Processing of ocean bottom data

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6.1 Introduction

In the last few years, a new type of seismic data acquisition is growing in popularity, being the acquisition at the ocean bottom. In this type of acquisition, (multi-component) geophones are put at the ocean bottom and a vessel with an airgun array, similar as in conventional marine acquisition, is moving across this ocean bottom cable (see Figure 6.1).

The advantages of this type of acquisition are:

- Multi-component measurements at the bottom provide more information on the S-waves below the ocean bottom. In this way converted modes and anisotropy might be taken into account.
- Ocean bottom data is less sensitive to ocean wave motion and other noise sources related to hydrophones positioned near the ocean surface.
- Ocean bottom data is very well suited for 4-D seismics since the geophones can be kept in position for a longer period of time.
- In certain situations the area cannot be fully covered with conventional acquisition, due to obstacles (i.e. oil producing platforms).
- Ocean bottom acquisition provides us with actually measured Common Focus Point (CFP) gathers, with the focusing points positioned at the bottom.

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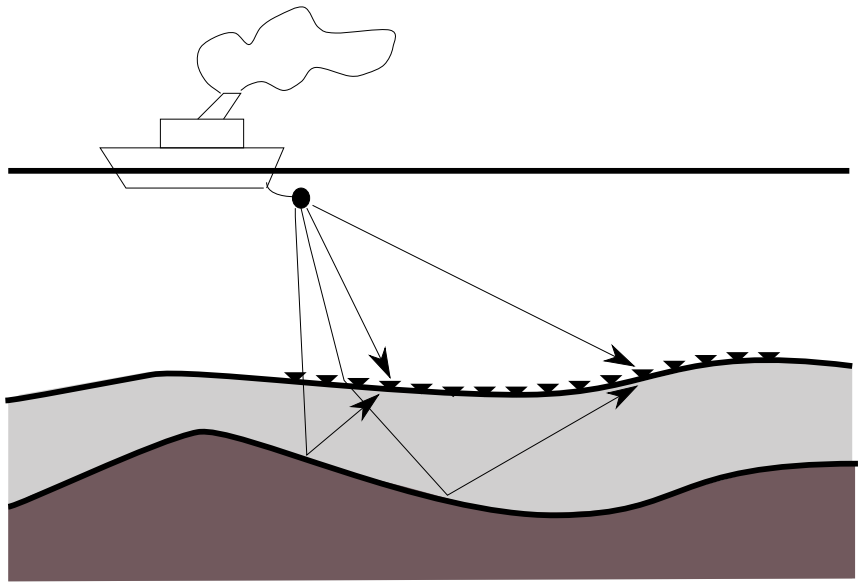


Fig. 6.1 *Ocean bottom seismic acquisition.*

In this chapter we propose a processing scheme for ocean bottom data. We will assume that at least two-component data is available [acoustic pressure $p(x,(y),z_1,t)$ and the vertical component of the particle velocity $v_z(x,(y),z_1,t)$, where z_1 is the depth level of the ocean bottom], but we will also consider the case of three- (and four-) component data [including the horizontal component(s) of the particle velocity $v_x(x,(y),z_1,t)$ (and $v_y(x,(y),z_1,t)$)]. The processing scheme for two-component data is based on the acoustic wave equation, whereas we will make use of the elastic wave equation for the processing scheme of three- (and four-) component data.

The processing steps that we propose are:

- Transformation of 3-D to 2-D amplitudes (optional).
- Decomposition into downgoing and upgoing (P- and S-) waves.
- Multiple elimination at the surface and at the ocean bottom.
- Prestack migration.
- AVA analysis.

The latter two steps are not different from those for surface marine or land data and will therefore not be discussed in this chapter.

6.2 Transformation of 3-D to 2-D amplitudes (optional)

The current practice of ocean bottom seismics is often restricted to acquisition along cables (pseudo 2-D data). For this situation one could either follow the $2\frac{1}{2}$-D approach, as proposed by Bleistein et al. (1987), or one could transform the '3-D amplitudes' into '2-D amplitudes' and proceed with 2-D processing (Wapenaar et al. 1992). We propose the latter procedure, because it also accounts for multiple reflections (whereas the $2\frac{1}{2}$-D approach does not).

The transformation algorithms for the acoustic pressure and the in-plane components of the particle velocity read

$$p^{2D}(x, z_1, t) = 2 \int_{|x|}^{\infty} p^{3D}(x', y = 0, z_1, t) \frac{x'}{\sqrt{x'^2 - x^2}} \mathrm{d}x', \tag{6.1}$$

$$v_x^{2D}(x, z_1, t) = 2 \int_{|x|}^{\infty} v_x^{3D}(x', y = 0, z_1, t) \frac{x}{\sqrt{x'^2 - x^2}} \mathrm{d}x', \tag{6.2}$$

$$v_z^{2D}(x, z_1, t) = 2 \int_{|x|}^{\infty} v_z^{3D}(x', y = 0, z_1, t) \frac{x'}{\sqrt{x'^2 - x^2}} \mathrm{d}x', \tag{6.3}$$

respectively (note the subtle difference between x and x' in the numerators of the kernels in equations 6.2 and 6.3). The superscripts $2D$ and $3D$ refer to the dimensionality of the geometrical spreading of the wave fields. Hence, the wave fields on the left- and right-hand sides of equations 6.1, 6.2 and 6.3 may be seen as line source and point source responses, respectively. For horizontally layered media these transformations are exact; for laterally inhomogeneous media they are preferably applied to CMP gathers, where x and x' should be interpreted as the offset. For more details, in particular about the numerical implementation of the integrals, we refer to the above mentioned paper in Geophysics. In this chapter we discuss $2-D$ as well as $3-D$ processing schemes. For the $2-D$ scheme we assume that the above discussed transformation has been applied. In both cases we omit the superscripts $2D$ and $3D$ for notational convenience.

6.3 Decomposition into downgoing and upgoing (P- and S-) waves

In this section we discuss a modification of the DELPHI decomposition approach (Wapenaar et al., 1990). A similar approach has been proposed by Amundsen and Reitan (1995).

6.3.1 Introduction of two-way and one-way wave vectors

After a Fourier transformation from the time to the frequency domain, the data are stored (per angular frequency component ω) in vectors $\vec{P}(z_1)$, $\vec{V}_z(z_1)$ and, when applicable, $\vec{V}_x(z_1)$ and $\vec{V}_y(z_1)$. This is done according to the usual DELPHI approach, hence, each element in a vector represents a lateral x (and y) position. We define a two-way wave field vector $\vec{Q}(z_1)$, according

to

$$\vec{Q}(z_1) = \begin{pmatrix} -\vec{\tau}_z(z_1) \\ \vec{V}(z_1) \end{pmatrix}, \tag{6.4}$$

where the traction vector $\vec{\tau}_z$ and the velocity vector $\vec{V}$ are defined as follows:

- for two-component data (acoustic approach)

$$\vec{\tau}_z(z_1) = -\vec{P}(z_1) \quad \text{and} \quad \vec{V}(z_1) = \vec{V}_z(z_1), \tag{6.5}$$

- for three- or four-component data (elastic approach)

$$\vec{\tau}_z(z_1) = \begin{pmatrix} \vec{0} \\ (\vec{0}) \\ -\vec{P}(z_1) \end{pmatrix} \quad \text{and} \quad \vec{V}(z_1) = \begin{pmatrix} \vec{V}_x(z_1) \\ (\vec{V}_y(z_1)) \\ \vec{V}_z(z_1) \end{pmatrix}, \tag{6.6}$$

  where the null-vectors $(\vec{0})$ account for the absence of shear stresses at the ocean bottom.

Analogous to equation (6.4) we define a one-way wave field vector $\vec{D}(z_1)$, according to

$$\vec{D}(z_1) = \begin{pmatrix} \vec{D}^+(z_1) \\ \vec{D}^-(z_1) \end{pmatrix}, \tag{6.7}$$

where the downgoing wave field vector $\vec{D}^+$ and the upgoing wave field vector $\vec{D}^-$ are defined as follows:

- for two-component data (acoustic approach)

$$\vec{D}^+(z_1) = \vec{P}^+(z_1) \quad \text{and} \quad \vec{D}^-(z_1) = \vec{P}^-(z_1), \tag{6.8}$$

  where vectors $\vec{P}^+$ and $\vec{P}^-$ contain the flux-normalized[3] downgoing and upgoing waves, respectively,

- for three- or four-component data (elastic approach)

$$\vec{D}^+(z_1) = \begin{pmatrix} \vec{\Phi}^+(z_1) \\ \vec{\Psi}^+(z_1) \\ (\vec{\Upsilon}^+(z_1)) \end{pmatrix} \quad \text{and} \quad \vec{D}^-(z_1) = \begin{pmatrix} \vec{\Phi}^-(z_1) \\ \vec{\Psi}^-(z_1) \\ (\vec{\Upsilon}^-(z_1)) \end{pmatrix}, \tag{6.9}$$

  where vector $\vec{\Phi}^\pm$ contains the flux-normalized down- or upgoing potential for P-waves, $\vec{\Psi}^\pm$ for SV-waves and $\vec{\Upsilon}^\pm$ for SH-waves.

[3] Flux-normalization means that the amplitudes are normalized in such a way that at each depth level z they are proportional to the square-root of the energy flux. The advantage of flux normalization is that the decomposed wave fields obey our one-way reciprocity theorems (for 1-D as well as 2-D and 3-D configurations; Wapenaar and Grimbergen, 1995).

