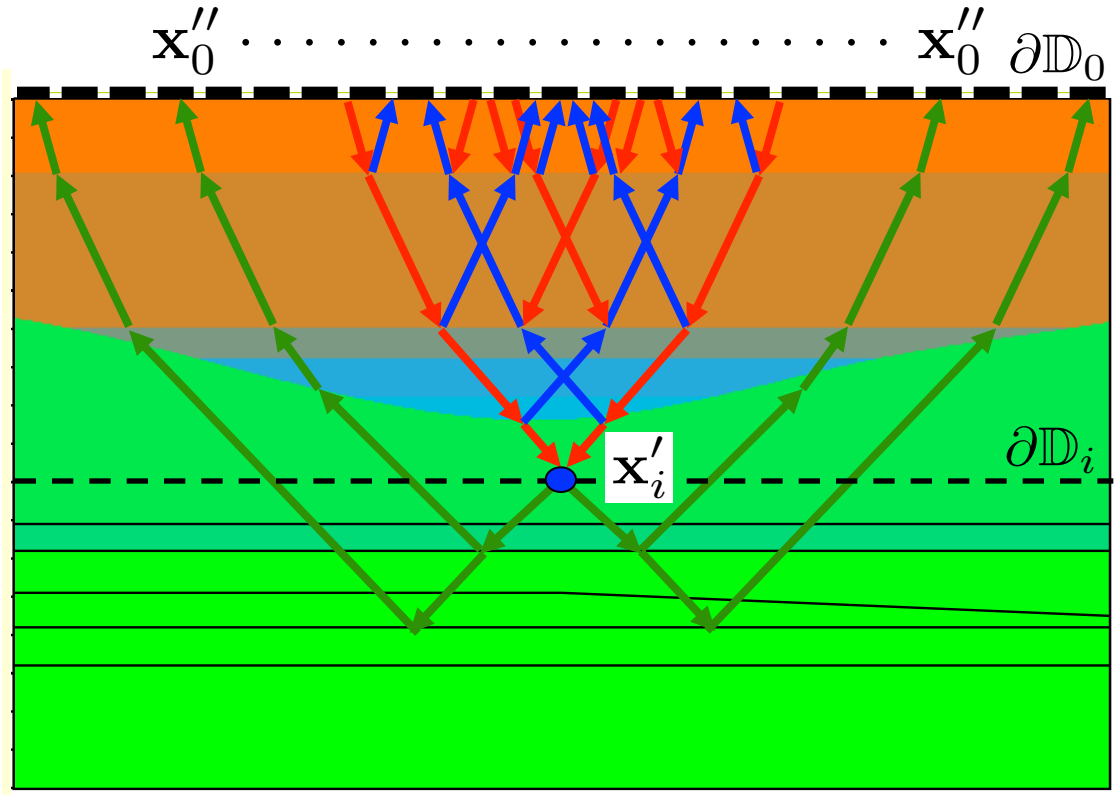


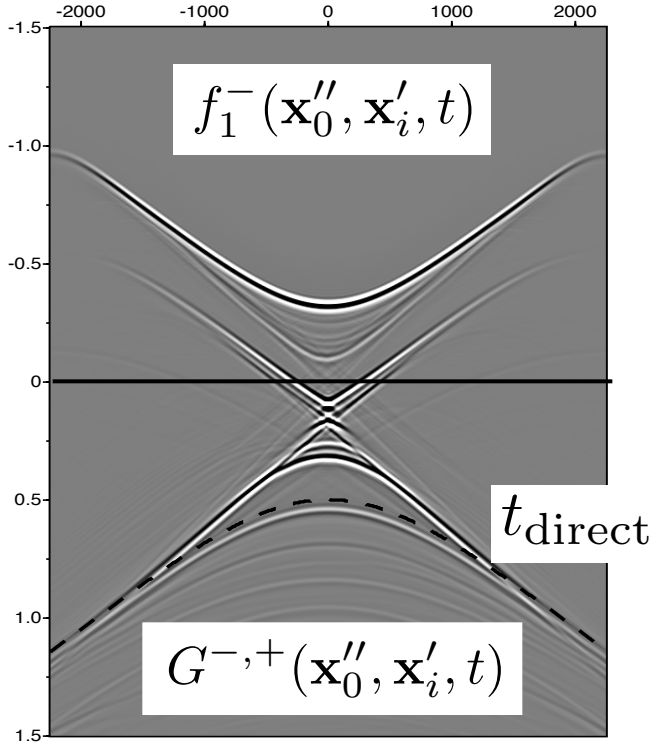


$$f_1^-(\mathbf{x}_0'', \mathbf{x}_i', t)$$

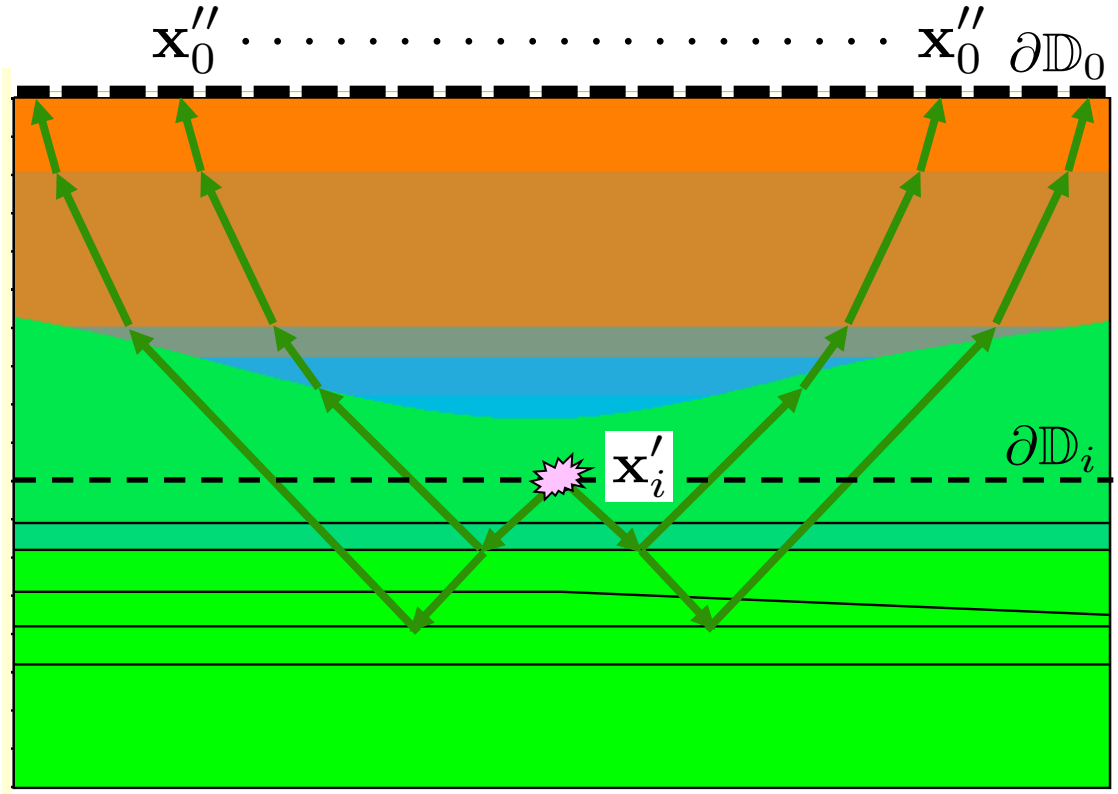


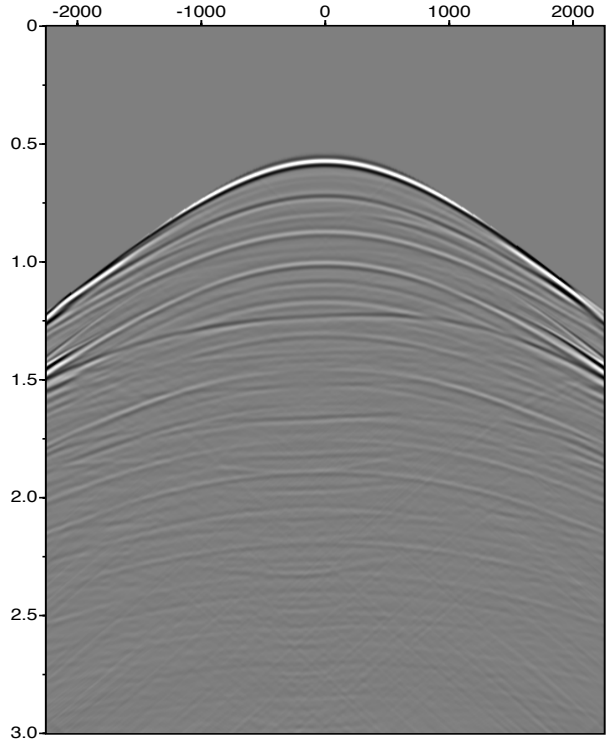
$$G^{-,+}(\mathbf{x}_0'', \mathbf{x}_i', t) + f_1^-(\mathbf{x}_0'', \mathbf{x}_i', t) = \int_{\partial\mathbb{D}_0} \mathrm{d}\mathbf{x}_0 \int_{-\infty}^{t} R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}_i', t') \, \mathrm{d}t'$$