6.3.2 General relation between two-way and one-way wave vectors

For arbitrary depth z the general relations between the two-way and one-way wave field vectors read:

- for composition

$$\vec{Q}(z) = \mathbf{L}(z)\vec{D}(z), \tag{6.10}$$

or

$$\begin{pmatrix} -\vec{\tau}_z(z) \\ \vec{V}(z) \end{pmatrix} = \begin{pmatrix} \mathbf{L}_1^+(z) & \mathbf{L}_1^-(z) \\ \mathbf{L}_2^+(z) & \mathbf{L}_2^-(z) \end{pmatrix} \begin{pmatrix} \vec{D}^+(z) \\ \vec{D}^-(z) \end{pmatrix}, \tag{6.11}$$

- for decomposition

$$\vec{D}(z) = \mathbf{L}^{-1}(z)\vec{Q}(z), \tag{6.12}$$

or

$$\begin{pmatrix} \vec{D}^+(z) \\ \vec{D}^-(z) \end{pmatrix} = \begin{pmatrix} -\{\mathbf{L}_2^-(z)\}^T & \{\mathbf{L}_1^-(z)\}^T \\ \{\mathbf{L}_2^+(z)\}^T & -\{\mathbf{L}_1^+(z)\}^T \end{pmatrix} \begin{pmatrix} -\vec{\tau}_z(z) \\ \vec{V}(z) \end{pmatrix}, \tag{6.13}$$

where ^{T} denotes transposition.

Note that for the acoustic situation in a laterally invariant medium, equations (6.11) and (6.13) may be rewritten in the rayparameter domain as

$$\begin{pmatrix} \tilde{P}(z) \\ \tilde{V}_z(z) \end{pmatrix} = \begin{pmatrix} \tilde{L}_1^+(z) & \tilde{L}_1^-(z) \\ \tilde{L}_2^+(z) & \tilde{L}_2^-(z) \end{pmatrix} \begin{pmatrix} \tilde{P}^+(z) \\ \tilde{P}^-(z) \end{pmatrix} \tag{6.14}$$

and

$$\begin{pmatrix} \tilde{P}^+(z) \\ \tilde{P}^-(z) \end{pmatrix} = \begin{pmatrix} -\tilde{L}_2^-(z) & \tilde{L}_1^-(z) \\ \tilde{L}_2^+(z) & -\tilde{L}_1^+(z) \end{pmatrix} \begin{pmatrix} \tilde{P}(z) \\ \tilde{V}_z(z) \end{pmatrix}, \tag{6.15}$$

with

$$\tilde{L}_1^{\pm}(z) = \sqrt{\frac{\varrho(z)}{2q(z)}} \quad \text{and} \quad \tilde{L}_2^{\pm}(z) = \pm\sqrt{\frac{q(z)}{2\varrho(z)}}, \tag{6.16}$$

where $\varrho(z)$ is the mass density and where the vertical slowness $q(z)$ is related to the rayparameter(s) p_x (and p_y), according to

$$q(z) = \sqrt{c^{-2}(z) - p_x^2 - (p_y^2)}, \tag{6.17}$$

where $c(z)$ is the acoustic propagation velocity. For similar relations for the elastic situation, see Appendix 6A.

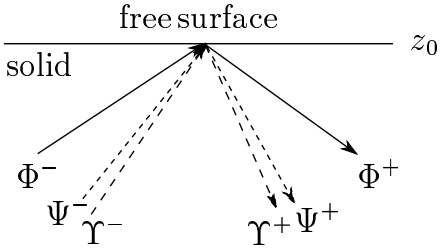


Fig. 6.2 *One-way waves at the free surface*

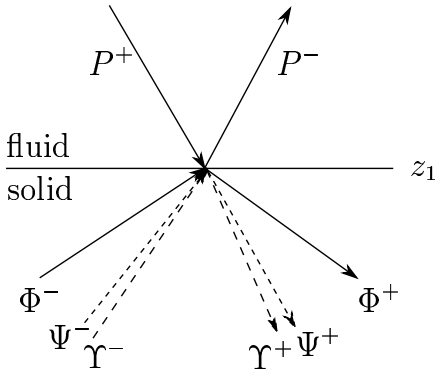


Fig. 6.3 *One-way waves at the ocean bottom*

6.3.3 Decomposition at the free surface

Before we discuss decomposition at the ocean bottom, we briefly review our algorithm for decomposition at the free surface. At the free surface $z = z_0$ we have $\vec{\tau}_z = \vec{0}$ (except at the source). Hence, applying equation (6.13) for $z = z_0$ we simply obtain

$$\vec{D}^{\pm}(z_0) = \pm\{\mathbf{L}_1^{\mp}(z_0)\}^T \vec{V}(z_0). \tag{6.18}$$

This equation states that the down/upgoing wave vector $\vec{D}^{\pm}$ at the free surface (see Figure 6.2 for the elastic approach) can be uniquely obtained from the particle velocity measurements $\vec{V}$ at the free surface. Note, however, that $\vec{D}^{+}$ here represents only the downgoing *reflected* waves; the downgoing *source* waves are ignored in equation (6.18), since we assumed $\vec{\tau}_z = \vec{0}$.

6.3.4 Decomposition at the ocean bottom

The decomposition at the ocean bottom is again directly obtained from equation (6.13) and consists of one decomposition just above and one just below the ocean bottom (see Figure 6.3 for the elastic approach):

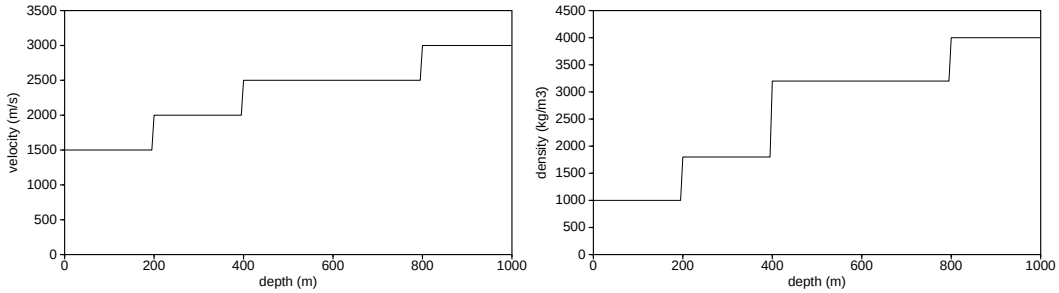


Fig. 6.4 *Velocity and density profile used for modeling numerical data*

- Decomposition just above the ocean bottom at $z = z_1^- = z_1 - \epsilon$, with $\epsilon \to 0$ [4]

$$\vec{D}^{\pm}(z_1^-) = \pm\{\mathbf{L}_2^{\mp}(z_1^-)\}^T \vec{\tau}_z(z_1) \pm \{\mathbf{L}_1^{\mp}(z_1^-)\}^T \vec{V}(z_1), \tag{6.19}$$

or, since the medium above the ocean bottom is an acoustic medium,

$$\vec{P}^{\pm}(z_1^-) = \mp\{\mathbf{L}_2^{\mp}(z_1^-)\}^T \vec{P}(z_1) \pm \{\mathbf{L}_1^{\mp}(z_1^-)\}^T \vec{V}_z(z_1), \tag{6.20}$$

or, in the rayparameter domain (assuming no lateral variations at z_1^-),

$$\tilde{P}^{\pm}(z_1^-) = \sqrt{\frac{q(z_1^-)}{2\varrho(z_1^-)}}\tilde{P}(z_1) \pm \sqrt{\frac{\varrho(z_1^-)}{2q(z_1^-)}}\tilde{V}_z(z_1). \tag{6.21}$$

- Decomposition just below the ocean bottom at $z = z_1^+ = z_1 + \epsilon$, with $\epsilon \to 0$

$$\vec{D}^{\pm}(z_1^+) = \pm\{\mathbf{L}_2^{\mp}(z_1^+)\}^T \vec{\tau}_z(z_1) \pm \{\mathbf{L}_1^{\mp}(z_1^+)\}^T \vec{V}(z_1). \tag{6.22}$$

In the elastic approach $\vec{\tau}_z$ and $\vec{V}$ are defined as in equation (6.6).