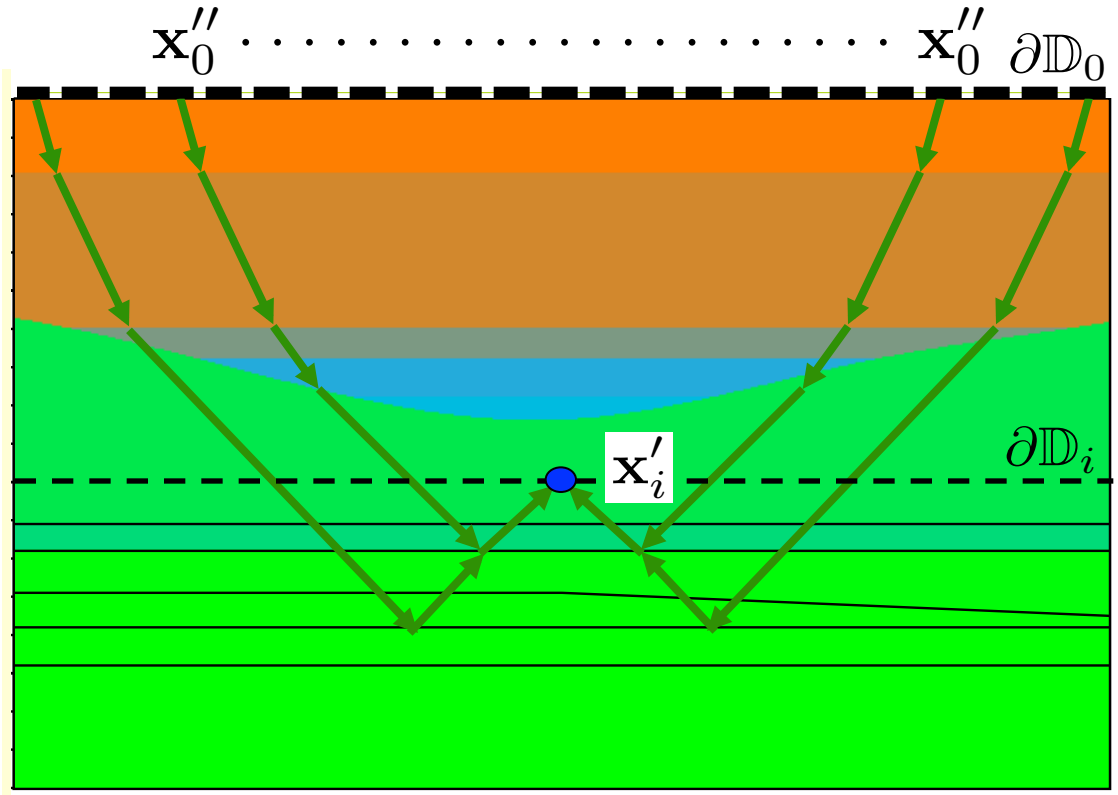


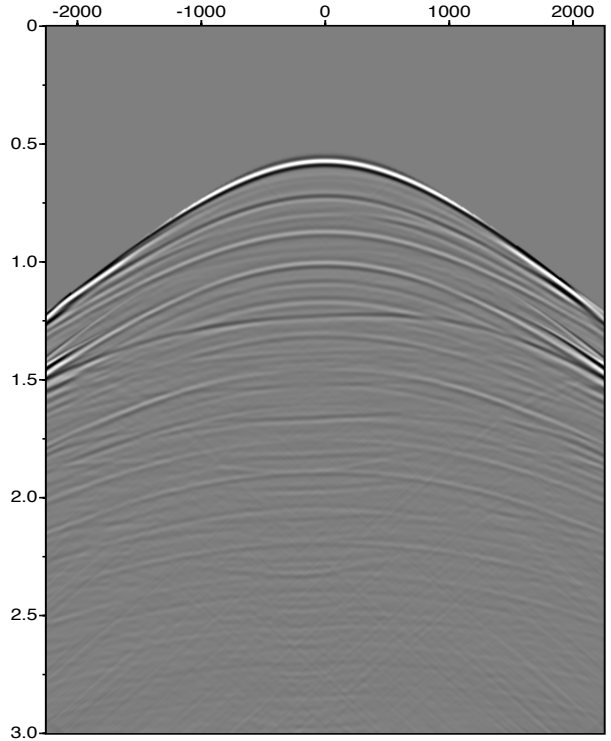
$$G^{-,+}(\mathbf{x}_0'', \mathbf{x}_i', t) + f_1^-(\mathbf{x}_0'', \mathbf{x}_i', t) = \int_{\partial\mathbb{D}_0} \mathrm{d}\mathbf{x}_0 \int_{-\infty}^{t} R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}_i', t') \mathrm{d}t'$$



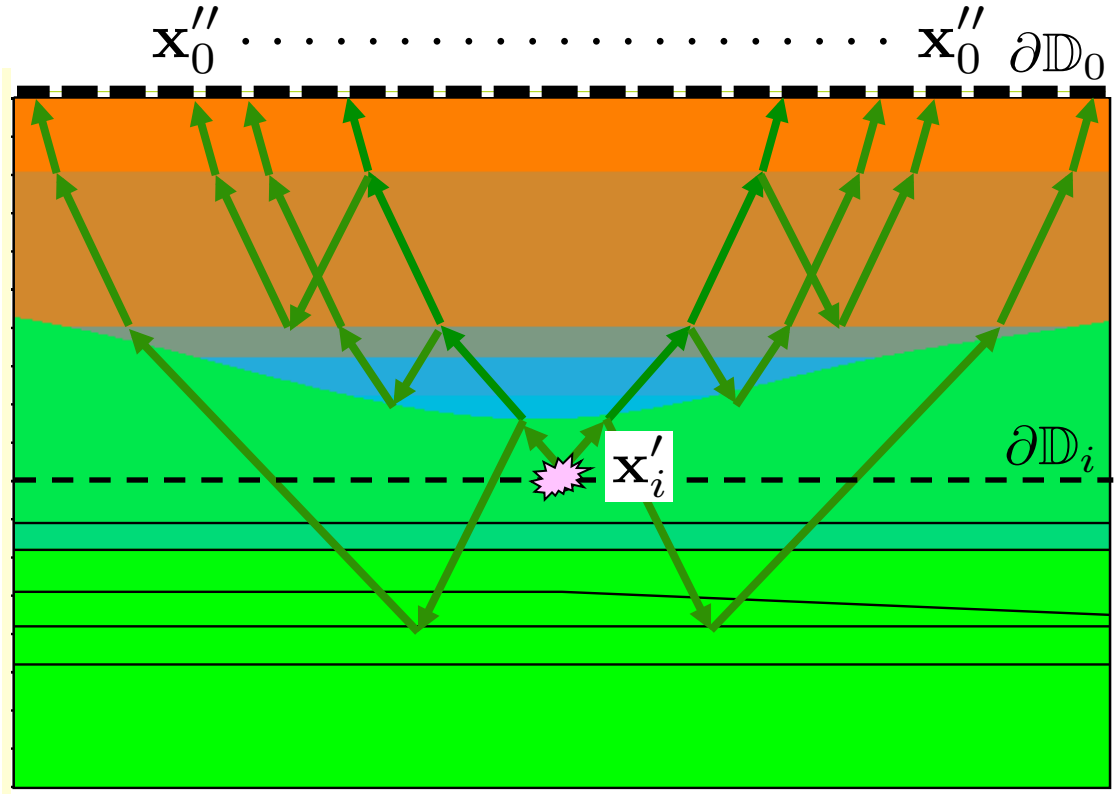


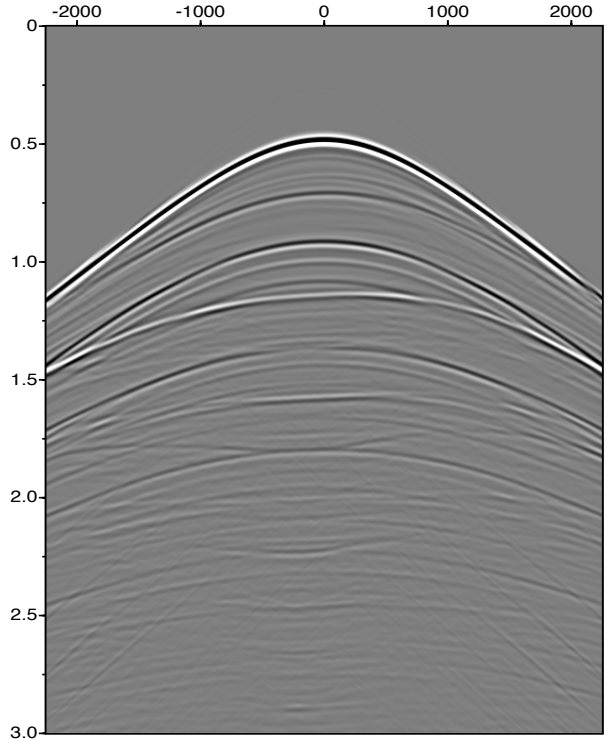
$$G^{-,+}(\mathbf{x}_0'', \mathbf{x}_i', t) + f_1^-(\mathbf{x}_0'', \mathbf{x}_i', t) = \int_{\partial\mathbb{D}_0} \mathrm{d}\mathbf{x}_0 \int_{-\infty}^{t} R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}_i', t') \mathrm{d}t'$$