In the acoustic approach equation (6.22) can be written as

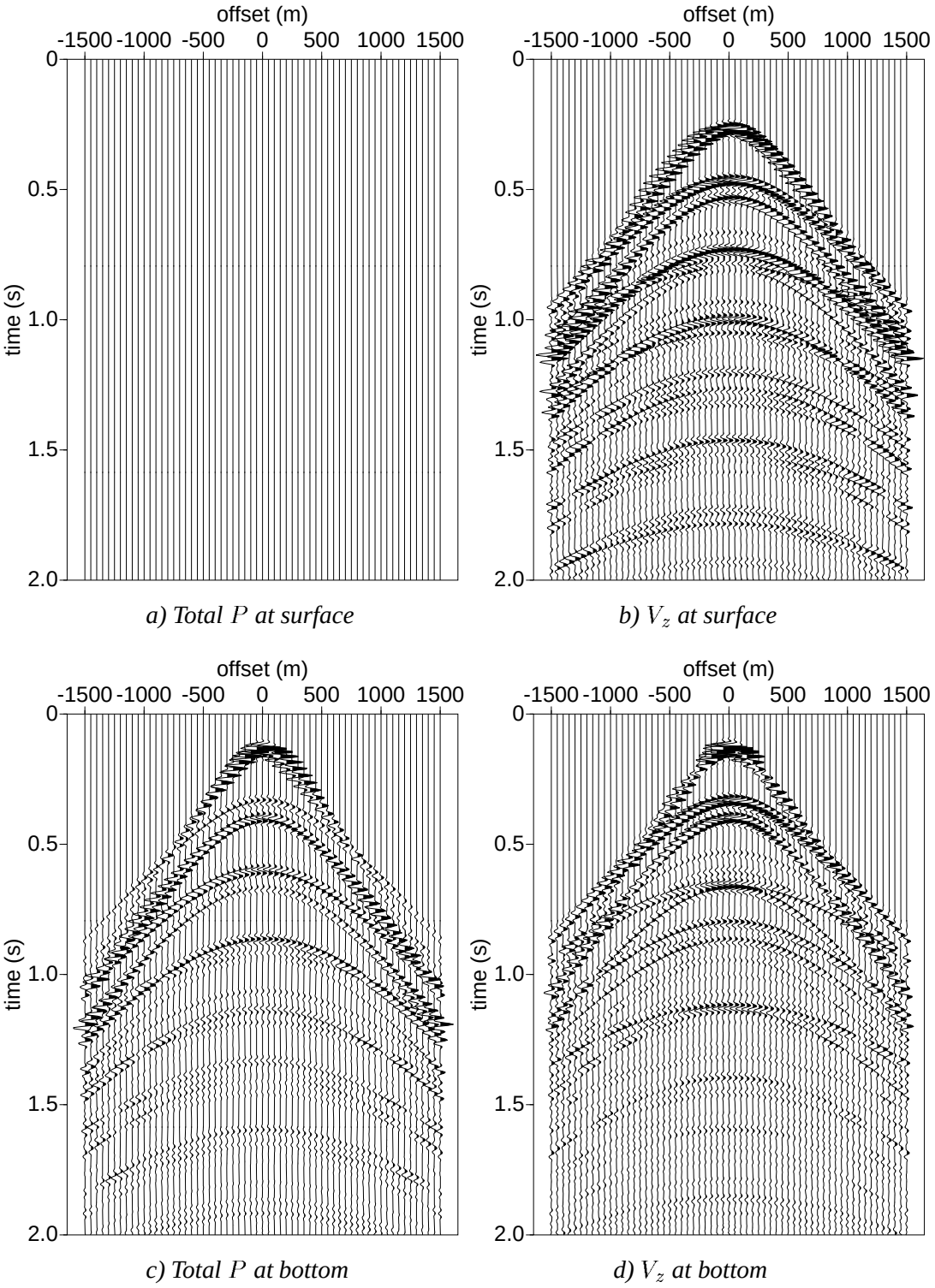
$$\vec{P}^{\pm}(z_1^+) = \mp\{\mathbf{L}_2^{\mp}(z_1^+)\}^T \vec{P}(z_1) \pm \{\mathbf{L}_1^{\mp}(z_1^+)\}^T \vec{V}_z(z_1), \tag{6.23}$$

or, in the rayparameter domain (assuming no lateral variations at z_1^+),

$$\tilde{P}^{\pm}(z_1^+) = \sqrt{\frac{q(z_1^+)}{2\varrho(z_1^+)}}\tilde{P}(z_1) \pm \sqrt{\frac{\varrho(z_1^+)}{2q(z_1^+)}}\tilde{V}_z(z_1). \tag{6.24}$$

Unlike in section 6.3.3, here $\vec{D}^+$ (or $\vec{P}^+$) represents the *total* downgoing wave field, including the transmitted downgoing source waves. This is made possible by the fact that $\vec{\tau}_z(z_1)$ can be measured relatively easy by the hydrophones at the ocean bottom (see equations 6.5 and 6.6), whereas the source induced $\vec{\tau}_z(z_0)$ at the free surface is not so easily measured. In section 6.5 we propose an alternative multiple elimination scheme that makes advantage of the fact that both the *total* $\vec{P}^+$ and $\vec{P}^-$ wave fields are available just below the ocean bottom.

[4] We use the notation $z_1^{\pm}$ only for quantities that are discontinuous at the ocean bottom. For continuous quantities we simply use z_1.



a) Total P at surface *b) V_z at surface*

c) Total P at bottom *d) V_z at bottom*

Fig. 6.5 *Numerical simulation of the total pressure and vertical velocity component at the surface level - a) and b) - and at the ocean bottom level - c) and d).*

6.3.5 Numerical example

To show the application of decomposition on ocean bottom data, we give here a numerical example. We consider a lateral homogeneous subsurface model as given by the profiles in Figure 6.4. In this model we simulate seismic data at the surface and at the ocean bottom. Figure 6.5 shows the pressure and vertical velocity component of the data as it would have been measured at the water surface (Figure 6.5a and b) and at the water bottom (Figure 6.5c and d).
Note that the free surface conditions have been taken into account, resulting in the generation of all surface-related multiples (internal multiples are also included) and in the fact that the total pressure at the surface (Figure 6.5a) is zero.

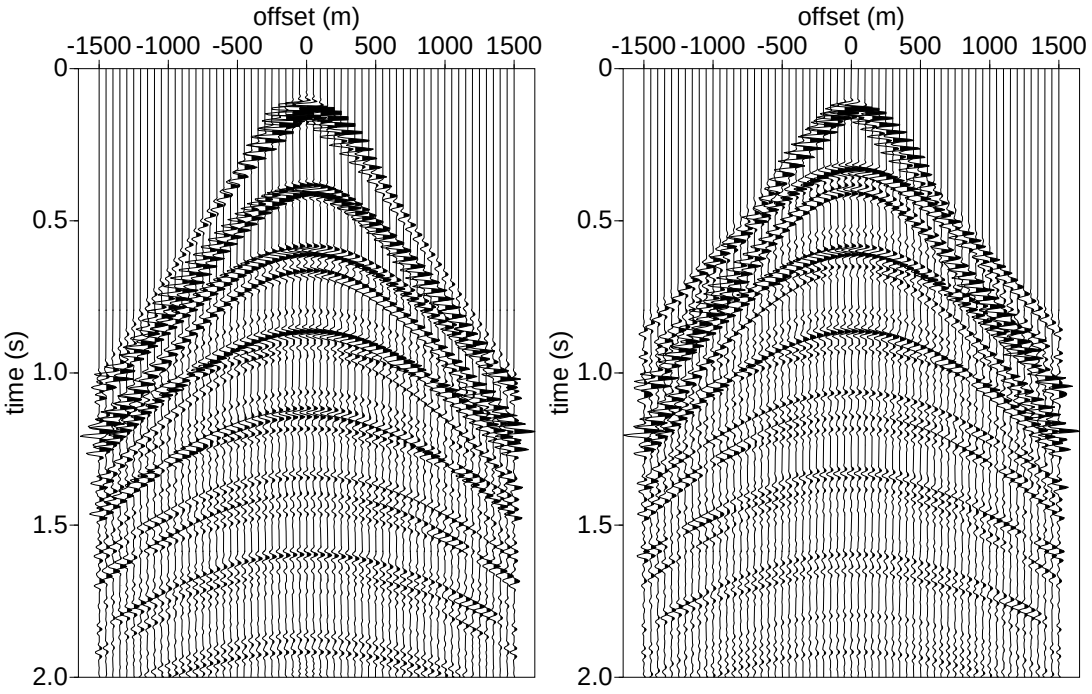
Decomposition

With the aid of equations (6.21) and (6.24) the decomposition into (flux-normalized) up- and downgoing wave fields can be applied for the situation just above or just below the ocean bottom (indicated by z_1^- and z_1^+ respectively). This is shown in Figure 6.6.
The situation we have here is different from normal surface data: at the ocean bottom we have both the *downgoing* as well as the *upgoing* wave field.
Note that the downgoing wave field just above the bottom (Figure 6.6a) contains the direct wave field from the source (first event) and all the downgoing surface-related multiples; no primary reflections are present here. This means that if we isolate the first event from the data we have two important wave fields: the downgoing source wavefield, with all its temporal and spatial characteristics, and all downgoing surface-related multiples! The absence of any primary reflection event in the downgoing wave field above the bottom can be used as criterion for a good decomposition.
Note also that for the upgoing wave field just below the bottom (Figure 6.6d) the primary reflection of the bottom is not present. This might be used as a criterion whether the decomposition was successful (i.e. whether the correct velocity and density of the second layer has been used).