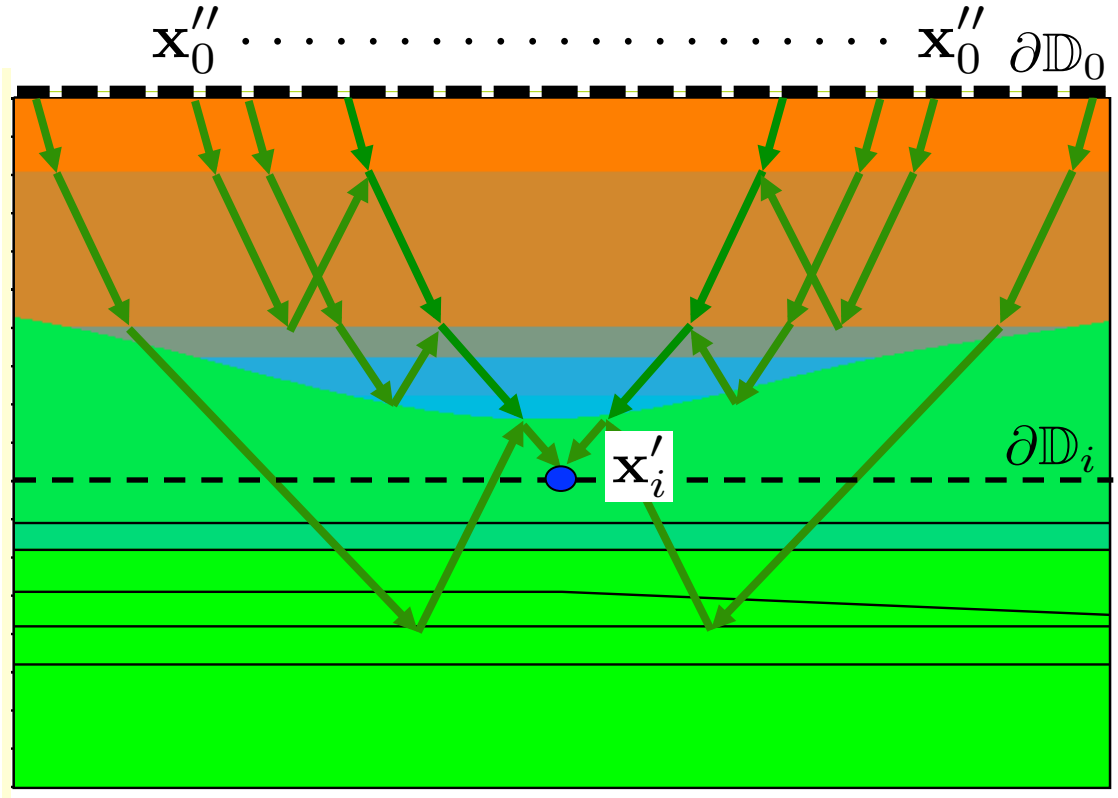


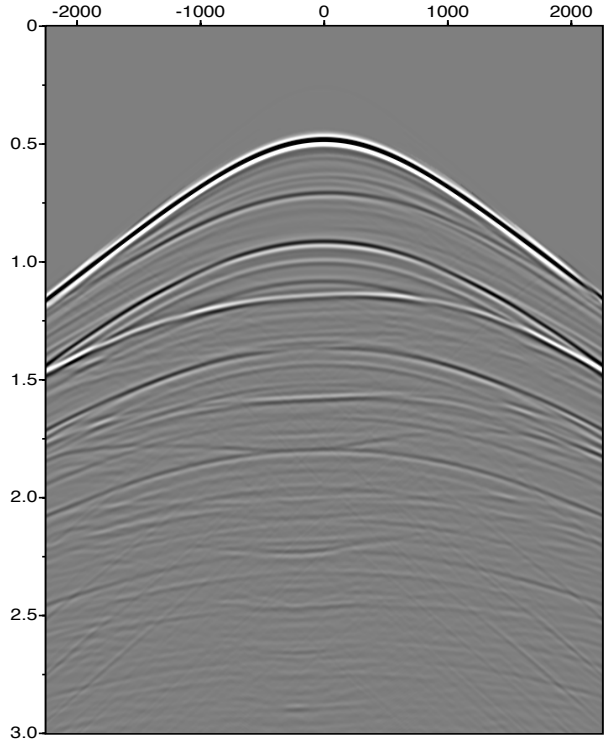
$$G^{-,+}(\mathbf{x}_i', \mathbf{x}_0'', t) + f_1^-(\mathbf{x}_0'', \mathbf{x}_i', t) = \int_{\partial\mathbb{D}_0} \mathrm{d}\mathbf{x}_0 \int_{-\infty}^{t} R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}_i', t') \mathrm{d}t'$$





$$G^{-,-}(\mathbf{x}_0'', \mathbf{x}_i', t) + f_1^+(\mathbf{x}_0'', \mathbf{x}_i', -t) = \int_{\partial\mathbb{D}_0} \mathrm{d}\mathbf{x}_0 \int_{-\infty}^{t} R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^-(\mathbf{x}_0, \mathbf{x}_i', -t') \mathrm{d}t'$$





$$-G^{+,+}(\mathbf{x}_i', \mathbf{x}_0'', t) + f_1^+(\mathbf{x}_0'', \mathbf{x}_i', -t) = \int_{\partial\mathbb{D}_0} \mathrm{d}\mathbf{x}_0 \int_{-\infty}^{t} R(\mathbf{x}_0'', \mathbf{x}_0, t-t') f_1^-(\mathbf{x}_0, \mathbf{x}_i', -t') \mathrm{d}t'$$

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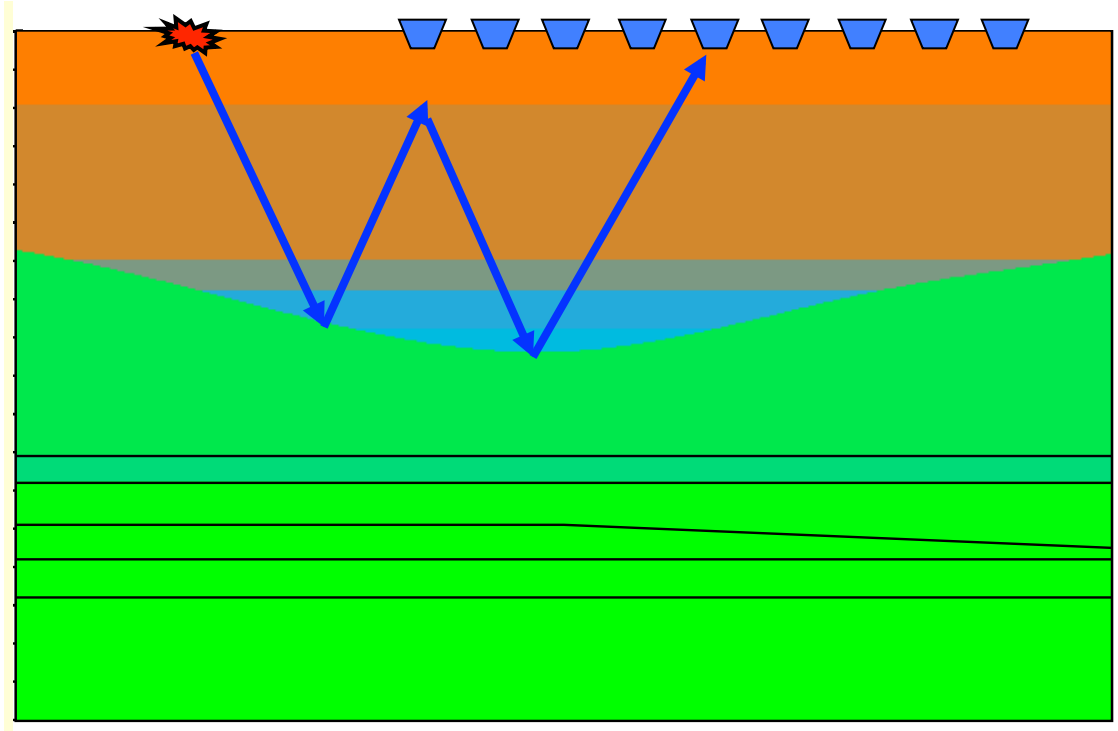

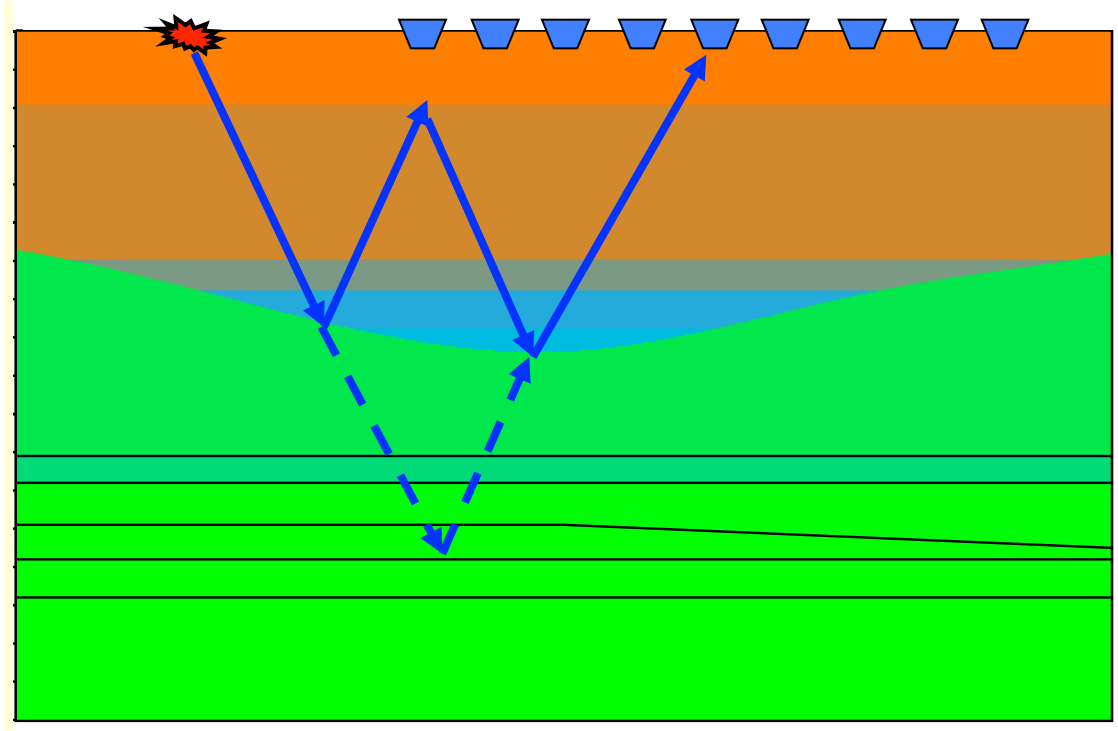

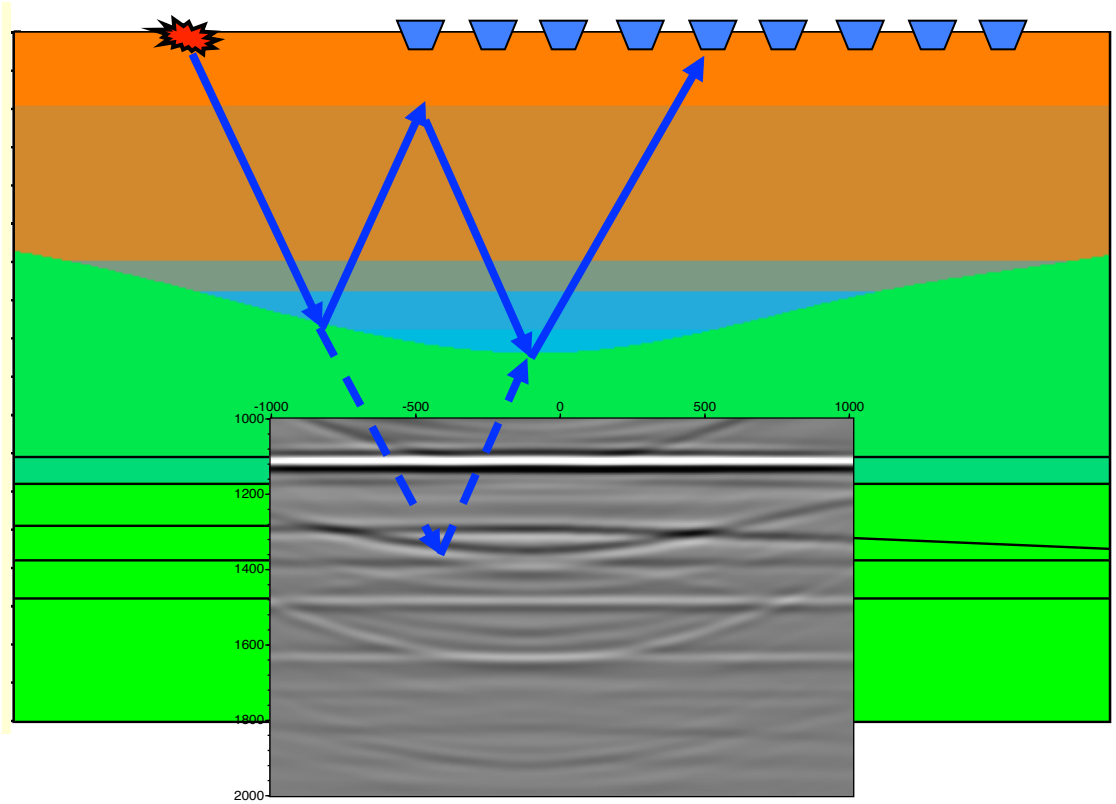
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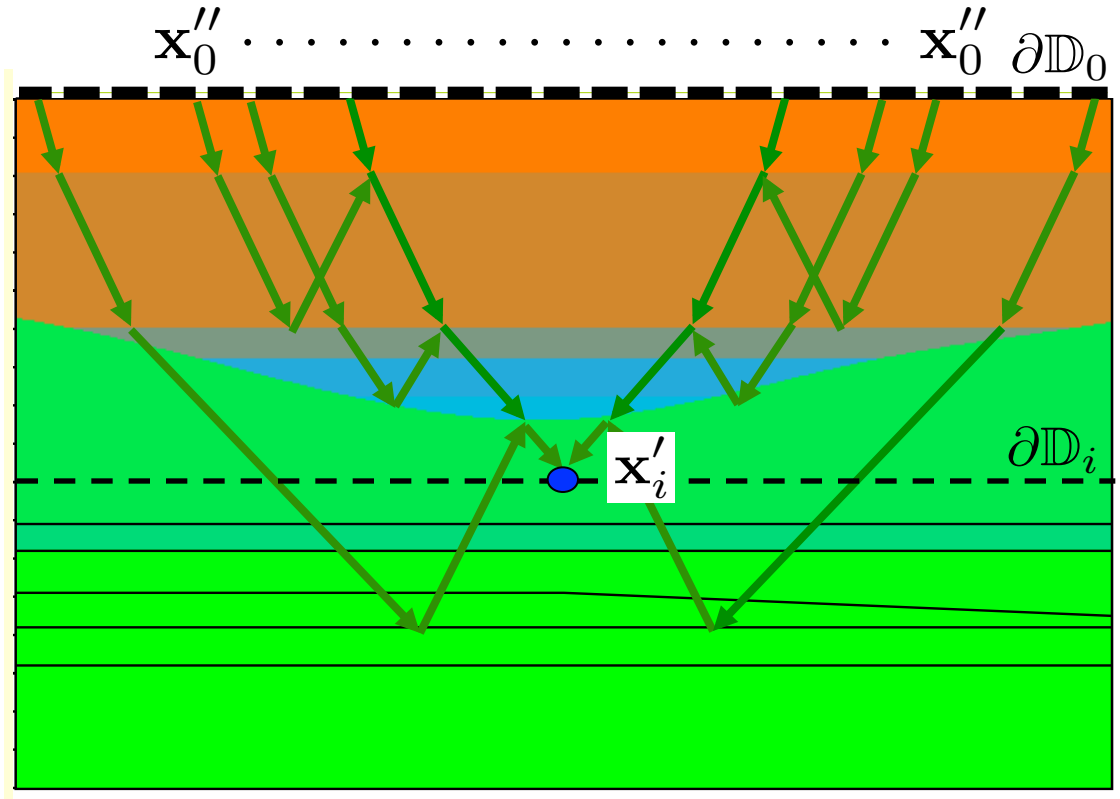
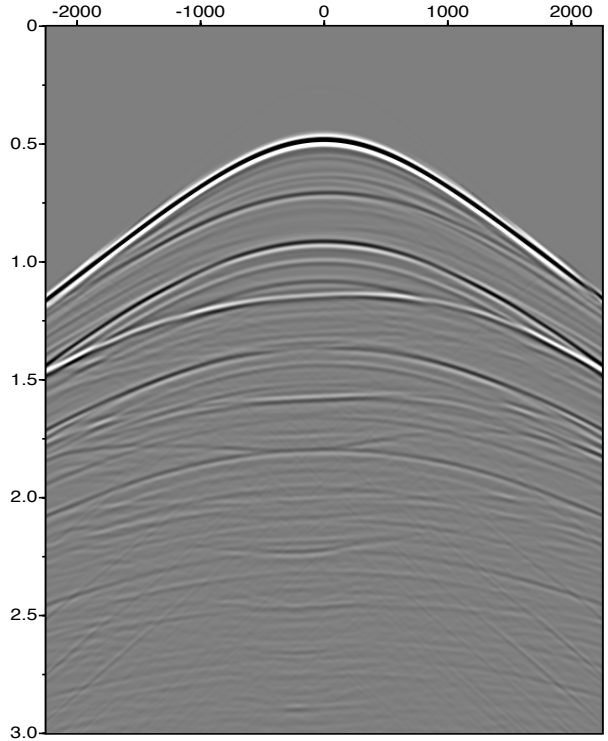
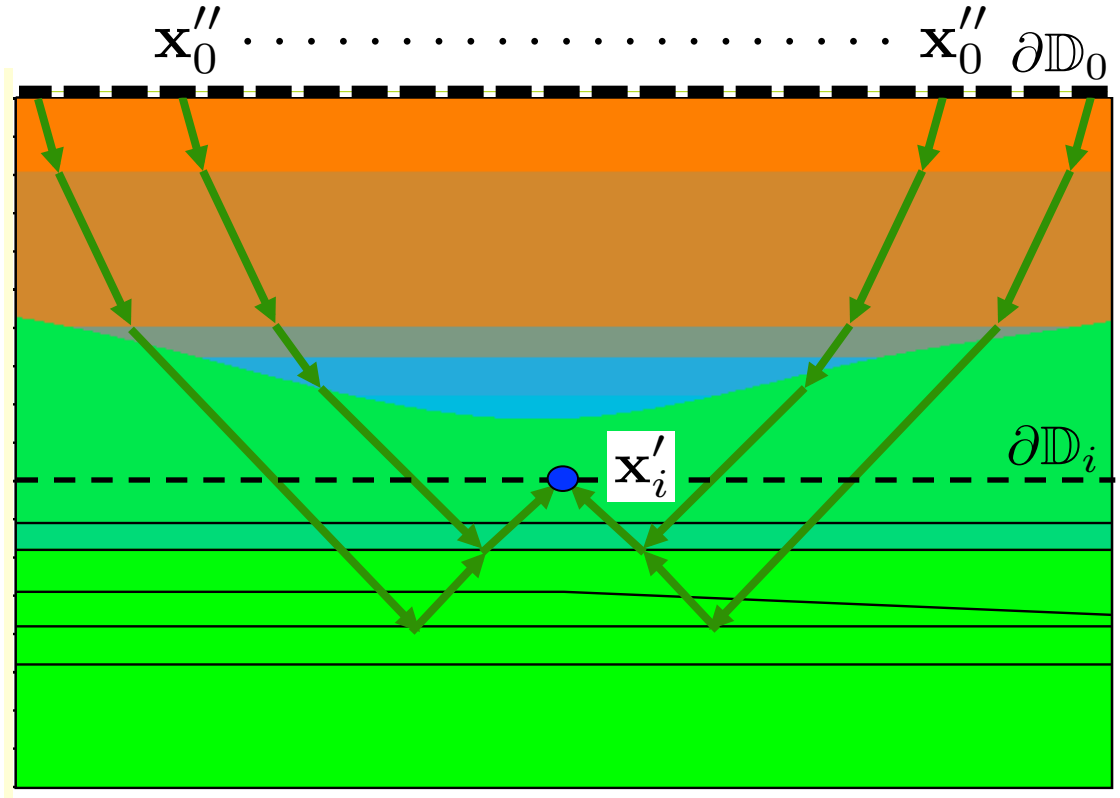
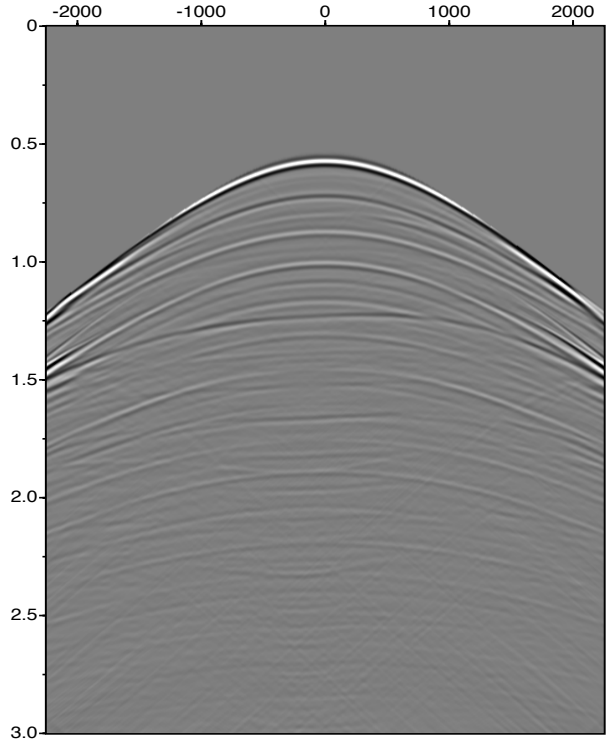