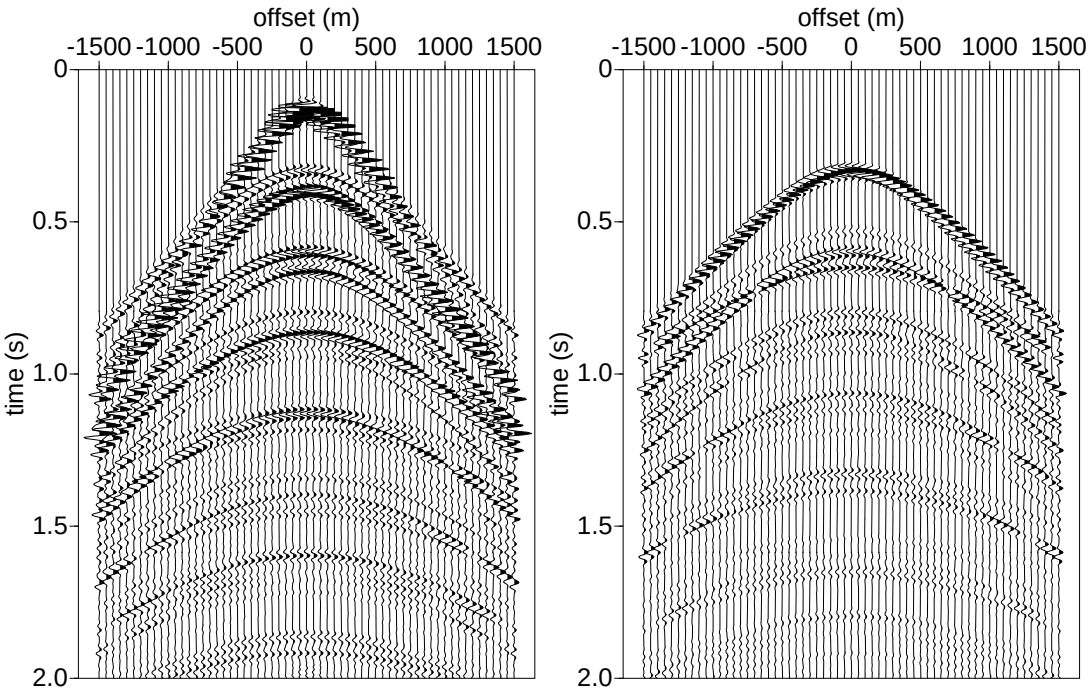
"Poor man's" multiple elimination

As noted the downgoing wave field just above the bottom contains the direct source wave field and the downgoing surface-related multiples, but no primary reflection events (which are all upgoing). Figure 6.8 shows the separation of this wave field into the two components, being the direct source wave field in Figure 6.8a and the multiples in Figure 6.8b. This splitting has been done by a simple "cutting" procedure.

The direct source wave field can be used to deconvolve the data for the temporal and spatial source array, in order to convert the data into point source responses. For this, the propagation through the water layer should be removed from the direct source wave field (which is a simple phase shift operator for a flat water bottom). On the other hand, the propagation through the water layer might be left in: a deconvolution for this downgoing source wave field corresponds to a redatuming at the source side of the source to the bottom level.



a) P^+ *just above bottom* *b)* P^- *just above bottom*



c) P^+ *just below bottom* *d)* P^- *just below bottom*

Fig. 6.6 *Decomposition of the total pressure and vertical velocity component into down- and upgoing waves just above the bottom - a) and b) - and just below the bottom - c) and d).*

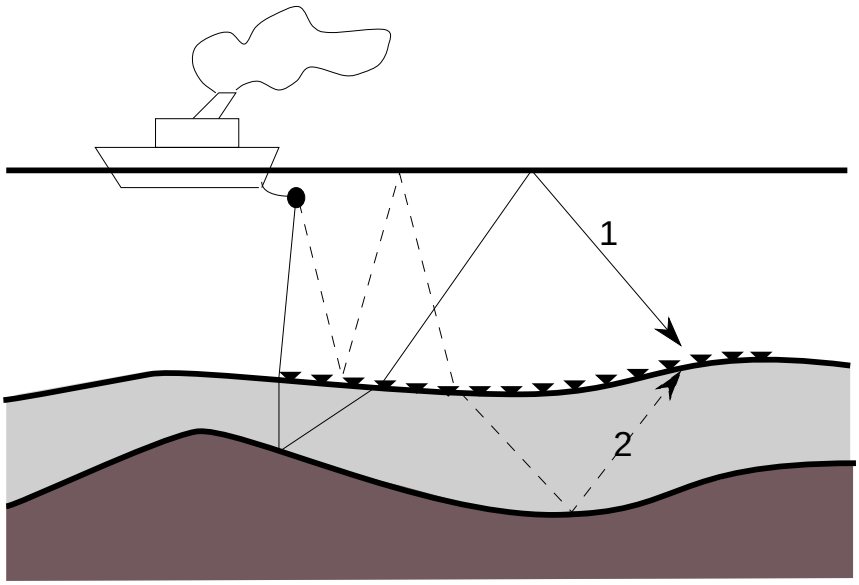


Fig. 6.7 *Two types of water layer reverberations recorded by the ocean bottom geophones and hydrophones.*

The downgoing multiples can be used for a simple multiple subtraction procedure, by multiplying them with the bottom reflectivity operator and subtracting this result from the upgoing wave field just above the bottom. However, doing this will not remove all surface-related multiples, but only those multiples with a reverberation in the water-layer as the last wave path. In Figure 6.7 two multiple paths are indicated; the first one is present in both the downgoing and (after reflection) upgoing wave field, the second is only present (after transmission) in the upgoing wave field and will not be removed by this subtraction procedure.

Although the downgoing multiple field is not complete, it can be used in a subtraction procedure from the upgoing wave field. For this, the downgoing multiples are multiplied with the bottom reflectivity operator (and therefore converted into upgoing multiples) and subtracted from the upgoing wave field ($\mathbf{P}^-(z_1^-)$). The result of this subtraction is displayed in Figure 6.8c. Clearly, several multiples (among which all the multiples that are captured in the water layer) have been removed. Furthermore, some reverberations have been reduced. In practice, the subtraction would be applied adaptively: the predicted multiples are fitted with the true multiples in the upgoing wave field in local space and time windows. Figure 6.8d shows this result for overlapping windows of 400 ms and 40 traces. In the deeper part of the data, more multiples could be adapted and an improved multiple subtraction result is achieved. Note that the primary events are unharmed by the adaptive subtraction method.

In the next section a proper multiple removal procedure is discussed.

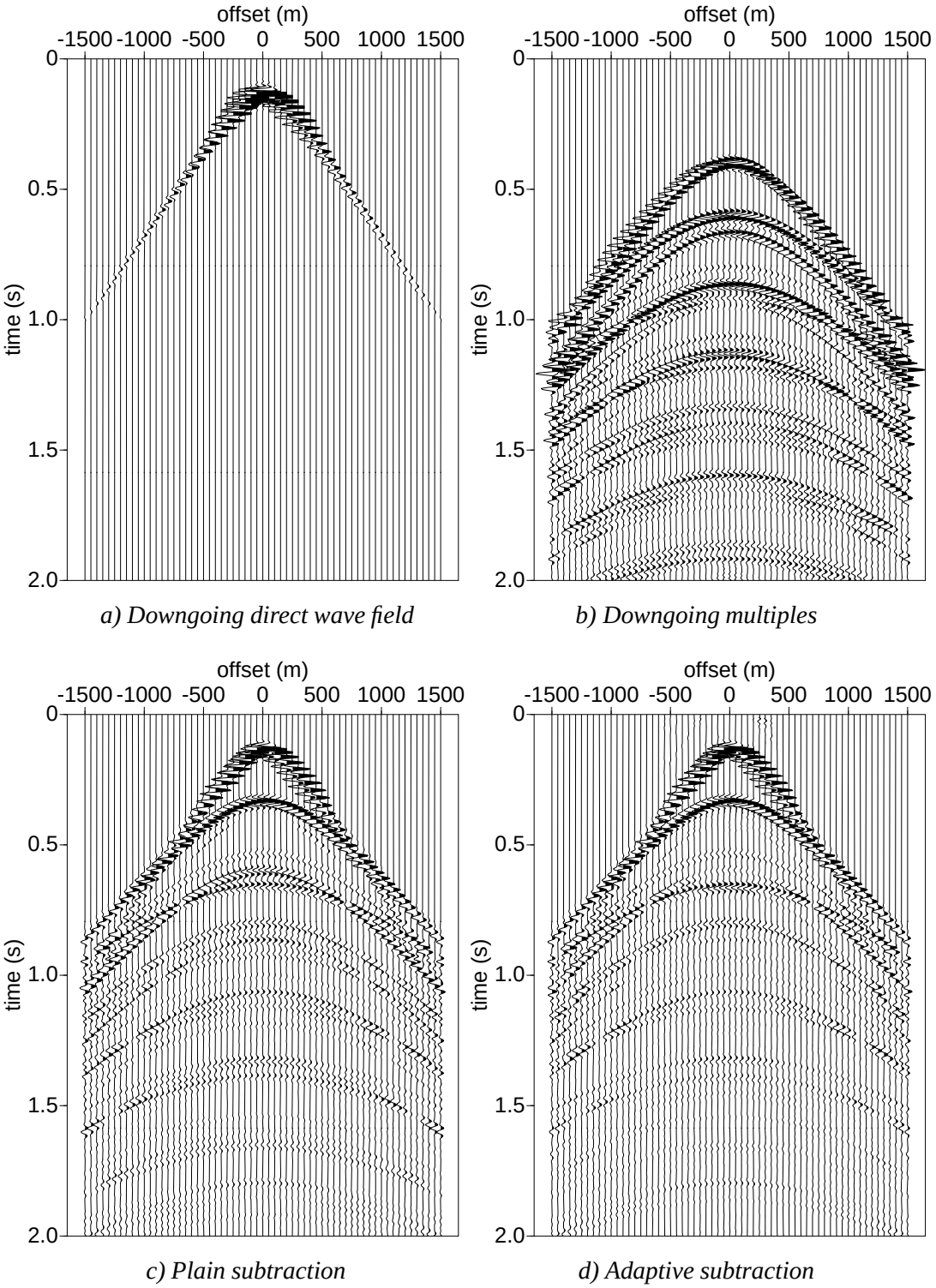


Fig. 6.8 *Downgoing wave field just above the bottom split by muting into a) the source wave field and b)the multiples. c) Plain subtraction of the multiples of b) from the upgoing wave field just above the bottom. d) Adaptive subtraction of the multiples of b).*

6.4 Elimination of surface-related and bottom-related multiples: the Delphi approach

The ocean bottom data can be considered as downward extrapolated surface data, or Common Focus Point gathers (as defined in the last DELPHI volumes) with receivers at the bottom. The surface-related and ocean bottom-related multiple elimination procedures that are normally applied on surface data can be rewritten for ocean bottom data as well. It turns out that to do this for both elimination procedures knowledge of the water layer has to be included. An alternative method that does not strictly require this knowledge, is discussed in section 6.5. In both sections we restrict ourselves to the acoustic approach, leaving the elastic extension for later research.