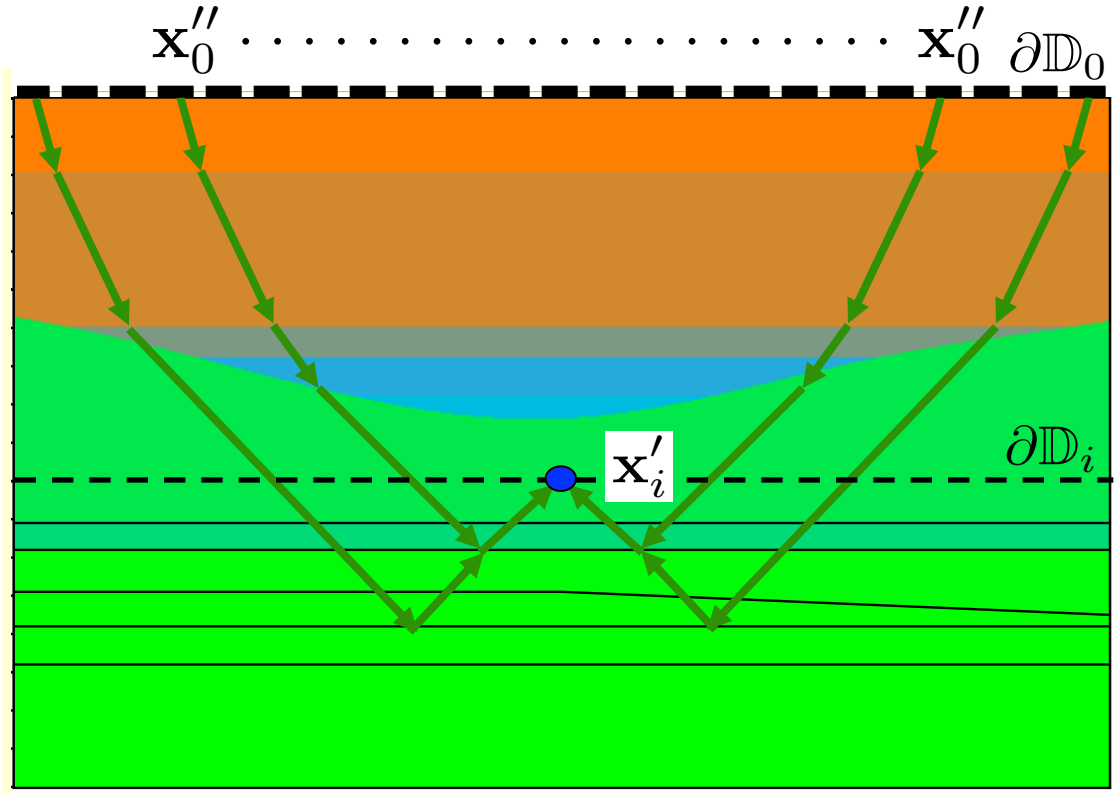


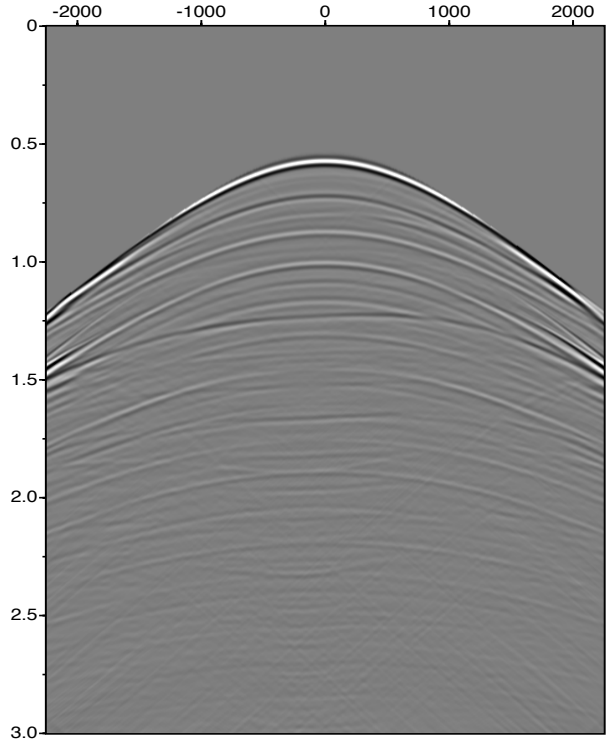
$$G^{+,+}(\mathbf{x}_i', \mathbf{x}_0'', t)$$



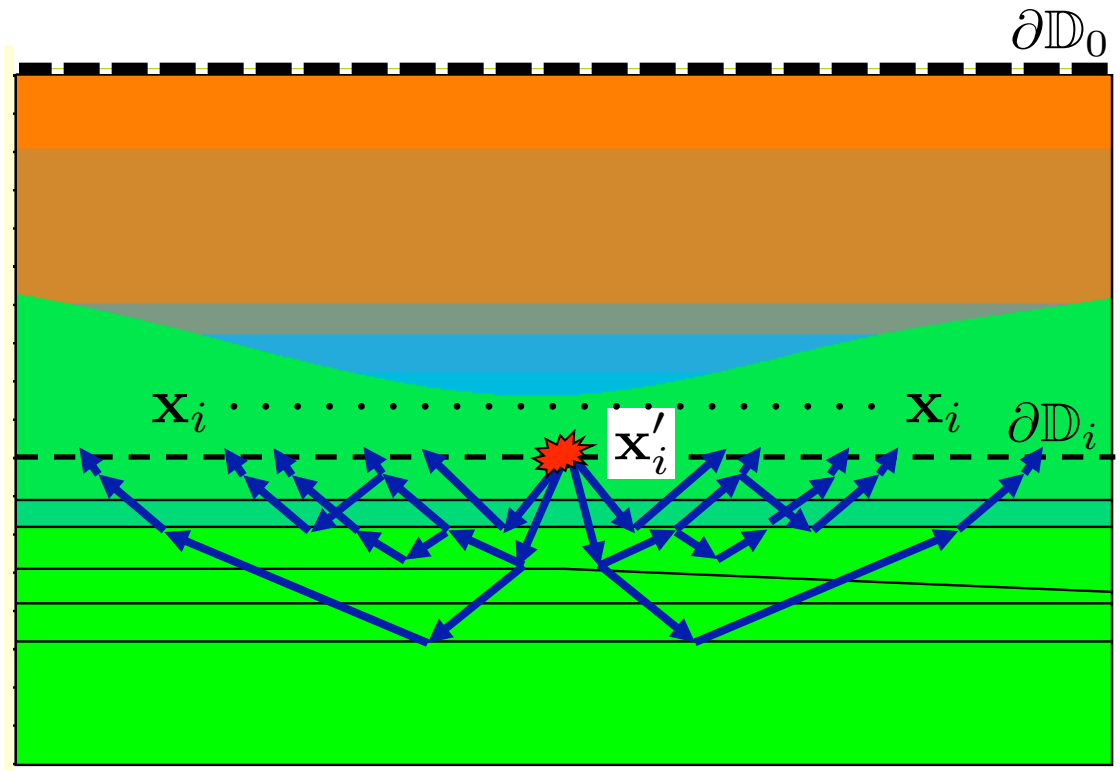


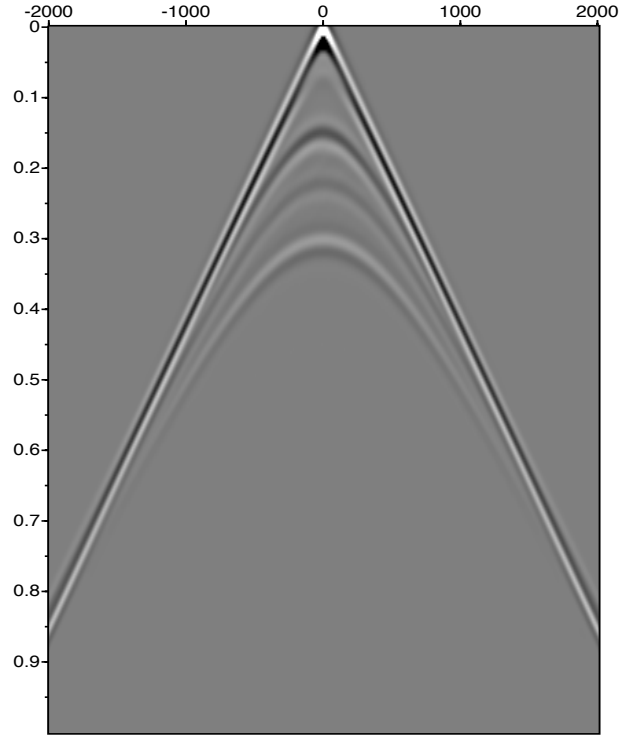
$$G^{-,+}(\mathbf{x}_i', \mathbf{x}_0'', t)$$