6.4.1 Theory for surface-related multiple elimination

For surface data, the surface-related multiple elimination scheme can be written as follows (Verschuur et al. 1992):

$$\mathbf{P}_0^-(z_0) = \mathbf{P}^-(z_0) - \mathbf{P}^-(z_0)\mathbf{A}^-(z_0)\mathbf{P}^-(z_0) + \mathbf{P}^-(z_0)[\mathbf{A}^-(z_0)\mathbf{P}^-(z_0)]^2 - ..., \tag{6.25}$$

where the surface operator, in the case of upgoing, flux-normalized wave fields, reads:[5]

$$\mathbf{A}^-(z_0) = \mathbf{L}_1^+(z_0)\mathbf{R}^-(z_0)S^{-1}(\omega). \tag{6.26}$$

In equation (6.25), $\mathbf{P}_0^-(z_0)$ denotes the surface data without surface-related multiples and $\mathbf{P}^-(z_0)$ the data with multiples.

As has been pointed out in previous DELPHI reports, the multiple elimination can be written as an adaptive, iterative procedure:

$$\{\mathbf{P}_0^-(z_0)\}^{(i+1)} = \mathbf{P}^-(z_0) - \mathbf{P}^-(z_0)\mathbf{A}^-(z_0)\{\mathbf{P}_0^-(z_0)\}^{(i)}, \tag{6.27}$$

in which for each iteration the surface operator $\mathbf{A}^-(z_0)$ can be estimated by a linear least-squares procedure. In practice, assuming that the free surface characteristics are known, only the inverse source signature $S^{-1}(\omega)$ need be estimated, which simplifies the adaptive procedure considerable.

In the situation of ocean bottom data, the above equations have to be modified using the following relation between the upgoing wave field at the surface and the wave field just above the ocean bottom:

$$\mathbf{P}^-(z_0) = \mathbf{W}^-(z_0, z_1)\mathbf{P}^-(z_1^-, z_0). \tag{6.28}$$

[5] It is assumed that $\mathbf{P}^-(z_0)$ represents the data after decomposition at the receiver side. Hence, it is the flux-normalized upgoing response of the physical sources at the surface z_0. The matrix $\mathbf{L}_1^+(z_0)$ in equation (6.26) accounts for the fact that no decomposition has taken place at the source side. When *no* flux normalization would be considered, this matrix would be the identity matrix and therefore it is generally absent in our multiple elimination schemes. We have included it here, because in this chapter we consider flux normalized waves.

Here $\mathbf{P}^-(z_1^-, z_0)$ denotes the upgoing wave fields at z_1^- just above the ocean bottom, due to sources at the surface z_0. Using this, equation (6.27) can be rewritten into:

$$\{\mathbf{P}_0^-(z_1^-, z_0)\}^{(i+1)} = \mathbf{P}^-(z_1^-, z_0) - \mathbf{P}^-(z_1^-, z_0)\mathbf{A}^-(z_0, z_1)\{\mathbf{P}_0^-(z_1^-, z_0)\}^{(i)}, \tag{6.29}$$

in which the "surface" operator is now defined as:

$$\mathbf{A}^-(z_0, z_1) = \mathbf{L}_1^+(z_0)\mathbf{R}^-(z_0)\mathbf{W}^-(z_0, z_1)S^{-1}(\omega). \tag{6.30}$$

For ocean bottom data to be used as surface multiple prediction operator, we need to add the propagation from the ocean bottom up to the surface level, as indicated in equations (6.29) and (6.30). Therefore, knowledge on water depth and velocity, in order to construct the propagation operator $\mathbf{W}^-(z_0, z_1)$, is required.

After enough iterations, equation (6.29) will converge to the following implicit relation:

$$\mathbf{P}_0^-(z_1^-, z_0) = \mathbf{P}^-(z_1^-, z_0) - \mathbf{P}^-(z_1^-, z_0)\mathbf{A}^-(z_0, z_1)\mathbf{P}_0^-(z_1^-, z_0), \tag{6.31}$$

or

$$\mathbf{P}_0^-(z_1^-, z_0) = \mathbf{P}^-(z_1^-, z_0)\mathbf{Q}(z_0), \tag{6.32}$$

with the "multiple prediction operator" $\mathbf{Q}(z_0)$ defined in terms of the data without multiples $\mathbf{P}_0^-(z_1^-, z_0)$ as:

$$\mathbf{Q}(z_0) = \mathbf{I} - \mathbf{A}^-(z_0, z_1)\mathbf{P}_0^-(z_1^-, z_0), \tag{6.33}$$

or in terms of the data with multiples $\mathbf{P}^-(z_1^-, z_0)$ (which brings us back to a similar relation as equation (6.25)):

$$\mathbf{Q}(z_0) = \mathbf{I} + \sum_{i=1}^{\infty}[-\mathbf{A}^-(z_0, z_1)\mathbf{P}^-(z_1^-, z_0)]^i. \tag{6.34}$$

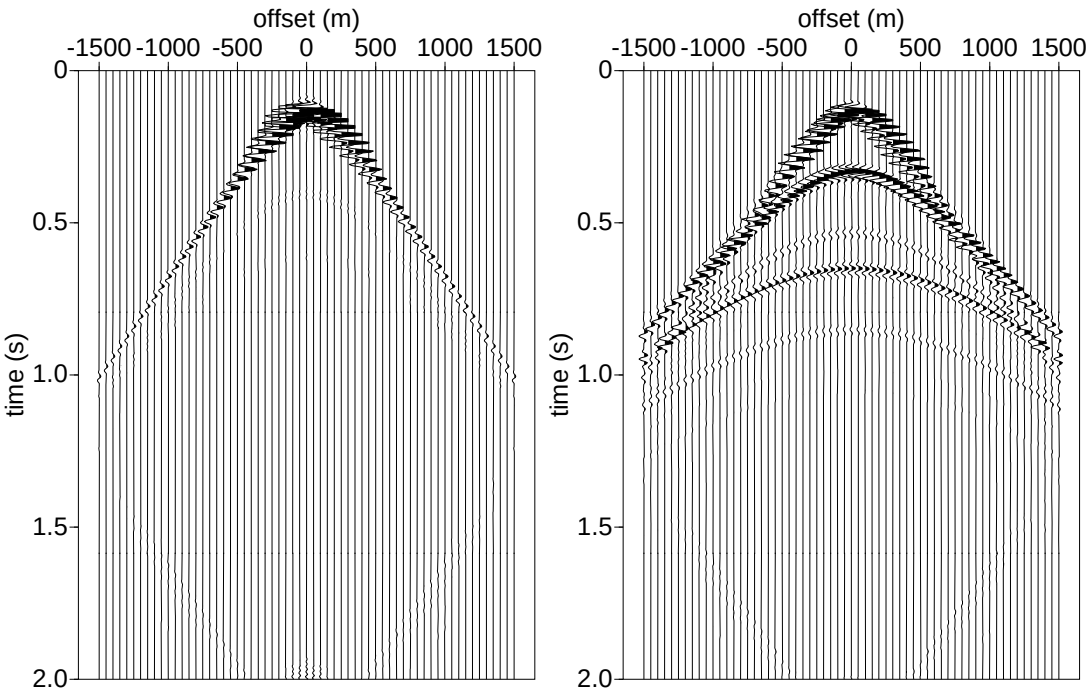
Note that as the multiple prediction operator $\mathbf{Q}(z_0)$ acts as a source operator at the surface, we can also apply it to other wave fields:

$$\mathbf{P}_0^{\pm}(z_1^-, z_0) = \mathbf{P}^{\pm}(z_1^-, z_0)\mathbf{Q}(z_0) \tag{6.35}$$

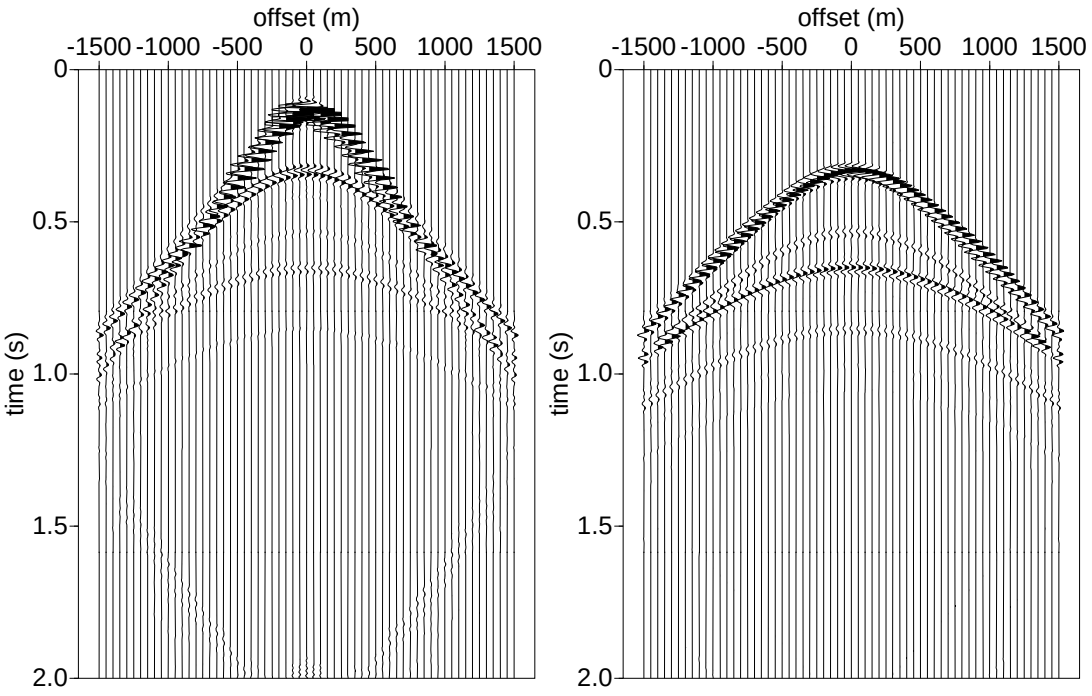
and

$$\mathbf{P}_0^{\pm}(z_1^+, z_0) = \mathbf{P}^{\pm}(z_1^+, z_0)\mathbf{Q}(z_0). \tag{6.36}$$

Each of these relations removes the effect of surface-related multiples to that specific wave field.



a) P^+ just above bottom *b) P^- just above bottom*



c) P^+ just below bottom *d) P^- just below bottom*

Fig. 6.9 *Surface-related multiple elimination result of the down- and upgoing waves just above the bottom - a) and b) - and just below the bottom - c) and d).*

6.4.2 Numerical example

The surface-related multiple elimination method, described by equations (6.29), (6.35) and (6.36), has been applied to the example data under consideration. The results for the wave fields just above and below the ocean bottom have been displayed in Figure 6.9. Clearly, the effect of multiple removal can be observed, yielding the downgoing source wave field in Figure 6.9a (similar to Figure 6.8a, but without using a muting process) and the primary reflectons and internal multiples in Figure 6.9b,c and d. Again the enormous effect of surface-related multiples on seismic data has been demonstrated: the major part of the energy has been removed from the data!
Note the difference between the "poor man's" result (Figure 6.8d) and the exact multiple elimination result (Figure 6.9b) for the upgoing wave field just above the ocean bottom.

6.4.3 Theory for ocean bottom-related multiple elimination

The surface-related multiple elimination, as described above, can be seen as the first multiple elimination step. Next, we want to remove the internal multiples, starting with the multiples related to the ocean bottom. To do this, we consider the upgoing wave field just below the ocean bottom after surface-related multiple elimination, $\mathbf{P}_0^-(z_1^+, z_0)$, being the result of applying equation (6.36).

The implicit relation for multiple elimination reads in theory (see Verschuur et al, 1992):

$$\mathbf{X}_0(z_1, z_1) = \mathbf{X}(z_1, z_1) - \mathbf{X}(z_1, z_1)\mathbf{R}^-(z_1)\mathbf{X}_0(z_1, z_1), \tag{6.37}$$

where $\mathbf{X}(z_1, z_1)$ describes the impulse response of the subsurface below the ocean bottom (i.e. assuming a homogeneous half-space for $z < z_1$, with the velocity of water) with all ocean bottom related multiples included, and $\mathbf{X}_0(z_1, z_1)$ the impulse response without ocean bottom related multiples. Of course, we want to replace $\mathbf{X}(z_1, z_1)$ and $\mathbf{X}_0(z_1, z_1)$ in this equation by one of the known or desired wave fields. This can be done by applying an inverse extrapolation to the seismic data at the source side from the surface to the ocean bottom level followed by a source wave field deconvolution:

$$\mathbf{X}(z_1, z_1) = \mathbf{P}^-(z_1^+, z_0)\mathbf{L}_1^+(z_0)\{\mathbf{T}^+(z_1)\mathbf{W}^+(z_1, z_0)\}^{-1}S^{-1}(\omega) \tag{6.38}$$

and

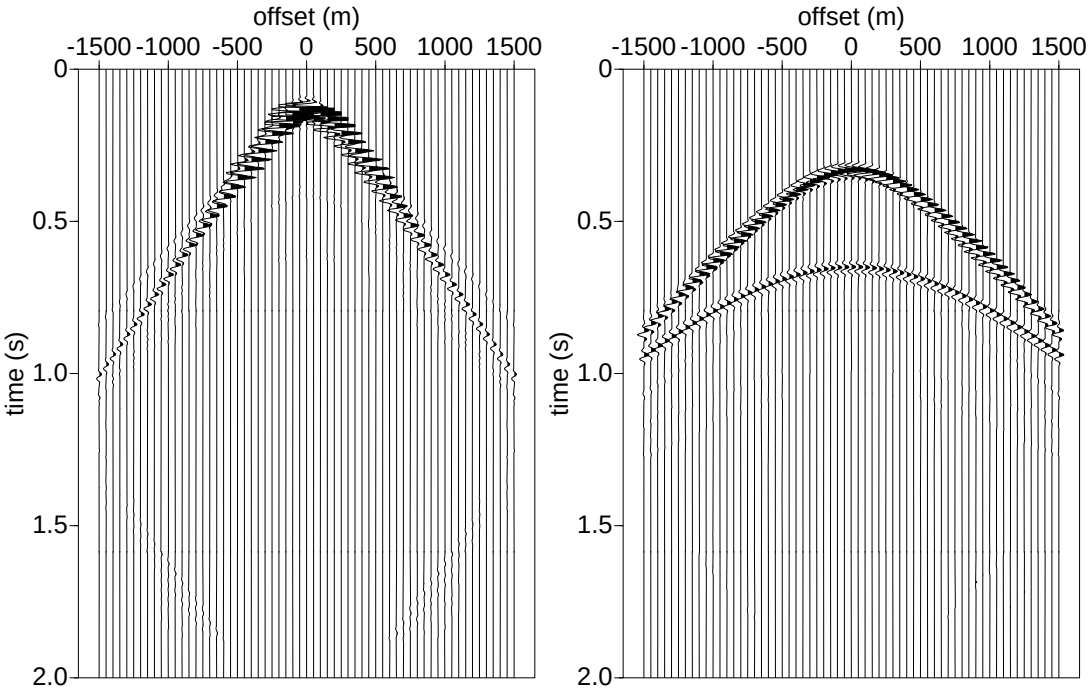
$$\mathbf{X}_0(z_1, z_1) = \mathbf{P}_0^-(z_1^+, z_0)\mathbf{L}_1^+(z_0)\{\mathbf{T}^+(z_1)\mathbf{W}^+(z_1, z_0)\}^{-1}S^{-1}(\omega), \tag{6.39}$$

with $\mathbf{T}^+(z_1)$ accounting for the transmission through the ocean bottom and $\mathbf{W}^+(z_1, z_0)$ describing propagation from the surface to the ocean bottom. With this, equation (6.37) can be rewritten as:

$$\mathbf{P}_0^-(z_1^+, z_0) = \mathbf{P}^-(z_1^+, z_0) - \mathbf{P}^-(z_1^+, z_0)\mathbf{A}^-(z_0, z_1)\mathbf{P}_0^-(z_1^+, z_0), \tag{6.40}$$

or as an iterative scheme:

$$\{\mathbf{P}_0^-(z_1^+, z_0)\}^{(i+1)} = \mathbf{P}^-(z_1^+, z_0) - \mathbf{P}^-(z_1^+, z_0)\mathbf{A}^-(z_0, z_1)\{\mathbf{P}_0^-(z_1^+, z_0)\}^{(i)}, \tag{6.41}$$



a) P^+ just below bottom *b) P^- just below bottom*

Fig. 6.10 *a) Downgoing wave field just below the bottom, after removing the surface- and ocean bottom-related multiples. b) Upgoing wave field just below the bottom, after removing the surface- and ocean bottom-related multiples.*

with the bottom operator $\mathbf{A}^-(z_0, z_1)$ now defined as:

$$\mathbf{A}^-(z_0, z_1) = \mathbf{L}_1^+(z_0)\{\mathbf{T}^+(z_1)\mathbf{W}^+(z_1, z_0)\}^{-1}\mathbf{R}^-(z_1)S^{-1}(\omega). \tag{6.42}$$

Similar to the situation with surface-related multiples, we can define a multiple prediction operator $\mathbf{Q}'(z_0)$ such that:

$$\mathbf{P}_0^-(z_1^+, z_0) = \mathbf{P}^-(z_1^+, z_0)\mathbf{Q}'(z_0) \tag{6.43}$$

with the multiple prediction operator $\mathbf{Q}'(z_0)$ now defined as:

$$\mathbf{Q}'(z_0) = \mathbf{I} - \mathbf{L}_1^+(z_0)\{\mathbf{T}^+(z_1)\mathbf{W}^+(z_1, z_0)\}^{-1}\mathbf{R}^-(z_1)S^{-1}(\omega)\mathbf{P}_0^-(z_1^+, z_0). \tag{6.44}$$

6.4.4 Numerical example

The method as described above is applied to the data after surface-related multiple elimination of Figure 6.9c and d, i.e. the downgoing and upgoing wave field just below the ocean bottom. In Figure 6.10 the result for the internal multiple elimination has been displayed. Figure

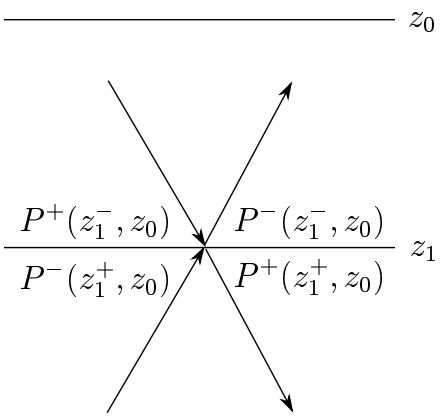


Fig. 6.11 *Wave fields after decomposition at the ocean bottom.*

6.10a shows that, similar to the surface situation, the downgoing wave field, after ocean bottom multiple elimination, only consist of the direct source wave field after transmission through the ocean bottom. The upgoing wave field, as displayed in Figure 6.10b, consists of the primaries of the second and third reflector and (although hardly visible) their internal multiples in the third layer.
To apply this method, we need knowledge of the water layer depth and velocity and also the reflectivity of the water bottom. The inverse source signature has been put in as a known quantity in all these examples; in practice this is not known and the multiple elimination scheme is then applied adaptively, estimating the inverse source signature by minimizing the energy in the multiple elimination result (see Verschuur et al, 1992).