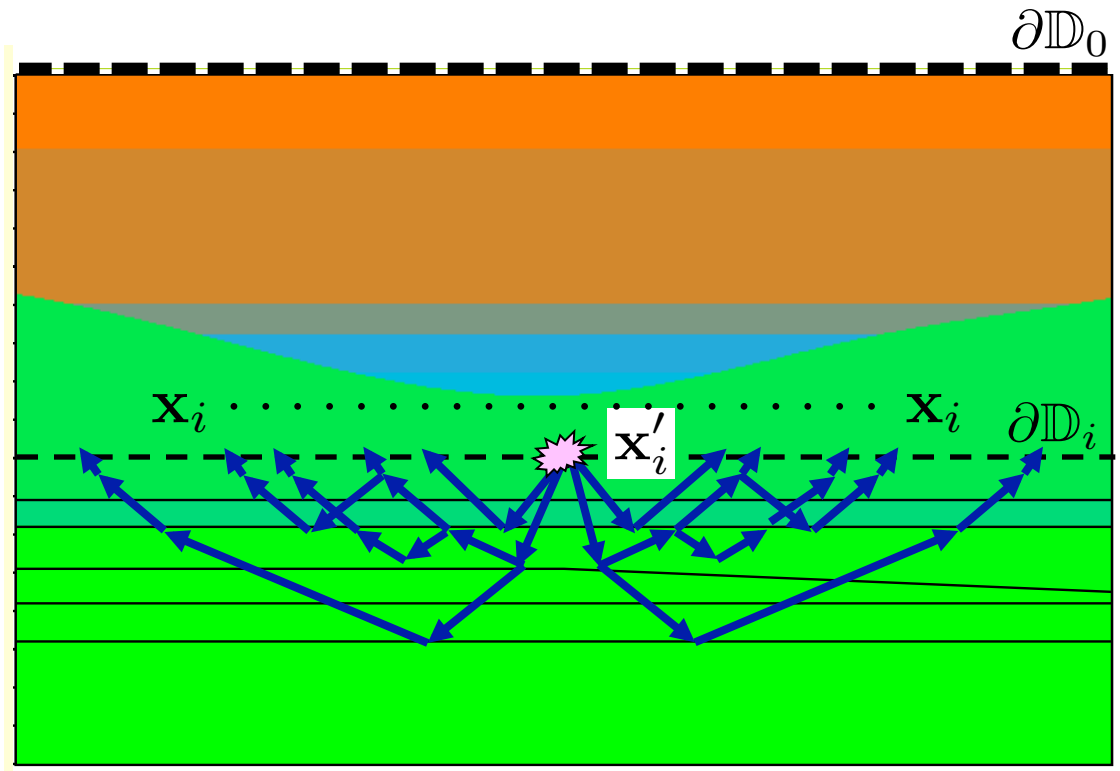


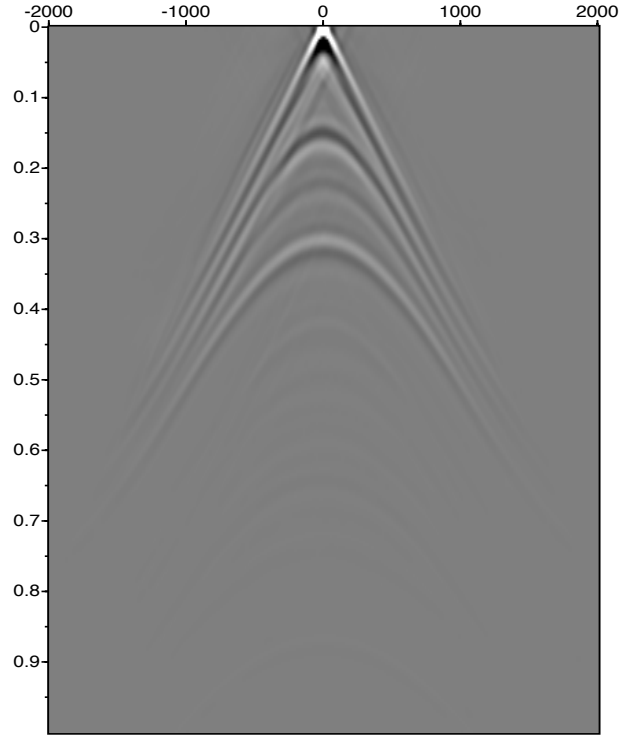
$$G^{-,+}(\mathbf{x}_i, \mathbf{x}_0'', t) = \int_{\partial\mathbb{D}_i} \mathrm{d}\mathbf{x}_i' \int_{-\infty}^{\infty} R(\mathbf{x}_i, \mathbf{x}_i', t') G^{+,+}(\mathbf{x}_i', \mathbf{x}_0'', t - t') \mathrm{d}t'$$



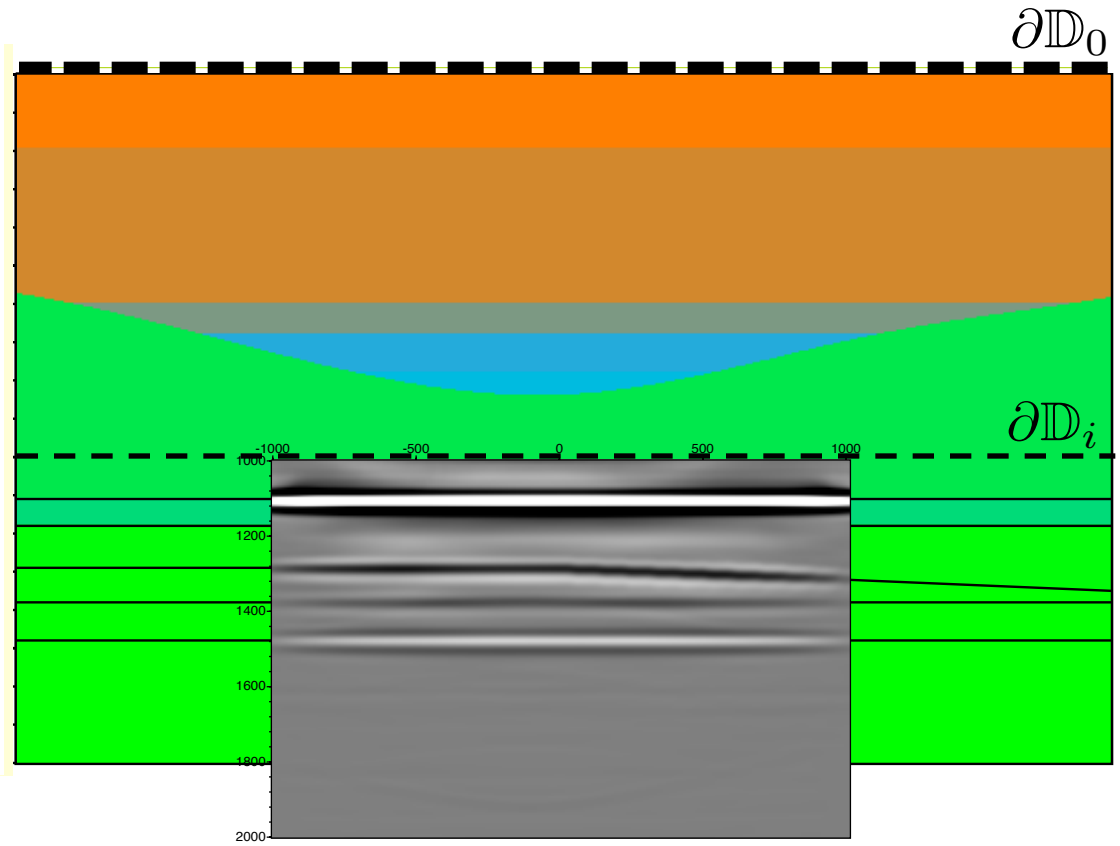


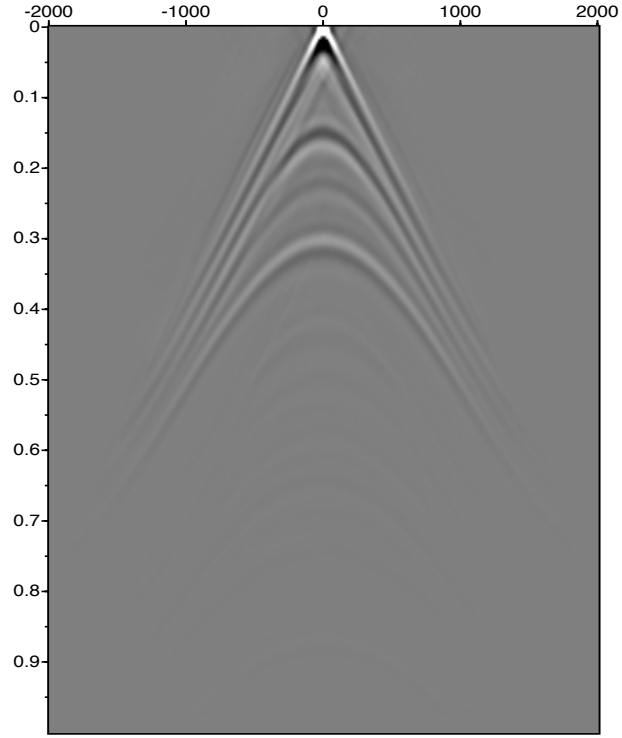
$$G^{-,+}(\mathbf{x}_i, \mathbf{x}''_0, t) = \int_{\partial\mathbb{D}_i} \mathrm{d}\mathbf{x}'_i \int_{-\infty}^{\infty} R(\mathbf{x}_i, \mathbf{x}'_i, t') G^{+,+}(\mathbf{x}'_i, \mathbf{x}''_0, t - t')\mathrm{d}t'$$