6.5 Elimination of surface-related and bottom related multiples: alternative approach

In this section we present an alternative approach to the elimination of surface-related and bottom related multiples. We make use of the fact that after decomposition at the ocean bottom, the *total*[6] $\vec{P}^+$ and $\vec{P}^-$ wave fields just below the ocean bottom are available.

6.5.1 Basic principle of ocean bottom multiple elimination

In this subsection we discuss multiple elimination at the ocean bottom. Again we only consider the acoustic approach. Figure 6.11 gives an overview of the wave fields that are available after acoustic decomposition of a single shot record at the ocean bottom.

In accordance with the usual DELPHI approach we store the results of the decomposition of all shot records in matrices $\mathbf{P}^{\pm}(z_1^-, z_0)$ and $\mathbf{P}^{\pm}(z_1^+, z_0)$. The wave fields just below the ocean

[6] This means that $\vec{P}^+$ contains also the (transmitted) downgoing source wave field.

bottom are related to each other, according to

$$\mathbf{P}^-(z_1^+, z_0) = \mathbf{X}_0(z_1, z_1)\mathbf{P}^+(z_1^+, z_0), \tag{6.45}$$

where $\mathbf{X}_0(z_1, z_1)$ is the multiple-free impulse response matrix of the inhomogeneous half-space below $z = z_1$. Hence, in principle multiple elimination involves nothing more than inverting equation (6.45), according to

$$\mathbf{X}_0(z_1, z_1) = \mathbf{P}^-(z_1^+, z_0)\{\mathbf{P}^+(z_1^+, z_0)\}^{-1}. \tag{6.46}$$

Note that with this simple algorithm the multiples related to the ocean bottom as well as those related to the ocean-surface are eliminated. In other words, the water layer is replaced by a homogeneous upper half-space with the water velocity. Also note that no knowledge of the thickness of the water layer is required. Of course in practice multiple elimination is not as simple as suggested by equation (6.46). In the following we show step by step how to arrive at a more practical scheme that avoids matrix inversion and how knowledge of the thickness of the water layer can be used to stabilize the process.

6.5.2 Ocean bottom multiple elimination in terms of a series expansion

In order to rewrite equation (6.46) as a series expansion, we introduce the following additional expression for $\mathbf{P}^+(z_1^+, z_0)$

$$\mathbf{P}^+(z_1^+, z_0) = \mathbf{T}^+(z_1)\mathbf{P}^+(z_1^-, z_0) + \mathbf{R}^-(z_1)\mathbf{P}^-(z_1^+, z_0), \tag{6.47}$$

where $\mathbf{T}^+(z_1)$ is the transmission matrix of the ocean bottom for downgoing waves and $\mathbf{R}^-(z_1)$ the reflection matrix for upgoing waves. Now equation (6.46) can be rewritten as

$$\mathbf{X}_0(z_1, z_1) = \mathbf{P}^-(z_1^+, z_0)\{\mathbf{T}^+(z_1)\mathbf{P}^+(z_1^-, z_0) + \mathbf{R}^-(z_1)\mathbf{P}^-(z_1^+, z_0)\}^{-1}, \tag{6.48}$$

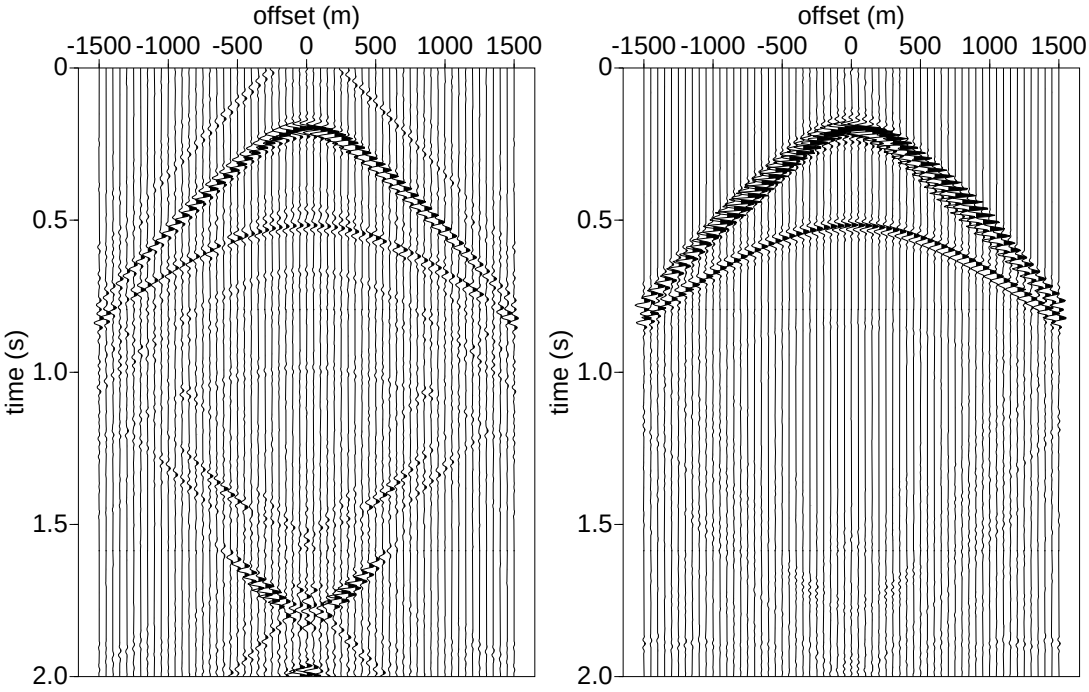
or

$$\begin{aligned}\mathbf{X}_0(z_1, z_1) \;=\; & \mathbf{P}^-(z_1^+, z_0)\{\mathbf{T}^+(z_1)\mathbf{P}^+(z_1^-, z_0)\}^{-1} \;\times \\ & [\mathbf{I} + \mathbf{R}^-(z_1)\mathbf{P}^-(z_1^+, z_0)\{\mathbf{T}^+(z_1)\mathbf{P}^+(z_1^-, z_0)\}^{-1}]^{-1},\end{aligned} \tag{6.49}$$

or

$$\begin{aligned}\mathbf{X}_0(z_1, z_1) \;=\; & \mathbf{P}^-(z_1^+, z_0)\{\mathbf{T}^+(z_1)\mathbf{P}^+(z_1^-, z_0)\}^{-1} \;\times \\ & \Big(\mathbf{I} + \sum_{i=1}^{\infty}[-\mathbf{R}^-(z_1)\mathbf{P}^-(z_1^+, z_0)\{\mathbf{T}^+(z_1)\mathbf{P}^+(z_1^-, z_0)\}^{-1}]^i\Big).\end{aligned} \tag{6.50}$$

The latter equation has the usual form of the DELPHI multiple elimination algorithm (Verschuur et al. 1992). The main complication here is the inverse matrix $(\mathbf{T}^+(z_1)\mathbf{P}^+(z_1^-, z_0))^{-1}$, which replaces the inverse source signature $S^{-1}(\omega)$ in the usual scheme. Whereas the inversion of $S(\omega)$ can be carried out in a stable sense between the minimum and maximum frequencies ω_{min} and ω_{max}, the inversion of $\mathbf{T}^+(z_1)\mathbf{P}^+(z_1^-, z_0)$ is unstable due to notches in the spectrum of the "illuminating wave field" $\mathbf{P}^+(z_1^-, z_0)$, which is contaminated by the water-layer reverberations. This is illustrated in Figures 6.12a and 6.13b.



a) X_0 from data with surface multiples *b) X_0 from data without surface multiples*

Fig. 6.12 *a) Estimated bandlimited subsurface impulse response at the ocean bottom, using the data with all multiples, according to equation (6.50). b) Estimated subsurface impulse response at the ocean bottom, using the data without surface-related multiples, according to equation (6.56).*

6.5.3 Stabilization by incorporating knowledge of the water layer

For the stabilization of the multiple elimination process, described by equation (6.50), we first eliminate the ocean-surface related multiples from the data matrix $\mathbf{P}^-(z_1^-, z_0)$, according to

$$\mathbf{P}_0^-(z_1^-, z_0) = \mathbf{P}^-(z_1^-, z_0)\{\mathbf{I} + \mathbf{L}_1^+(z_0)\mathbf{R}^-(z_0)\mathbf{W}^-(z_0, z_1)\mathbf{P}^-(z_1^-, z_0)S^{-1}(\omega)\}^{-1}, \tag{6.51}$$

or

$$\mathbf{P}_0^-(z_1^-, z_0) = \mathbf{P}^-(z_1^-, z_0) \overbrace{\Big(\mathbf{I} + \sum_{i=1}^{\infty}[-\mathbf{L}_1^+(z_0)\mathbf{R}^-(z_0)\mathbf{W}^-(z_0, z_1)\mathbf{P}^-(z_1^-, z_0)S^{-1}(\omega)]^i\Big)}^{\mathbf{Q}(z_0)}, \tag{6.52}$$

see equations (6.32) and (6.34).