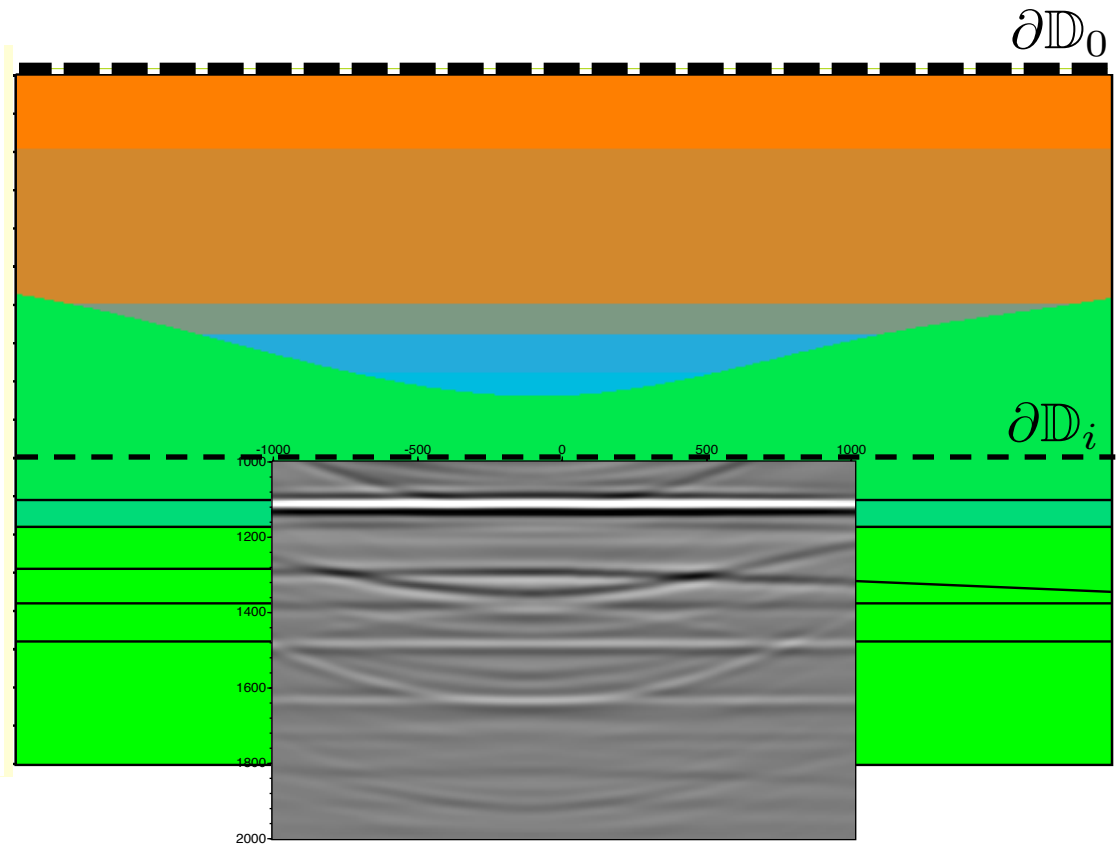


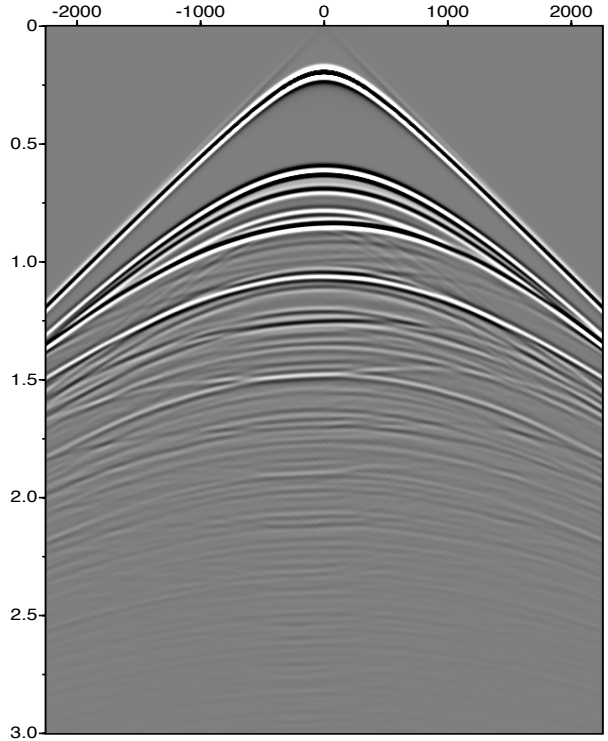
$$G^{-,+}(\mathbf{x}_i, \mathbf{x}''_0, t) = \int_{\partial\mathbb{D}_i} \mathrm{d}\mathbf{x}'_i \int_{-\infty}^{\infty} R(\mathbf{x}_i, \mathbf{x}'_i, t') G^{+,+}(\mathbf{x}'_i, \mathbf{x}''_0, t - t')\mathrm{d}t'$$