When we apply the "multiple prediction operator" $\mathbf{Q}(z_0)$ to all one-way wave field matrices

at the ocean bottom, according to

$$\mathbf{P}_0^{\pm}(z_1^-, z_0) = \mathbf{P}^{\pm}(z_1^-, z_0)\mathbf{Q}(z_0) \quad \text{and} \quad \mathbf{P}_0^{\pm}(z_1^+, z_0) = \mathbf{P}^{\pm}(z_1^+, z_0)\mathbf{Q}(z_0), \tag{6.53}$$

then, in analogy with equations (6.45) and (6.47), the following relations hold at the ocean bottom

$$\mathbf{P}_0^-(z_1^+, z_0) = \mathbf{X}_0(z_1, z_1)\mathbf{P}_0^+(z_1^+, z_0), \tag{6.54}$$

$$\mathbf{P}_0^+(z_1^+, z_0) = \mathbf{T}^+(z_1)\mathbf{P}_0^+(z_1^-, z_0) + \mathbf{R}^-(z_1)\mathbf{P}_0^-(z_1^+, z_0). \tag{6.55}$$

Hence, in analogy with equation (6.50), multiple elimination at the ocean bottom is now accomplished by

$$\begin{aligned} \mathbf{X}_0(z_1, z_1) \;=\; & \mathbf{P}_0^-(z_1^+, z_0)(\mathbf{T}^+(z_1)\mathbf{P}_0^+(z_1^-, z_0))^{-1} \;\times \\ & \Big(\mathbf{I} + \sum_{i=1}^{\infty}[-\mathbf{R}^-(z_1)\mathbf{P}_0^-(z_1^+, z_0)(\mathbf{T}^+(z_1)\mathbf{P}_0^+(z_1^-, z_0))^{-1}]^i\Big). \end{aligned} \tag{6.56}$$

Note that the inversion of the matrix $\mathbf{T}^+(z_1)\mathbf{P}_0^+(z_1^-, z_0)$ is a much more stable process than the inversion of the matrix $\mathbf{T}^+(z_1)\mathbf{P}^+(z_1^-, z_0)$ in equation (6.50), because the spectrum of the illuminating wave field $\mathbf{P}_0^+(z_1^-, z_0)$ does not contain notches due to water-layer reverberations. This is illustrated in Figures 6.12b and 6.34c. The inversion of $\mathbf{T}^+(z_1)\mathbf{P}_0^+(z_1^-, z_0)$ can be further simplified by rewriting it as

$$\{\mathbf{T}^+(z_1)\mathbf{P}_0^+(z_1^-, z_0)\}^{-1} = \mathbf{F}^+(z_0, z_1)\{\mathbf{T}^+(z_1)\mathbf{P}_0^+(z_1^-, z_0)\mathbf{F}^+(z_0, z_1)\}^{-1}, \tag{6.57}$$

with

$$\mathbf{F}^+(z_0, z_1) = \{\mathbf{W}^+(z_1, z_0)\}^{-1} \approx \{\mathbf{W}^+(z_1, z_0)\}^H, \tag{6.58}$$

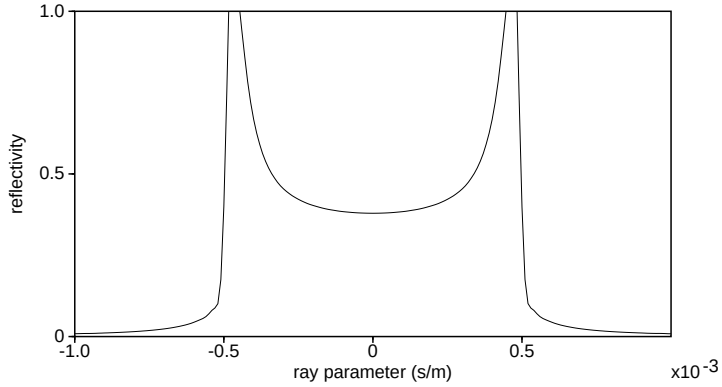
where ^{H} denotes transposition and complex conjugation. Since $\mathbf{F}^+(z_0, z_1)$ removes the downward propagation effects in the water layer, the matrix to be inverted (i.e., $\mathbf{T}^+(z_1)\mathbf{P}_0^+(z_1^-, z_0)$ $\mathbf{F}^+(z_0, z_1)$), is now nearly a diagonal matrix.

Summarizing, in this section we proposed a two-step procedure in which the multiples related to the ocean-surface are removed prior to the multiples related to the ocean bottom. Since the first step is not strictly necessary (see the previous sub-section) it can be seen as a "data-preconditioning" that stabilizes the second step.

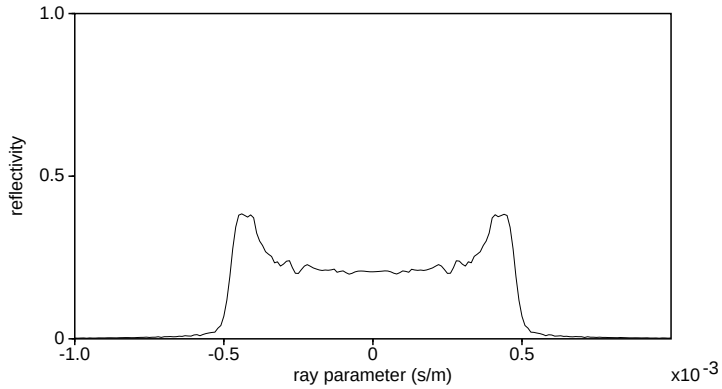
It can be shown that $\mathbf{X}_0(z_1, z_1)$, obtained with equation (6.56), is related to the multiple free data $\mathbf{P}_0^-(z_1^+, z_0)$, obtained with the method in section 6.4.3, according to

$$\mathbf{P}_0^-(z_1^+, z_0) = \mathbf{X}_0(z_1, z_1)\mathbf{T}^+(z_1)\mathbf{W}^+(z_1, z_0)S(\omega), \tag{6.59}$$

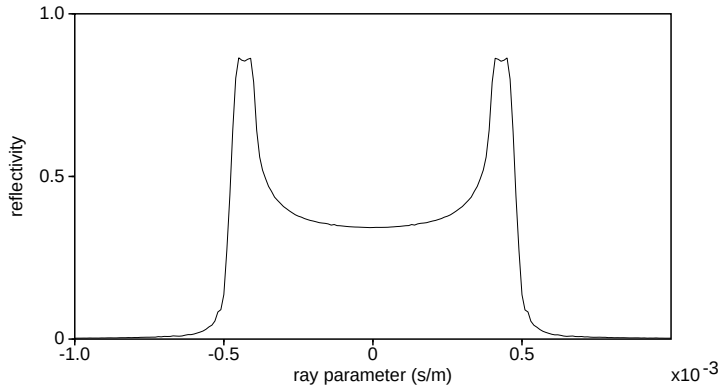
under the condition that the exact operator $\mathbf{W}^+(z_1, z_0)$ is used in section 6.4.3.



a) Theoretical reflectivity for second reflector



b) Estimated reflectivity from Figure 6.12a.



c) Estimated reflectivity from Figure 6.12b.

Fig. 6.13 *Reflectivity of the second interface as a function of ray-parameter: a) theoretical values; b) result extracted from Figure 6.12a and c) from Figure 6.12b.*

6.6 Conclusions

- **Decomposition**
  It has been shown that the data measured by hydrophones and geophones at the ocean bottom provides sufficient information for a full decomposition into downgoing and upgoing waves above as well as below the ocean bottom, assuming the medium parameters just above and below the bottom are known. Acoustic decomposition requires P- and V_z-measurements and yields $P^{\pm}$ above and below the bottom; elastic decomposition requires in addition measurements of V_x and V_y and yields $P^{\pm}$ above the bottom and the P- and S-wave potentials $\Phi^{\pm}$, $\Psi^{\pm}$ and $\Upsilon^{\pm}$ below the bottom.

- **Multiple elimination**
  We have restricted ourselves to acoustic multiple elimination schemes. First we have shown that the usual DELPHI approach to multiple elimination can be extended in a straightforward way to the situation for ocean bottom data, by including knowledge of the extrapolation operator $\mathbf{W}^{+}(z_1, z_0)$ for the waterlayer. This applies to surface- as well as bottom-related multiple elimination. Next we have proposed an alternative scheme that makes use of the information about the downgoing source wave field in the decomposed wave fields at the ocean bottom and thus avoids that $\mathbf{W}^{+}(z_1, z_0)$ should be known. We have shown that approximate knowledge of $\mathbf{W}^{+}(z_1, z_0)$ can be advantageous to stabilize the process. The pros and cons of both methods will be further investigated. Also the extension to elastic multiple elimination will get ample attention.

6.7 References

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