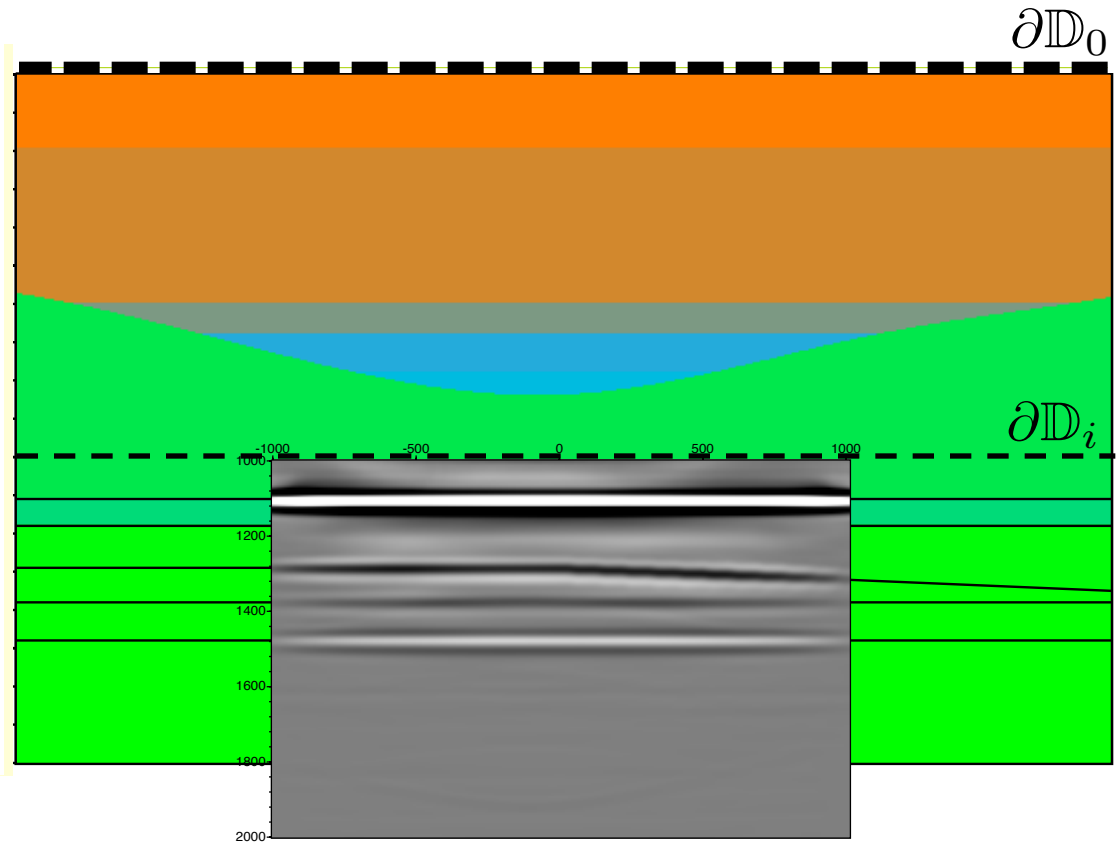


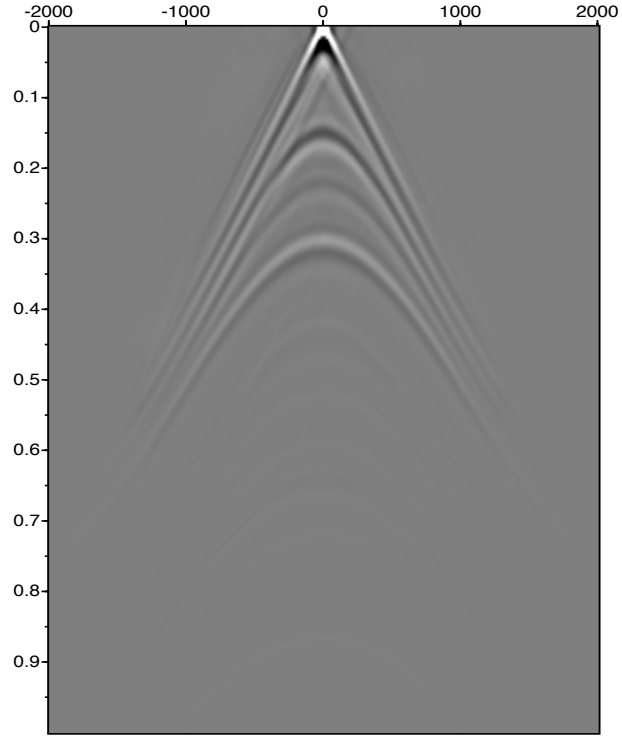
Marchenko imaging





Standard imaging





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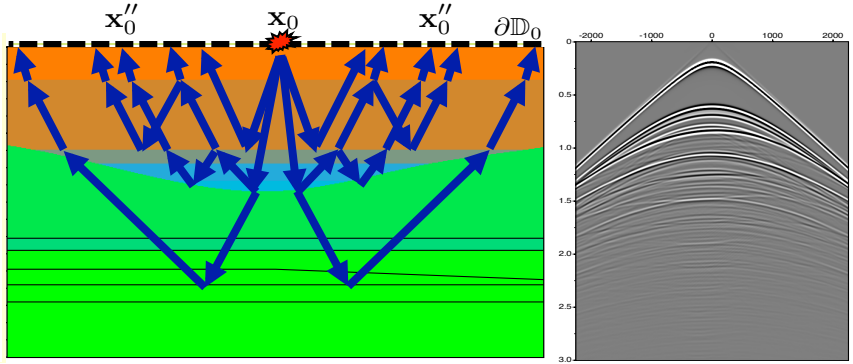
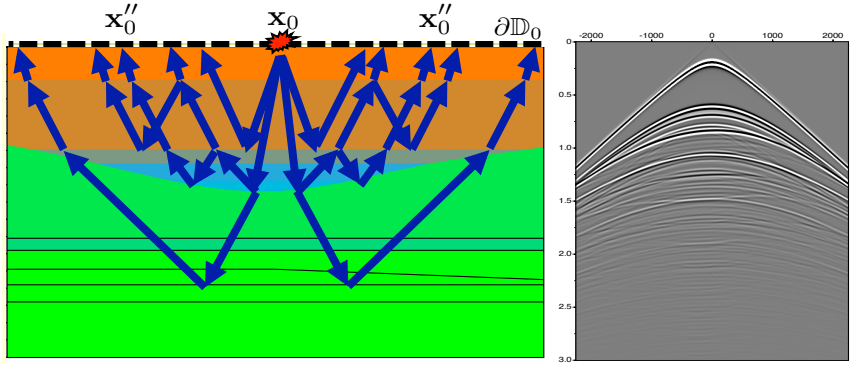
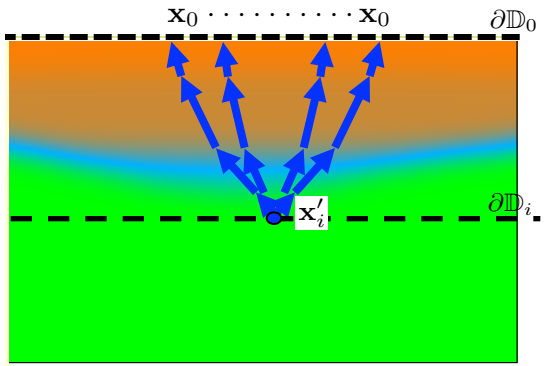
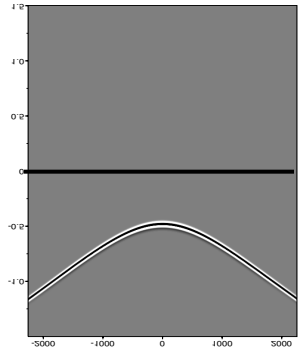


Reflection data

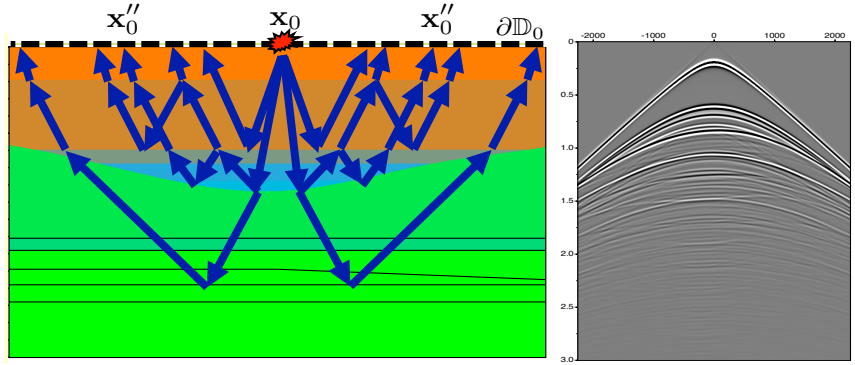


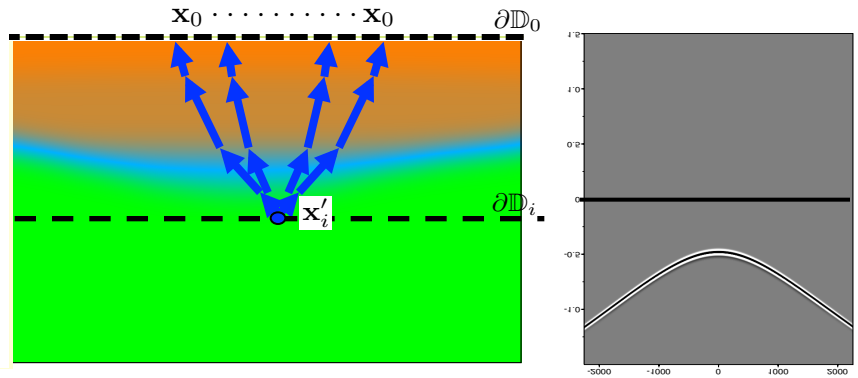




Reflection data

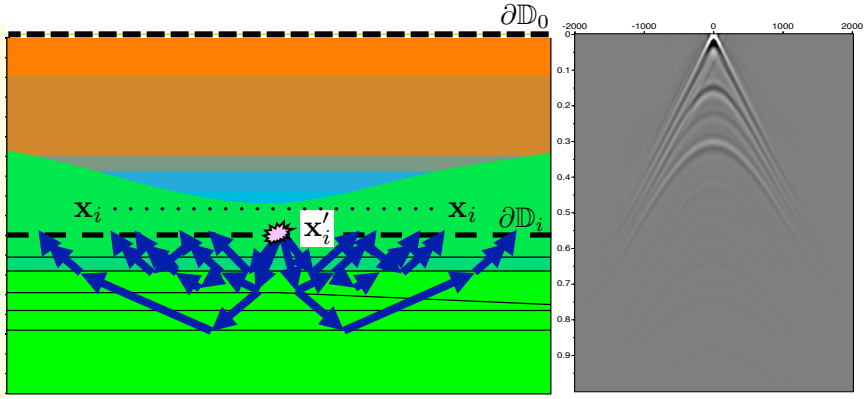
First arrivals



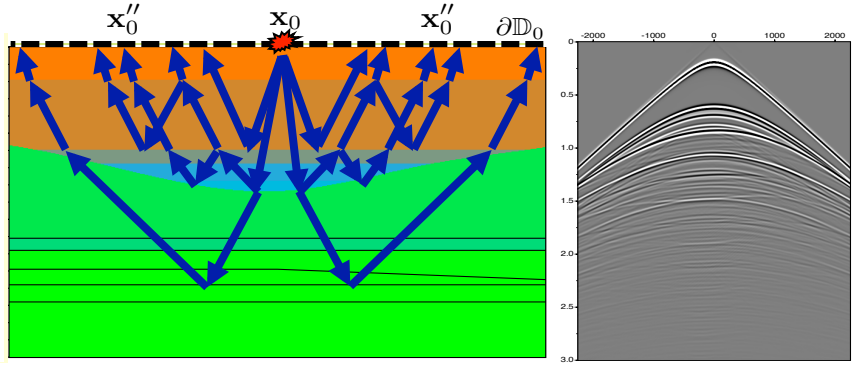


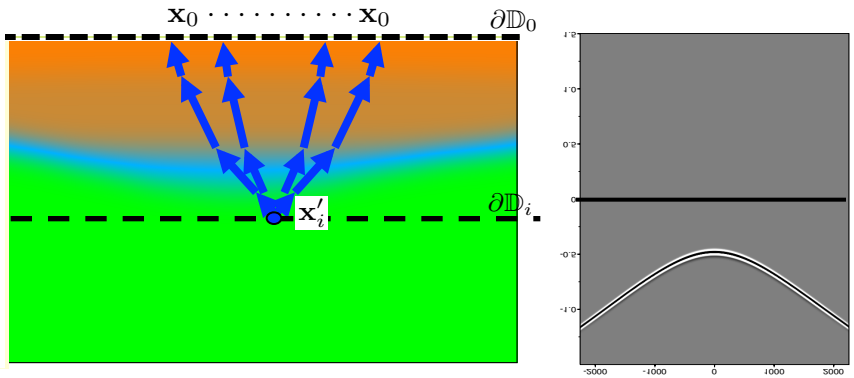
Reflection data

First arrivals



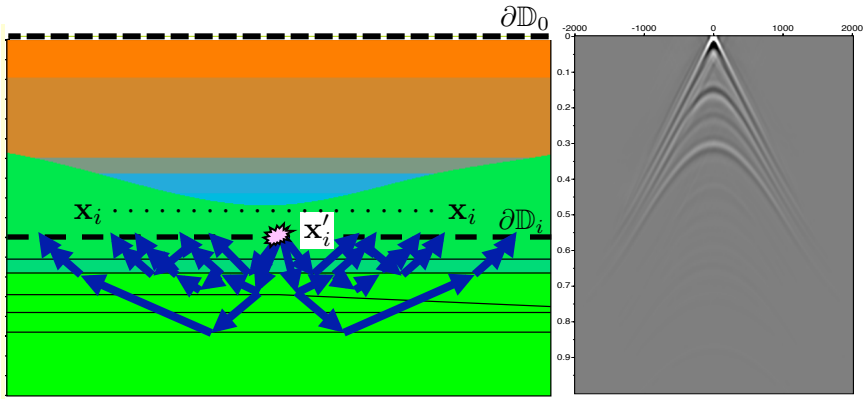
Data-driven redatuming, AVO analysis

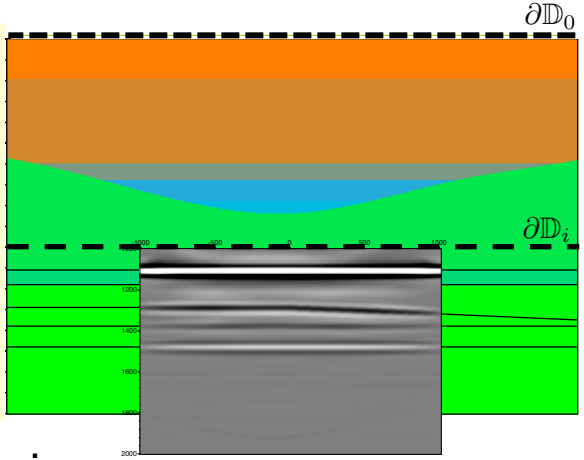




Reflection data

First arrivals





Data-driven redatuming, AVO analysis and imaging